A Statistical Process Monitoring Control for an Integrated Production, Maintenance, and Quality Policy of a Forecasting Production Problem

Aminu Sahabi Abubakar, Hajej Zied, Aime C Nyoungue and Ayoub Tighazoui

May 10, 2023

CIGI QUALITA MOSIM 2023
A Statistical Process Monitoring Control for an Integrated Production, Maintenance, and Quality Policy of a Forecasting Production Problem

AMINU SAHABI ABUBAKAR [1], ZIED HAJEJ [2], AIME C. NYOUNGUE[3], AYOUB TIGHAZOUI[4]

Université de Lorraine
LGIPM, F-57000 - Metz, France
[1]abubakar1@univ-lorraine.fr
[2]zied.hajej@univ-lorraine.fr
[3]aime.nyoungue@univ-lorraine.fr
[4]ayoub.tighazoui@unistra.fr

***Résumé* – Cet article propose une nouvelle stratégie consistant en une maintenance préventive corrective, imparfaite et parfaite pour une prévision des problèmes de production et de maintenance sous contraintes de qualité. La relation entre la demande, la production et la maintenance varie d'une période à l'autre. Pour résoudre ce problème, nous avons développé une maintenance intégrée basée sur des taux de production optimaux, des niveaux de stocks et une politique de contrôle qualité pour un système de production devant satisfaire une demande prévisionnelle sous un niveau de service donné et sur un horizon fini. Le modèle intégré implique un nouveau système de commutation entre une planification de maintenance parfaite et imparfaite basée sur les techniques de qualité développées. Diagramme de processus statistique avec un signal d'alerte (surveillance) indiquant des causes de variation spéciales attribuables. Nous utilisons les intervalles d'échantillonnage (h), la taille des échantillons (n) et la limite de la carte de contrôle (kp) comme niveaux de décision. Ensuite la probabilité d'être sous ou hors contrôle, la corrélation entre la dégradation et la variabilité de production de la machine. Les prévisions dans les décisions de production visaient à améliorer la fiabilité et à réduire les éléments non conformes, minimisant ainsi les coûts totaux attendus. Une approche d'optimisation séquentielle basée sur la simulation est utilisée pour optimiser les paramètres de décision de la politique de contrôle.**

***Abstract* – This paper proposed a new strategy consisting of corrective, imperfect, and perfect preventive maintenance for a forecasting production and maintenance problems under quality constraints. The relationship between demand, production, and maintenance varies from one period to another. To address this problem, we developed integrated maintenance based on optimal production rates, inventory levels, and a quality inspection policy for a production system that must satisfy a forecasting demand under a given service level and during a finite horizon. The integrated model involves a new switching system between perfect and imperfect maintenance planning based on the developed quality techniques. A statistical process chart with an alert (surveillance) signal indicating special assignable causes of variation. We use the sampling intervals (h), samples sizes (n), and the control chart limit (kp), as decision levels. Then the probability of being in or out of control, the correlation between the degradation and Production variability of the machine. The forecasts in production decisions aimed to improve reliability and reduce the non-conformal items, thereby, minimizing the expected total costs. A simulation-based sequential optimization approach is used to optimize the decision parameters of the control policy.**

***Mots clés* - Production, Qualité, Maintenance, Suivi statistique des processus, Causes attribuables.**
***Keywords* – Production, Quality, Maintenance, Statistical Process monitoring, Assignable causes.**

---

1 INTRODUCTION

The world is changing rapidly, especially in the recent decade faced with a series of challenges affecting the demand, production, supply, and cost of products. The recent decade has experienced and is presently experiencing tremendous changes by constantly advancing new technologies within companies and is currently shaping the customer's demand behavior. Also, the need for higher quality products at competitive prices. Indeed, these must-have consequences on the production process and product quality. Therefore, to remain competitive, companies must cope with these multiple and random challenges. This has resulted in companies seeking optimal solutions that could optimally contribute to the overall performance of their production systems. It is more challenging when the Customers are becoming more exigent, while production systems are unreliable (subject to random breakdowns). Hence the production system becomes more complex and subject to multiple uncertainties, (Abubakar et al., 2022). Determining the best production planning and maintenance strategy for industrial organizations has always been difficult. There is a need to minimize the costs and satisfy customers' multiple requirement (Hajej et al., 2018). The strategic maintenance planning and optimization ensure production system reliability and availability by stabilizing the process also reducing costs associated with non-conformal items (Bahria et al., 2021). The maintenance and quality control policy for unreliable production system under dynamic inspection to ensure an optimal solution to the growing production problem (Ait-El-Cadi et al., 2021). The (Bouslah et al., 2016), investigated production, preventive maintenance, and quality control using a sampling method. The production lot size, the sampling plan, the safety stock, and overhaul planning are under quality constraints. (Sett et al., 2017), considered production with stock planning, they were able to advance past works and integrated controls related to the least essential accessibility of the hardware; they got the ideal esteem of stock level time at which buffer stock builds up the age of preventive support activity (security stock). (Si et al., 2018), suggested a reliability and maintenance structure for a two-state process optimizing decision variables in industrial systems to identify discrete time-frames for Preventive Maintenance tasks. Nevertheless, they also neglected inventory scarcity. Production and quality issues were attended to by (Abubakar et al., 2020) in their work which Integrated the model of production quality. The work studied a randomly failing manufacturing system that has to satisfy customer random demand at given service and quality requirements. Quality issues are treated by (Pakurár et al., 2019). They worked on integrated optimization of Production planning, maintenance, and quality control policy without considering the effects of inventory control shortages. (Guo et al., 2022) investigated the concepts of production integrated to capacitated lot sizing, and maintenance. Different production and maintenance problems were studied by other authors considering different types of constraints, (Liu et al., 2020). (Addeh et al., 2018), studied the improved design of control charts, then control chart plus periodic preventive maintenance. The integration of control chart with periodic preventive maintenance gets the attention of (Ben-Daya & Rahim, 2000; Salmasnia et al., 2017), who researched the integration of the X-bar control chart and periodic preventive maintenance strategy. (Hajej et al., 2021) developed a joint control of production, maintenance, and quality for a multi-warehouse supply chain management system, with the aid of a statistical process monitoring tool. (Fakher et al., 2018) studied a multi-period multi-product incapacitated lot-sizing context that integrates production, maintenance, and quality for an imperfect process. (Rivera-Gómez et al., 2021) proposed an integrated production maintenance and quality control policy for an unreliable single product manufacturing system subject to degradation. Depending on the defectives proportion determined during the inspection, and the application of preventive maintenance. A minimal repair is carried out at failure to restore the production system to its previous status (ABAO). It has been established that the integration of these fundamental production functions will result in significant system optimization to increase reliability and minimize cost. However, planning and controlling these functions jointly and optimally represents a challenge for industrial companies and are often treated separately in literature or with single assignable causes in the production process.

2 PRODUCTION AND MAINTENANCE

2.1 Notations

The following notations and decision variables were used in this paper;

U_{max}: maximal production rate of machine *M*
U_{min}: minimal production rate of machine *M*
$u(k)$: production rate of machine during period *k* (k=0, 1, …,H)
$U= \{u(0), u(1), ..., u(H-1)\}$
τ_i: delivery time for warehouse w_i
L: Number of warehouses
Δt: length of a production period
H: number of production periods in the planning horizon
$H.\Delta t$: length of the finite planning horizon
$Q_i(k)$: delivery rate during period *k* (k=0, 1, …, H-1) for each warehouse
$Q_i=\{Q_i(0), Q_i(1), ..., Q_i(H-1)\}$
$\hat{d}_i(k)$: average demand at *k* (k=0, 1, …, H) for each customer
$V_{di(k)}$: variance of demand at *k* (k=0, 1,, H) for each customer
$S(k)$: inventory level of *S* at the end of period *k* (k=0, 1,…, H)
$w_i(k)$: inventory level of *Si* (i:0…L) at the end of period *k* (k=0, 1, …, H) for each warehouse

θ_i: probability related to each customer (*i)* satisfaction level.
C_p: unit production cost of machine *M*
C_h: inventory holding cost of one product unit during one period at the first store *S*.
C_{hi}: inventory holding cost of one product unit during one period at the ware-house S_i (i:0…L)
C_{TM}: Cost of total maintenance
$C_{pm\text{-}imp}$: Cost of preventive maintenance imperfect
C_{cm}: Cost of corrective maintenance action.
C_{perf}: cost of Perfect maintenance
k_p: Standard deviations between the CL and the control limits
μ_p: Preventive maintenance duration
μ_c: Corrective maintenance duration
mu: monetary unit
N_{imperf}: Number of imperfect maintenance
N_{imperf}: Number of perfect maintenance
C_i: Unit cost of inspection
C_r: Unit cost of one defective unit
Decision variables:
h: Sampling interval
i: The number of sampling intervals in a perfect cycle time
k_p: The Control chart limits
n: The sample size.

2.2 Problem Description

More than ever before, there is now the need to develop improved industrial strategies to guarantee quality at the best cost. In this model, we consider more practical scenarios to reflect the real manufacturing situations where the process can be in or out of control under different scenarios and due to several assignable causes. This model integrated an optimal production plan with maintenance and quality monitoring in the presence of multiple assignable causes. An 'x–bar' Control chart with an embedded alert signal is used no monitor a quality characteristic and notifies when the process shifts to an out-of-control state. We considered an imperfect manufacturing system involving a single machine production system linked to a multi-warehouse to satisfy random demand (Figure 1). The process begins in-control state, producing conforming products with the capacity of having a variable production rate $u(k)$ and sometimes goes out of control under three possible scenarios; I, II, & III. The different scenarios interpreted from the rule of seven are also applied in a meaningful way in Quality Management, used in conjunction with Control Charts. The Rule of Seven, as involved in Quality Management, says that "A run of seven or more consecutive points in a control chart, either above the mean, or below the mean, or continuously increasing or decreasing, may indicate the process may be out-of-control".

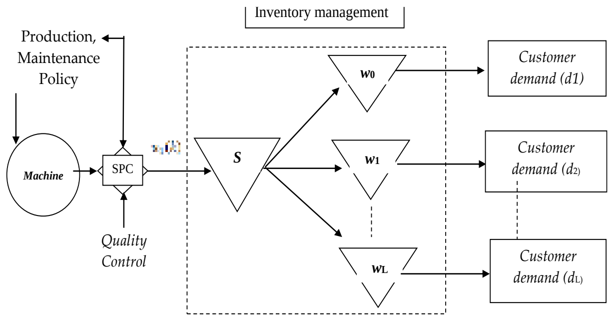

Figure 1. Production and Supply Chain Management.

The production machine is subjected to a random breakdown, and the degradation rate of the machine is influenced by the production rates; consequently, the failure rate $\lambda(t)$ increases with time and the production rate $U(k)$. Hence, affects the production process's reliability and generates the production of non-conforming units. In this study, the decision variables related to the control chart are n, h, and k_c.

2.3 Different scenarios and associated maintenance actions

To reduce the probability of shifting to out-of-control, quality monitoring of products is essential to employ a proper maintenance strategy by facilitating the process degradation. The following are the three possible scenarios of the unreliable process due to real situations that likely occur in a production run.

2.3.1 Scenario I:

The sample average of the quality indicator is between the control limits $(\text{LCL} < \bar{x}_s < UCL)$ for all samples. The production process always remains in the in-control state from the start to the end of the production cycle. In this situation, the maintenance strategy is to switch between imperfect and perfect maintenance. Indeed, imperfect preventive maintenance (with number N_{imp} over $H.\Delta t$) is applied periodically. Perfect preventive maintenance follows that; after several imperfect preventive maintenances, the perfect actions are performed to guarantee the manufacturing process's reliability and sustainability, as illustrated by Figure 2.

(a) Periodic IMPERFECT preventive maintenance following degradation of the machine.

(b) After several imperfect preventive maintenances, then PERFECT preventive maintenance is implemented as well as new (AGAN).

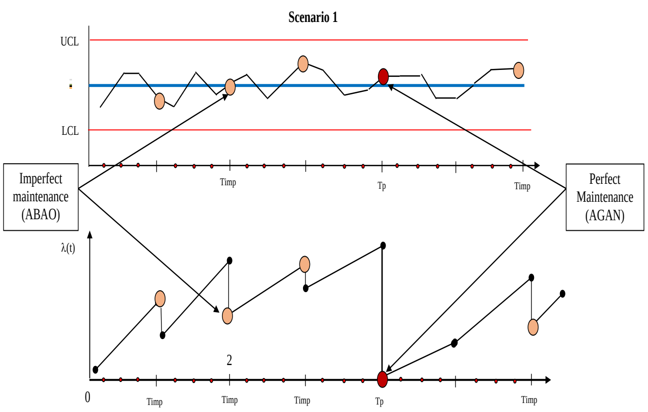

Figure 2. Periodically imperfect and perfect PM scenario I.

2.3.2 Scenario II :

The process shifts to the out-of-control state between the j^{th} and $(j+1)^{th}$ sampling due to the occurrence of i^{th} assignable cause A_i $(i = 1, 2, \ldots, S)$, and consequently, the mean value of the quality characteristic changes from μ to $\mu + \delta_i \cdot \mu$. In that event, operators seek to detect the assignable cause and perform perfect maintenance with an average duration of μ_p, as illustrated in Figure 3. Then, the process is restored to an as-good-as-new state and periodically reschedule the maintenance plan according to the production rate. Items produced during the last sampling interval transition to the 'out of control' state (between the samples $(J-1)$ and J) are all rejected.

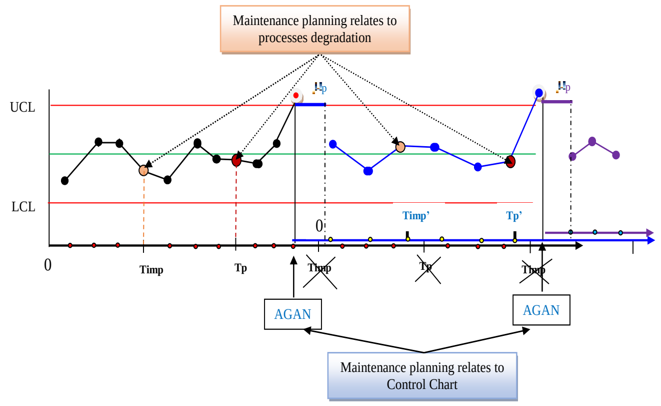

Figure 3. Perfect maintenance with duration μ_p for scenario II

2.3.3 Scenario III

The process starts in the in-control state, and the process instability is generated during the presence of a series of more than 7 points on the same side at time j^{th} to $(j+7)^{th}$ sampling(Figure 4). Consequently, the process starts to become unstable and eventually will manage the non-product quality. The operators seek to detect the assignable cause and conduct the corrective maintenance with an average duration of μ_c. Then, the process is restored to an as-good-as-new situation and periodically reschedule the maintenance plan according to the production rate—all items produced during the last sampling interval $(j+7)^{th}$ are rejected. sample average of the quality indicator is between the control limits.

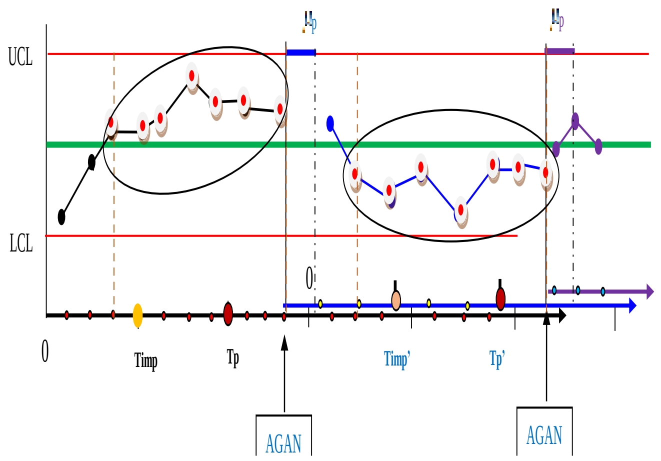

Figure 4 Corrective maintenance with duration μ_c scenario III

3 PROBLEM FORMULATION

The objective of this study is to determine the optimal plan of production $u(k)$ to satisfy the forecasted demand under a given service and quality level. Then, according to the economic plan of production, determination of the optimal parameters of the control chart, as well as the optimal number of maintenance strategies (N_{perf} & N_{imperf}). The performance of the proposed control chart is assessed by determining the in-control and out-of-control average run lengths. The objective is to minimize the expected costs related to production, inventory, maintenance, and quality including sampling and rejection costs during the finite time horizon. We assume that the horizon is portioned equally into H periods with a length equal to Δt. At the beginning of each production period, a stock to ensure random warehouses discrete satisfaction $Q_i(k)$ is created, and each warehouse to ensure a random demand $d(k)$ under a given service level.

- **Production model**

The inventory level of the leading stock S is characterized by a dynamic balance equation given as follows.

$$S(k) = S(k-1) + u(k).\Delta t - \sum_{i-1}^{L} Q_i(k) \quad (1)$$

Where $S(0) = 0$. So,

$$S(k) = \sum_{i=1}^{k-1} u(i).\Delta t - \sum_{j=1}^{L} Q_j(k) \quad (2)$$

The generated area of the inventory level evolution during a period k $(k = 1, \dots \dots, H)$ is given as follows:

$$Z_s(k) = max(S(k-1), 0).\Delta t + {}^1\!/_2 . u(k)\Delta t^2$$
$$(k = 1, \dots, H) \quad (3)$$

Consequently, the total holding cost of the principle stock HC during the finite horizon $H.\Delta t$ product is given by the following expression:

$$HC_s = \sum_{k-1}^{H} C_h \times Z_s(k) \quad (4)$$

In the inventory balance equation for each warehouse, the quantity of products incoming in each warehouse w_i at period k is the product quantity that left the principle stock s at the period index $k - \tau_i/_{\Delta t}$ and represented by $Q_i(k - \frac{\tau_i}{\Delta t})$. Thus, the inventory level of each warehouse level w_i at period k equals the warehouse level of w_i at period $k-1$ plus the number of products that arrives at w_i (i.e. $Q_i(k - \tau_i)$) minus the customer demand d_i at period k. The following relation gives the balanced equation:

$$w_i(k) = \begin{cases} w_i(k-1) + Q_i(k-\tau_i) - d_i(k) & If\ k \geq \tau_i \\ w_i(k-1) - d(k) & otherwise \end{cases}$$

with $k = \{0,1, \dots, H\}$, $\tau_i \geq 1, (i = 1, \dots, L)$ (5)

The generated area of the inventory level evolution for each warehouse during a period is given as follows:

$$Z_{wi}(k) = max(w_i(k-1), 0).\Delta t + {}^1\!/_2 . Q_i(k - \tau_i)\Delta t^2 \quad (6)$$

The following expression gives the inventory holding cost for all warehouses.

$$HC_w = \sum_{k-1}^{H} C_h \times Z_w(k) \quad (7)$$

Consequently, the total holding cost during the finite horizon $H.\Delta t$ expressed as follows:

$$HC = C_h \times \sum_{k=1}^{H} \begin{pmatrix} \max(S(k-1), 0) \cdot \Delta t + \frac{1}{2} \cdot u(k) \cdot \Delta t^2 \\ + \max(w_i(k-1), 0) \cdot \Delta t + \frac{1}{2} \cdot Q_i(k - \tau_i) \cdot \Delta t^2 \end{pmatrix} \quad (8)$$

Concerning the production cost during the finite horizon $H.\Delta t$, the production cost for period k

$$PC = C_p \times \sum_{k-1}^{H} u(k).\Delta t \quad (9)$$

The consequence of a delay characterizes the delay penalties to satisfy all the demands. If a delay occurred at the end of period k caused a shortage recovered during the next period $(k+1)$.

Penalties are determined as a function of the required duration $d_w(.)$ to produce the missed quantity at the end of each period, given by the following expression:

$$PC = C_d \times (\sum_{k-1}^{H} (\sum_{k-1}^{H} dw_i)),$$

Where $dw_i = \frac{\lfloor \min(w_i(k), 0) \rfloor}{Q(k+1-\tau_i)}$ (10)

Minimizing the different costs mentioned above is done by respecting the following constraints.

The service level requirements constraint for each warehouse and during all periods (probabilistic rule).

$$Prob[w_i(k) \geq 0] \geq \theta_i \quad (11)$$

$(k = 1, \dots, H-1)$ and $(i = 1, \dots, l)$

The bounds of the production level at each period k.

$$u_{min} \leq u(k) \leq u_{max} \quad (12)$$

So, the problem is defined as follows:

$$Min\ F = PC + HC + DC + DPC + TMC \quad (13)$$

- **Maintenance and quality model**

The cost models of maintenance and quality consider all possible scenarios presented above. Three scenarios are possible depending on the production plan obtained and its influence on the degradation degree. The control chart's efficiency can be measured using the probability of not detecting an adjustment when taking a sample of size n. The effectiveness of the chart is all the greater when this probability is low.

Scenario I`:

The probability that the process 'in-control state is formulated:

$$\alpha_1 = Pr(LCL < \bar{x}_s < UCL)$$

$$\alpha_1 = F\left(\mu + k_p \cdot \frac{\sigma}{\sqrt{n}}\right) - F(\mu - k_p \cdot \frac{\sigma}{\sqrt{n}}) \quad (14)$$

With F as the distribution function of the reduced centered normal law.

Let ARL_1 be the average run length characterized by the average number of successive samples where the process is stable during the finite horizon $H.\Delta t$ (scenario 1) expressed as follows:

$$ARL_1 = \frac{1}{\alpha_1}$$

Scenario II:

For scenario 2, when the process is unstable, the average can vary, taking μ_1 as the value. For a sample j, We note δ_j the expression of the adjustment of the mean in several standard deviations:

$$\delta_j = \frac{\mu_1 - \mu}{\sigma} \quad (15)$$

Let ARL_2 be the average run length characterized by the average number of successive samples to detect the control limits' shift (scenario 2), as expressed by (Montgomery 2004).

$$ARL_1(\delta_j) = \frac{1}{1 - \alpha_2(\delta_j)} \quad (16)$$

$\alpha_2(\delta_j) = P_j(\delta_j)$ is being the probability of non-detection of the shift to the control limits.

$$\alpha_2(\delta_j) = \{Prob(LCL \le \bar{x}_s \le UCL \mid \mu_1 = \mu + \delta_j \cdot \sigma)\}$$

$$\alpha_2(\delta_j) = F\left(\frac{UCL-\mu_1}{\frac{\sigma}{\sqrt{n}}}\right) - F\left(\frac{LCL-\mu_1}{\frac{\sigma}{\sqrt{n}}}\right)$$

$$\alpha_2(\delta_j) = F(k_p - \delta_j \times \sqrt{n}) - F(-k_p - \delta_j \times \sqrt{n}) \qquad (17)$$

The average restoration cycle duration for scenario 2, RCD_2, is given by

$$RCD_2 = ARL_2(\delta_j) \times I_s + \mu_P \qquad (18)$$

Scenario III:

When the process is in control, the probability of any point falling above (or below) the centerline is ½. Then, from the Multiplicative Law of probability for independent events, the probability of seven consecutive points falling, say, above the centerline equals (1/2)7 = (1/128). Likewise, the probability of seven successive points falling below the centerline is (1/2)7 = (1/128). Therefore, by the Additive Law of Probability, the probability of seven straight points falling on the same side of the centerline.

$$\alpha_3 = P(7\ points\ below\ the\ center\ line) | LCL \le \bar{x}_s \le UCL$$

$$= (P(7\ points\ above\ the\ center\ line) | LCL \le \bar{x}_s \le UCL) + (P(7\ points\ below\ the\ center\ line) | LCL \le \bar{x}_s \le UCL) \qquad (19)$$

Let ARL₄ be the average run length presented as the average number of successive samples to detect seven consecutive points on the same side of the centerline while the process is in control (scenario 4), expressed as follows;

$$ARL_3 = \frac{1}{\alpha_3} \qquad (20)$$

The average restoration cycle duration for scenario 4, RCD_4, is given by

$$RCD_3 = ARL_3 \times I_s + \mu_P \qquad (21)$$

- **Total maintenance cost**

The maintenance strategy optimization minimizes the expenses of perfect and imperfect preventive maintenance and corrective maintenance actions. The objective of the maintenance strategy is to determine the optimal parameters of the control chart, which are: the sample size n, the sampling interval h, the control limits coefficient k_c, as well as the optimal number N_{imp}* and N_p* of preventive maintenance actions and the adequate time between them, (T_{imp}* and T_p*) in the 'in-control state (Figure 5), and for each new cycle after the perfect maintenance action applied due to assignable causes.

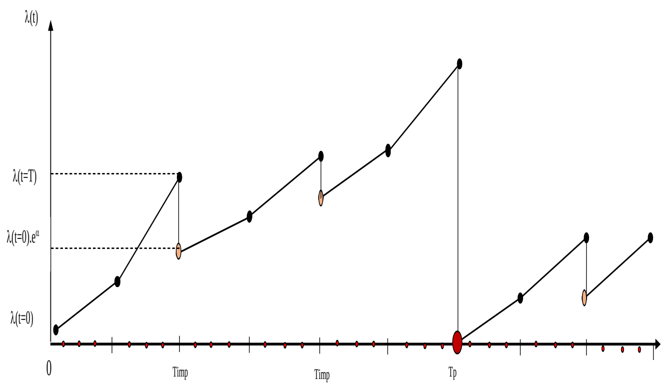

Figure 5. Switching the imperfect & perfect maintenance

Formally, the failure rate in the case of switching between imperfect and perfect maintenance actions is expressed as follows:

Let

$$T_p = Int[\frac{H}{N_p}] \times \Delta t$$

$$T_{imp} = Int[\frac{T_p}{N_{imp}}]$$

The average numbers of failures are respectively defined differently for each scenario.

$$\lambda_k(t) = \left(1 - \left\lfloor \frac{k-1}{\left(\left\lfloor \frac{k-2}{T_{imp}} \right\rfloor + 1\right) \times T_{imp}} \right\rfloor\right)\left(1 - \left\lfloor \frac{k-1}{\left(\left\lfloor \frac{k-2}{T_p} \right\rfloor + 1\right) \times T_p} \right\rfloor\right)\underbrace{\left(\lambda_{k-1}(\Delta t) + \frac{u(k)}{U_{max}} \lambda_n(t)\right)}_{A} + \left(1 - \left\lfloor \frac{k-1}{\left(\left\lfloor \frac{k-2}{T_p} \right\rfloor + 1\right) \times T_p} \right\rfloor\right) \times \left\lfloor \frac{k-1}{\left(\left\lfloor \frac{k-2}{T_{imp}} \right\rfloor + 1\right) T_{imp}} \right\rfloor \times \underbrace{\lambda_{\left(k - \left\lfloor \frac{T_{imp}}{\Delta t} \right\rfloor\right)}\left(\left(\left\lfloor \frac{k \times \Delta t}{T_{imp}} \right\rfloor - 1\right) \times T_{imp}\right) \times e^{\alpha}}_{B} + \left\lfloor \frac{k-1}{\left(\left\lfloor \frac{k-2}{T_p} \right\rfloor + 1\right) \times T_p} \right\rfloor \times \underbrace{\frac{u(k)}{U_{max}} \lambda_n(t)}_{C} \quad \forall\, t \in [0, \Delta t] \qquad (22)$$

preventive maintenance (failure rate B different to zero), i.e., at each $q \times T_{imp}$ with ($p = 1, \ldots, N_{imp}$ otherwise equals to 1.

- **Total cost of quality**

The total cost of quality is the sum cost of sampling and the cost of the non-conforming products for each scenario.

- cost of sampling

$$C_S = C_s \times n \times ARL_1 + C_s \times n \times ARL_2(\delta_j) + C_s \times n \times ARL_3$$

- Cost of non-conforming products.

$$C_{NC} = C_r \times \left(\sum_{m=1}^{M_2}\left(u\left(\left\lfloor \frac{ARL_{2m}}{\Delta t} \right\rfloor\right) \times \left(\frac{ARL_{2m}(\delta_j)}{\Delta t} - I_s\right)\right) + \sum_{m=1}^{M_3}\left(u\left(\left\lfloor \frac{ARL_{3m}}{\Delta t} \right\rfloor\right) \times \left(\frac{ARL_{3m}}{\Delta t} - I_s\right)\right)\right) \qquad (23)$$

Therefore, the average total cost of quality is given by:

$$C_Q = C_{FA} + C_{NC} + C_{sampling} \qquad (24)$$

- **Total maintenance and quality cost**

So, the total cost of maintenance and quality is given as follows;

$$C_{TM/Q} = C_M + C_Q \qquad (25$$

4 NUMERICAL EXPERIMENT

We consider a supply chain problem composed of a production system, one main, stock, and two warehouses ((L=2) to satisfy random demand. The production system is composed of one machine and produces one type of product over a finite planning horizon H=24 months in which the period length

Δt=1 month. Assumed the standard deviation of each demand of a product is the same for all periods and each demand σ_di ({i:1,2,}) = 200 and the initial inventory level, we assume that S(0)=0. The average demand for customers of warehouses 1 and 2: $d_1(k) = d_2(k) = 300\ \{k: 0, \dots, H-1\}$

Lower and upper boundaries of production capacities: $u_{min} = 200$, $u_{max} = 500$ and $c_p = 12mu, c_h = 1.2\ mu/k, c_{si} = 1.2\ mu/k$ $\{i: 1, \dots, L = 2\},\ s_i(0) = 0\ with\ \{i: 1, 2\}$, $\mu_0 = 5,\ \sigma_0 = 1.2$, $c_i = 30\$/product, c_r = 50\$$/item, $\delta = 0.75$.

Customer satisfaction degree (θ_i), is equal to $92\%\ (i = 1, 2, .\, l)$.

Table 1. Customer demand

d₁(0)	d₁(1)	d₁(2)	d₁(3)	d₁(4)	d₁(5)
450	465	450	435	450	420
$d_1(6)$	$d_1(7)$	$d_1(8)$	$d_1(9)$	$d_1(10)$	$d_1(11)$
480	420	480	450	405	420

Table 2. The optimal production plan

u(1)*	*u*(2)*	*u*(3)*	*u*(4)*	*u*(5)*	*u*(6)*
440	290	350	320	320	420
u*(7)	u*(8)	u*(9)	u*(10)	u*(11)	u*(12)
410	420	360	230	240	440

Table 3. The optimal delivery plan (*Q₁*)

$Q_1^*(1)$	$Q_1^*(2)$	$Q_1^*(3)$	$Q_1^*(4)$	$Q_1^*(5)$	$Q_1^*(6)$
250	250	250	250	220	130
$Q_1^*(7)$	$Q_1^*(8)$	$Q_1^*(9)$	-	-	-
180	110	230	-	-	-

Table 4. The optimal delivery plan (Q1)

$Q_1^*(1)$	$Q_1^*(2)$	$Q_1^*(3)$	$Q_1^*(4)$	$Q_1^*(5)$	$Q_1^*(6)$
230	180	200	130	130	90
$Q_1^*(7)$	$Q_1^*(8)$	$Q_1^*(9)$	-	-	-
140	210	120	-	-	-

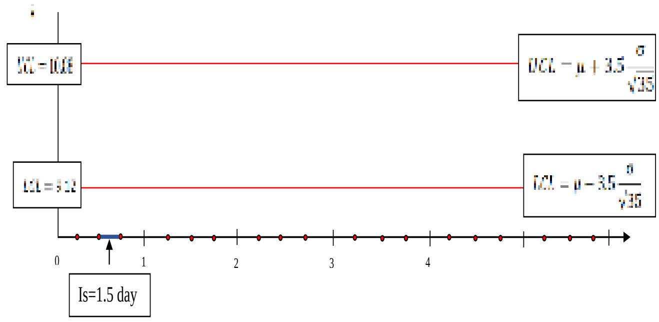

Figure 6. Control Chart limits and deviations

Table 5. The optimal Control Chart Parameters

*n**	*h**	*Kp**	Nimp	Np
35	1.5	3.5	7	1
ARL_1	ARL_2	ARL_3		
2	4	7		

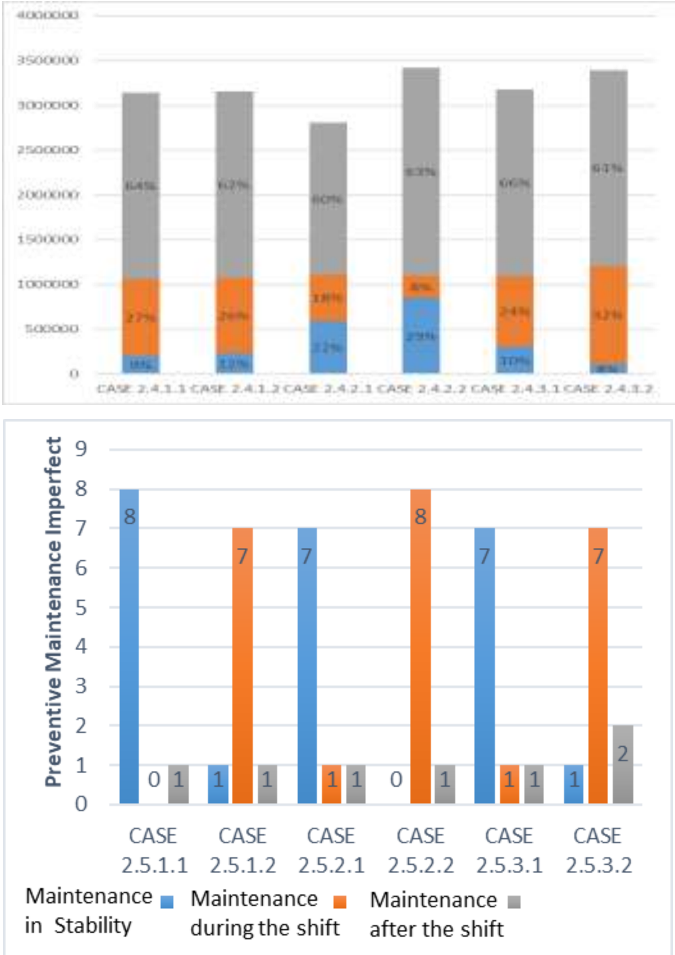

Figure 7. The impact of maintenance strategies on cost

We can consider that the best production, maintenance, and quality control strategy consists of taking one sample of size 35 every 1.5 days. Furthermore, regarding the control chart's design, the optimal number of standard deviations between the Centerline and the control limits is 3.5 (Figure 6) and conducting 7 number of imperfect as well as 1 number perfect maintenance (Figure 7), according to the optimal production plan as represented in Tables 2, inventory (Tables 3 & 4). The optimal total cost of production is commensurate with 2 ARL_1, 4 ARL_2, and 7 ARL_3.

5 Conclusion

The proposed production maintenance and quality strategy is according on the forecasted production based on the customer satisfaction degree. We develop a stochastic, dynamic model of an integrated production problem. The problem was solved using a numerical and analytical approach that solves the probabilistic into a deterministic function. And, determines the parametrized structure of the resulting optimal control policy. A simulation-based approach is used to optimize the parameters of the control charts. We determined and present in Table 5 the optimal decision variables characterized by the sample size, sampling interval, control limits coefficients, and the number of perfect, and imperfect PM. The best combination of the machine's degradation depends on production rates and the control chart parameters developed to reduce the non-conforming items and guarantee the production system's reliability. The main finding of the study showed the strong interrelation of the three key production functions and that the integrated model led to a significant increase in

process reliability, reduction in the non-quality products, and total cost minimization.
Future works will consider a multi-product system monitored by multi-inspection quality checks.

6 References

Abubakar, A. S., Nyoungue, A., & Hajej, Z. (2020). Integrated Production, Maintenance and Control Chart of Supply Chain Management Under Quality Constraint. *2020 IEEE International Conference on Industrial Engineering and Engineering Management (IEEM)*, 1215–1219. https://doi.org/10.1109/IEEM45057.2020.9309985

Abubakar, A. S., Zied, H., & Nyoungue, A. C. (2022). An optimal production, maintenance and quality problem, with improved statistical process chart of a supply chain under service and quality requirements. *IFAC-PapersOnLine*, *55*(10), 1746–1751. https://doi.org/10.1016/j.ifacol.2022.09.650

Addeh, A., Khormali, A., & Golilarz, N. A. (2018). Control chart pattern recognition using RBF neural network with new training algorithm and practical features. *ISA Transactions*, *79*, 202–216. https://doi.org/10.1016/j.isatra.2018.04.020

Ait-El-Cadi, A., Gharbi, A., Dhouib, K., & Artiba, A. (2021). Integrated production, maintenance and quality control policy for unreliable manufacturing systems under dynamic inspection. *International Journal of Production Economics*, *236*, 108140. https://doi.org/10.1016/j.ijpe.2021.108140

Bahria, N., Harbaoui Dridi, I., Chelbi, A., & Bouchriha, H. (2021). Joint design of control chart, production and maintenance policy for unreliable manufacturing systems. *Journal of Quality in Maintenance Engineering*, *27*(4), 586–610. https://doi.org/10.1108/JQME-01-2020-0006

Ben-Daya, M., & Rahim, M. A. (2000). Effect of maintenance on the economic design of $\bar{x}$-control chart. *European Journal of Operational Research*, *120*(1), 131–143. https://doi.org/10.1016/S0377-2217(98)00379-8

Bouslah, B., Gharbi, A., & Pellerin, R. (2016). Integrated production, sampling quality control and maintenance of deteriorating production systems with AOQL constraint. *Omega*, *61*, 110–126. https://doi.org/10.1016/j.omega.2015.07.012

Fakher, H. B., Nourelfath, M., & Gendreau, M. (2018). Integrating production, maintenance and quality: A multi-period multi-product profit-maximization model. *Reliability Engineering & System Safety*, *170*, 191–201. https://doi.org/10.1016/j.ress.2017.10.024

Guo, Y., Hrouga, M., Liu, J., Afifi, S., Allaoui, H., & Zhang, Y. (2022). JOINT OPTIMIZATION OF CAPACITATED LOT-SIZING WITH LOST SALES AND NON-CYCLICAL PREVENTIVE MAINTENANCE. 국제산업공학저널, *29*(3). https://doi.org/10.23055/IJIETAP.2022.29.3.8013

Hajej, Z., Nyoungue, A. C., Abubakar, A. S., & Mohamed Ali, K. (2021). An Integrated Model of Production, Maintenance, and Quality Control with Statistical Process Control Chart of a Supply Chain. *Applied Sciences*, *11*(9), 4192. https://doi.org/10.3390/app11094192

Hajej, Z., Rezg, N., & Gharbi, A. (2018). Quality issue in forecasting problem of production and maintenance policy for production unit. *International Journal of Production Research*, *56*(18), 6147–6163.

Liu, B., He, K., & Xie, M. (2020). Integrated production and maintenance planning for a deteriorating system under uncertain demands. *IFAC-PapersOnLine*, *53*(3), 222–226. https://doi.org/10.1016/j.ifacol.2020.11.036

Pakurár, M., Haddad, H., Nagy, J., Popp, J., & Oláh, J. (2019). The Service Quality Dimensions that Affect Customer Satisfaction in the Jordanian Banking Sector. *Sustainability*, *11*(4), 1113. https://doi.org/10.3390/su11041113

Rivera-Gómez, H., Gharbi, A., Kenné, J.-P., Ortiz-Zarco, R., & Corona-Armenta, J. R. (2021). Joint production, inspection and maintenance control policies for deteriorating system under quality constraint. *Journal of Manufacturing Systems*, *60*, 585–607. https://doi.org/10.1016/j.jmsy.2021.07.018

Salmasnia, A., Abdzadeh, B., & Namdar, M. (2017). A joint design of production run length, maintenance policy and control chart with multiple assignable causes. *Journal of Manufacturing Systems*, *42*, 44–56. https://doi.org/10.1016/j.jmsy.2016.11.003

Si, W., Love, E., & Yang, Q. (2018). Two-state optimal maintenance planning of repairable systems with covariate effects. *Computers & Operations Research*, *92*, 17–25. https://doi.org/10.1016/j.cor.2017.11.007