Modeling and Estimation of Acoustic Pulse Energy Dissipation in a Waveguide with Elastic Walls

Fyodor Legusha, Vyacheslav Maslov and Iurii Popov

June 3, 2020

Modeling and estimation of acoustic pulse energy dissipation in a waveguide with elastic walls

Legusha Fedor
Physics department
State Marine Technical University
St.-Peterburg, Russia
email: legusha@yandex.ru

Vyacheslav Maslov
Acoustic department
Krylov State Research Centre
St.-Peterburg, Russia
email: krylov@ksrc.ru

Iurii Popov
Acoustic department
Krylov State Research Centre
St.-Peterburg, Russia
email: 2360460yk@gmail.com

***Abstract* – In this paper we study the wave processes in the boundary acoustic layers of the multimode waveguide systems. Wave propagation in a waveguide with "imperfectly" rigid walls is considered. Thermoacoustic effects that occur when a wave propagates in a near-surface layer with a finite impedance of the inner surface of the waveguide are considered. The estimation of changes in the energy dissipation of a propagating wave of rigid walls is performed.**

Keywords* – *waveguide, energy dissipation, acoustic waves, acoustic impedance

I. Introduction

Elastic vibrations and acoustic waves are widely used in engineering. At the same time, depending on the scope of application, the requirements for the accuracy of the description and the reliability of the results obtained increase many times. These requirements lead to the need for wave propagation to take into account physical phenomena of the second order of smallness. In a number of applied problems of acoustics, the determining factor is the registration or emission of sound signals without distortion introduced by inhomogeneous surface waves. Such tasks are related to the development of miniature electronic devices (cell phones, microphones, hearing AIDS). In hydroacoustics, the influence of inhomogeneous surface waves is manifested in the development of miniature hydrophones, the use of elastic PI, etc. All these problems are combined with the problem of describing physical phenomena in the boundary layers responsible for acoustic energy dissipation. In practice, it is extremely cumbersome to solve precise hydrodynamic equations that allow us to take into account the influence of viscous and heat waves in surface layers and calculate energy dissipation. In this case, the final result depends largely on the accuracy of the description of physical conditions, which are not always known with sufficient accuracy. In this regard, an acoustic waveguide with rigid walls was chosen to study the physical processes that determine acoustic energy dissipation.

A physical and mathematical model of a cylindrical waveguide with rigid walls can be chosen with sufficient accuracy to describe second-order smallness physical phenomena. The geometry and boundary conditions of the waveguide ensure the propagation of plane harmonic sound waves at a constant speed and shape preservation. Take into account that the loss of energy propagating in water can be considered negligible ~0,0002 [dB/m]. Accordingly, the energy dissipation of a plane wave in a waveguide should be determined only by losses at the ends of the waveguide and thermoacoustic transformations in the surface layers. For the medium, we can assume that entropy is constant and not consider the mechanisms of dissipative losses for a propagating wave.

The theory of wave propagation in waveguides is well known and methodologically implemented in acoustic measuring pipes (waveguides), which, depending on the purpose and technical data, can also be called impedance pipes, pulse pipes, and Kundt pipes. Acoustic research in measuring tubes is based on the theory of sound wave propagation in waveguides, described in fundamental works [2 – 4]. The theoretical basis allows measurements in liquid and air media when signals are propagated in noise, tone, or pulse modes. However, physical phenomena occurring in layers near the walls of the waveguide are not taken into account in these studies. This is due to the fact that measurements are usually indirect and absolute values of the wave energy are not required in the experiment. Another necessary condition is the approximation of ideal "acoustically" rigid walls in the frequency range below the first critical frequency. This condition guarantees the propagation of only one plane wave at a constant speed and preserving its shape. Failure to meet this condition leads to a superposition of non-planar waves in the waveguide propagating at different speeds. Accounting for these forms is difficult to analyze. Their excitation in the waveguide does not correspond to the excitation conditions in a free medium. The condition of "absolutely" hard walls for a real waveguide used in acoustic measurements is usually fulfilled with a sufficient degree of accuracy. This greatly simplifies the task of estimating the plane wave dissipation energy during the experiment.

To study these phenomena on the example of a real cylindrical waveguide, we used short acoustic pulses emitted by a cylindrical piston installed at one end of the waveguide. A sufficiently wide spectrum of the propagating pulse created additional conditions for evaluating the influence of "imperfect" boundary conditions on the waveguide walls. Based on the results obtained, the analysis of wave energy dissipation at the level of second-order smallness values was performed.

II. Statement Problem

Consider a cylindrical waveguide of radius R (fig. 1), length L, filled with water. It is known that the type of waves propagating along the tube is determined by the nature of the excitation of vibrations. Under the conditions of the problem, we assume that a flat piston is installed in the initial section $z = 0$. Vibrations of the piston emit a sound wave in the waveguide under the action of a given pulse from an electromechanical vibrator. The position of the point in the waveguide is determined in cylindrical coordinates (r, φ, z).

The radiated pulse P_1 of the signal passed the distance of the section of the pipe filled with water $L = 6.5$ m, experienced a reflection from the "soft" border (the border of the media "water-air"), then P_2 passed the section filled with water in the opposite direction and was registered at the radiation site. The total distance covered was 13 m. At the point where the pulse was registered, it experienced a reflection from the "hard" border (the border was created by a massive iron cover). The experiment used a pulse with sinusoidal filling, which allowed the calculation to use the formula for the harmonic wave. As a result of repeated reflection, the pulse faded. The amplitudes of the first and subsequent reflected pulses are shown on fig. 2.

Energy dissipation was estimated by the pulse attenuation. The attenuation coefficient can be estimated from the change in the pulse amplitude over a known time (or a known distance). Taking into account the speed of sound $c_0 = 1480$ m/s and the length of the traveled distance of 13 m, the period of registration of re-reflected pulses is $\tau_i = 0.0088$ s. The amplitude of the damped oscillations in the plane wave approximation is determined by the formula

$$p(t) = p_i \cdot \exp(-i\omega t \pm k_0 x) \cdot \exp(\pm \gamma x), \quad (1)$$

where p_i – is the amplitude of the initial pulse (at the registration point and the rigid wall);
$\alpha = \gamma + ik_0$ – constant distribution;
γ – dissipation (attenuation of the amplitude per unit path length);
k_0 – wave number.

Dissipation can be defined using the ratio:

$$\gamma = (1/2L)\times\ln(p_i/p_{i+1}), \quad (2)$$

where i – the sequential number of the reflected pulse.

Table 1 is shown the amplitudes of reflected pulses in a cylindrical waveguide with different carrier frequency of the tone signal.

The energy calculation shows that, on average, in the frequency range, the amplitude of each reflected pulse decreases almost twice as compared to the previous one, i.e. about 6 dB. Accordingly, the pulse loses approximately $(W–W_{i+1})/W_i = 1 – (A_{i+1}/A_i)^2 = 1– (1/2)^2 = 3/4$ parts of initial energy. At low frequencies, the attenuation was higher, apparently due to the fact that at these frequencies the acoustic impedance of the "hard" wall is a complex value. In this case, its mass was not large enough to fully reflect the wave, and the connection with the waveguide itself and the technological binding led to the radiation of part of the energy to the external environment. It was difficult to account for such energy losses in this experiment (additional recording tools must be used). Therefore, we consider only the values at frequencies of 5000 and 5800 Hz as control values, where in the theoretical calculation the wall can be considered absolutely "rigid" with infinite impedance. Table 1 is shown that the wave attenuation is 0.22–0.24[dB/m].

TABLE 1. Calculation of the average dissipation

Frequency, Hz	Amplitudes of reflected pulses, мB				⟨γ⟩, m^{-1}
	1	*2*	*3*	*4*	
800	42	12.5	4.1	1.9	0,05
2000	29.9	16.1	8.5	4.5	0.04
4000	24.2	12.9	6.9	3.5	0.04
5000	11.6	6.0	3.3	2	0.03
5800	13.5	6.3	3.8	2.3	0.03

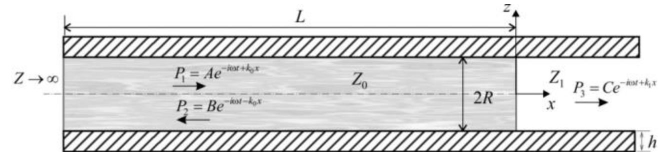

Fig. 1. A cylindrical waveguide filled with water

III. A Theoretical Model

To begin with, let's consider the problem without taking into account viscous and heat losses. In this setting, instead of the Navier-Stokes equation, we consider the solution of the Helmholtz wave equation (3).

$$1/r\times(r\cdot\partial p/\partial r) + 1/r^2\times(\partial^2 p/\partial\varphi^2)+ \partial^2 p/\partial z^2+k_0^2 p = 0. \quad (3)$$

In general, the equation allows for the existence of an infinite number of sound waves in the waveguide, which have different shapes and propagation speeds. Using a piston with a diameter close to the diameter of the pipe R as a radiator allows you to limit the range of possible solutions to the function, excluding the azimuth component from the solution

$$p(r,\varphi,z,t) = \sum_n p_n(r,z)\exp(-i\omega t), \quad (4)$$

where $p_n(r,z,t) = A_n J_0(k_{rn}r)\times\exp(-i(\omega t-k_{zn}z))$.

The presence of sufficiently thick steel walls allows [3] to consider them "absolutely" rigid in the problem of sound wave propagation. In other words the impedance of the inner surface of the pipe z_t is assumed to be infinitely large compared to the impedance of the medium $z_0 = \rho c_0$.

$$z_t = \frac{1}{(\rho c_0)} \frac{p}{V_r}\bigg|_R \to \infty. \quad (5)$$

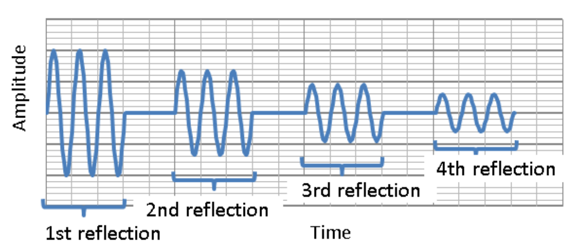

Fig. 2. Registered pulses

This condition implies a solution for propagating waves in a waveguide.

$$\left(\frac{\partial p}{\partial n}\right)\Bigg|_{r=R} = J_1\left(k_{rn} r\right) = 0. \qquad (6)$$

The zeros of the Bessel function $k_{rn}R = \pi\alpha_{0n}$; $\alpha_{00} = 0$; $\alpha_{01} = 1.220$; $\alpha_{02} = 2.223$; …determine the solution of equation (6). This solution admits the presence of a plane propagating wave always in the waveguide, corresponding to the solution at $\alpha_{00} = 0$:

$$p_0(r,z,t) = A_0 e^{-i(\omega t - k_0 z)}.$$

The presence of non-planar propagating sound waves for the considered waveguide $R = 0.075$[m] is possible at frequencies higher than some critical one, at which the component of the wavenumber in the axial direction becomes a real value:

$$k_{zn} = \sqrt{k_0^2 - k_{rn}^2};\ k_{rn} = \frac{\pi\alpha_{0n}}{R}.$$

For the considered waveguide of radius R, the first critical frequency is $f \approx 5900$[Hz]. It is the frequency range below this critical frequency that is of interest from a practical point of view, since it allows us to limit the solution only for a plane wave in a multimode waveguide.

Knowing the propagating modes of vibrations, it is possible to calculate the passage and reflection of the acoustic wave from the ends of the studied waveguide and estimate the energy loss. According to the experimental conditions, on the one hand, the pulse was reflected from the "soft" border, on the other from the "hard" one. For a plane wave, the reflection coefficient can be calculated using a simple formula using the ratio of the impedances of the two media. For a "soft" border, we get:

$$REF = \frac{p_{ref}}{p_{pass}} = \frac{Z_0 - Z_1}{Z_0 + Z_1} = \frac{1.48\cdot10^6 - 3.3\cdot10^2}{1.48\cdot10^6 + 3.3\cdot10^2} = 0.9996. \qquad (7)$$

Assuming that the energy flux is proportional to the square of the amplitude, we can conclude that the proportion of the pulse energy (99,9 % of the energy must be reflected back into the medium Z_0). The complex part of the impedance will determine the energy loss due to an inhomogeneous surface wave. In this calculation, we consider the effect of this wave on additional attenuation negligible and do not evaluate it.

Thus, the main part of the absorbed wave energy must be determined by the boundary conditions on the walls of the waveguide. It is obvious that the accuracy of the boundary condition (3) will largely determine the propagation conditions not only of the plane wave, but also the conditions of energy dissipation.

Note that the impedance of the inner surface of a real waveguide is a finite value. In this case, the boundary condition (3) can be rewritten as mixed boundary conditions:

$$\left(\frac{\partial p}{\partial n} + \frac{i\omega\rho}{z_t}p\right)\Bigg|_{r=R} =$$
$$\left[\sum_n \left(k_{rn}J_0'\left(k_{rn}r\right) + \frac{i\omega\rho}{z_t}J_0\left(k_{rn}r\right)\right)A_n e^{-i\left(\omega t + \sqrt{k_0^2 - k_{rn}^2}\,z\right)}\right]_{r=R} = 0. \qquad (8)$$

In the approximation of $i\omega\rho/z_t \ll 1$, the roots of equation (8) must be close to the values of the roots (6). In other words, the boundary frequency above which non-planar waves must be taken into account will still be close. At low frequencies, it is possible to limit ourselves to considering only the zero waveform. Decomposition of the Bessel functions (8) into a Taylor series near the root $kr_0R \approx \pi\alpha_{00}$ in the equation leads to the expression

$$-\frac{1}{R}\left(\pi\alpha_{00}\cdot J_1(\pi\alpha_{00}) + \left(k_{r0}R - \pi\alpha_{00}\right)\frac{d\left[k_{r0}R\cdot J_1\left(k_{r0}R\right)\right]}{d\left(k_{r0}R\right)}\Bigg|_{\pi\alpha_{00}} + \ldots\right) +$$
$$+\frac{i\omega\rho}{z_t}\left(J_0(\pi\alpha_{00}) - \left(k_{r0}R - \pi\alpha_{00}\right)J_1(\pi\alpha_{00}) + \ldots\right) = 0 .$$

From this expression, wave numbers of the zero mode can be obtained directly in the approximation of the final value of the impedance on the wall surface.

$$k_{r0} = \sqrt{\frac{ik_0\rho c_0}{z_t R}},\quad k_{z0} = k_0\left(1 + i\frac{\rho c_0}{2z_t R k_0}\right). \qquad (9)$$

From equations (4) and (9), we can write an expression for the sound pressure and vibrational velocity of a wave in a waveguide with "imperfectly" rigid walls in cylindrical coordinates:

$$p(r,\varphi,z,t) = A_0 J_0\left(k_{r0}r\right)e^{-i(\omega t - k_{z0}z)} = A_0\left(1 + r^2\frac{ik_0\rho c_0}{z_t R}\right)e^{-\frac{\rho c_0}{2z_t R}z}e^{ik_0 z - i\omega t},$$
$$V_z(r,\varphi,z,t) = A_0\frac{1}{\omega\rho}\left(1 + r^2\frac{ik_0\rho c_0}{z_t R}\right)\left(ik_0 - \frac{\rho c_0}{2z_t R}\right)e^{-\frac{\rho c_0}{2z_t R}z}e^{ik_0 z - i\omega t},$$
$$V_r(r,\varphi,z,t) = -A_0\frac{1}{\omega\rho}\frac{2r}{z_t R}e^{-\frac{\rho c_0}{2z_t R}z}e^{ik_0 z - i\omega t}. \qquad (10)$$

It can be seen $z_t \to \infty$ that when equation (10) passes into the equation of a plane wave.

From the obtained expressions (10) it can be seen that the front of the propagating wave is quasi-plane, the wave has attenuation and its phase velocity differs from the phase velocity of the wave in free space. $V_z(r, \varphi, z, t)$ – the vibrational velocity in the radial direction is not zero on the surface of the waveguide walls. Obviously, when calculating viscous and heat waves in the boundary layer, it is necessary to take into account the influence of the radial component of the velocity.

The phase velocity and amplitude of the resulting quasi-plane wave can be estimated from the expression (7):

$$c_{z0}(\omega) = \frac{c_0}{\left(1 + i\frac{\rho c_0^2}{2z_t R\omega}\right)};\quad A_0(r,\omega) = A_0\left(1 + r^2\frac{ik_0\rho c_0}{z_t R}\right). \qquad (11)$$

The wave impedance of a quasi-plane wave when radiated by a piston has the form:

$$z_{z0} = \frac{p}{V_z} = \frac{\omega\rho}{ik_0 - \dfrac{\rho c_0}{2 z_t R}} = \rho c_0 \frac{\omega}{i\omega - \dfrac{\rho c_0^2}{2 z_t R}} \,. \quad (12)$$

In this statement of the problem ($i\omega\rho / z_t << 1$), the values $p(r, \varphi, z, t)$, $V(r, \varphi, z, t)$, c_{z0}, z_{z0} will determine the desired second-order smallness correction for a propagating quasi-plane wave in a real waveguide with walls close to "absolutely" rigid.

Assuming that the average velocity over the surface of the piston in the waveguide must be a constant value for vibrations, we introduce a function $u_{z0}(t)$ that depends only on time. The sound pressure for the transition wave propagating in the waveguide, taking into account the approximations made when solving equation (4), can be expressed as:

$$p(r,z,t) \approx \frac{1}{2\pi}\int_{-\infty}^{+\infty} \omega\rho \frac{1 + r^2 \dfrac{ik_0\rho c_0}{z_t R}}{ik_0 - \dfrac{\rho c_0}{2 z_t R}} e^{-\frac{\rho c_0}{2 z_t R} z} e^{ik_0 z - i\omega t} d\omega \int_{-\infty}^{+\infty} u_{z0}(\tau) e^{-i\omega\tau} d\tau \,. \quad (13)$$

In the limit case $z_t \to \infty$, equation (9) will take the known form:

$$p\,(z,t) = \rho c_0 u_{z0}\,(t - z/c_0).$$

It is obvious that the structure of the acoustic field in a cylindrical waveguide with "imperfectly" rigid walls will be determined by the impedance of the wall surface, the frequency and function of the piston vibrations. The given formula (13) allows performing an analytical calculation of a propagating pulse in a hydroacoustic waveguide of any shape and obtaining a vector of velocities and displacements near the waveguide boundaries.

IV. Numerical model of propagation of a short sinusoidal signal

Obviously, in the simplest setting, the tone signal can be considered. In this case, the integral according to the formula (13) is reduced to calculating the signal amplitude at a single frequency, which from the physical point of view corresponds to the propagation of a sinusoidal signal with a certain amplitude and phase velocity determined by the equations (11–12). During the experiment, the effect of the impedance of the waveguide walls is reduced only to determining the correction to the phase velocity of the piston wave. The most interesting physical processes are that occur during the propagation of short (broadband) signals. The results of the analysis of such processes are extremely sensitive to the accuracy of setting the envelope of the pulse front, the accuracy of setting the initial and boundary conditions. To improve the accuracy of the comparative analysis, a numerical model of a cylindrical waveguide was constructed, which was used in experimental studies (fig. 2).

Figure 3 shows a finite element model of a cylindrical waveguide (for clarity, the b sector is cut out in the model). A model of a piston radiator is located at one end of the model, and a free border (water-air border) is set at the other end. The

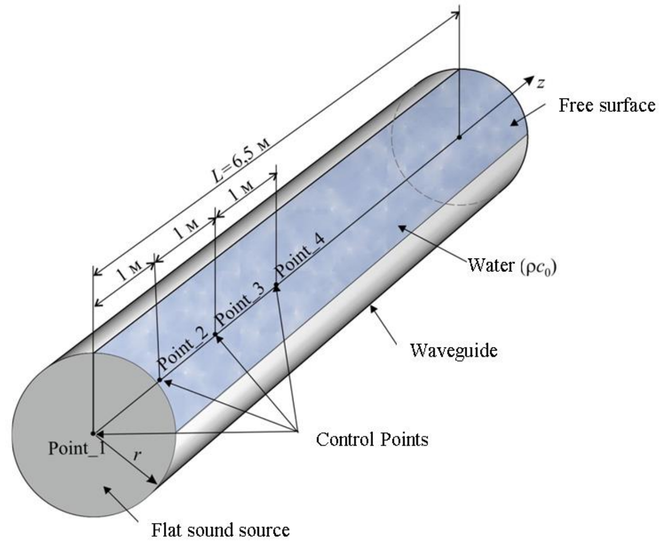

Fig. 3. Numerical model of a cylindrical waveguide

impedance boundary conditions corresponding to the impedance of the inner surface of the waveguide are set on the walls of the waveguide. Under the conditions of the problem to be solved, they could take values corresponding to elastic impedance, purely real impedance, and absolutely rigid boundary impedance. To analyze the results obtained and compare them with experimental data, the model identified four control points (Point_1, Point_2, Point_3, Point_4) located 1 meter apart. In the experimental hydroacoustic waveguide during the experiment, three small hydrophones were located near the control point Point_1.

The shape of the experimentally obtained echo signal (fig. 4) under the influence of a short pulse close to the Delta function was affected by random fluctuations. In addition, the actual envelope of the pulse was known fairly approximately, which made some additional errors in the calculation.

For further research, a short pulse equal to one period of oscillation at a frequency f = 2000 [Hz]

Fig. 5 shows experimental data of a propagating echo signal obtained in a waveguide when a short sinusoidal pulse is applied to the piston (fig. 4). For clarity, a time interval is shown that includes a signal from the emitter and a signal that passes along the entire length of the waveguide and is reflected from its second end with a free boundary.

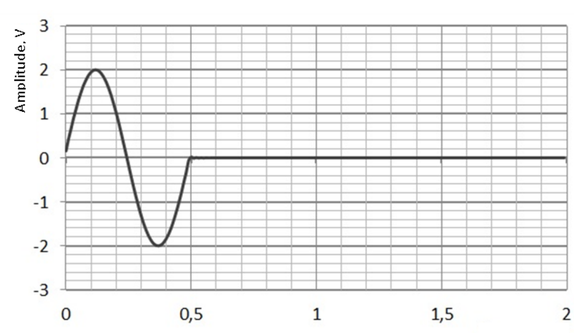

Fig. 4. A single sinusoidal pulse applied to the electrodynamic vibrator

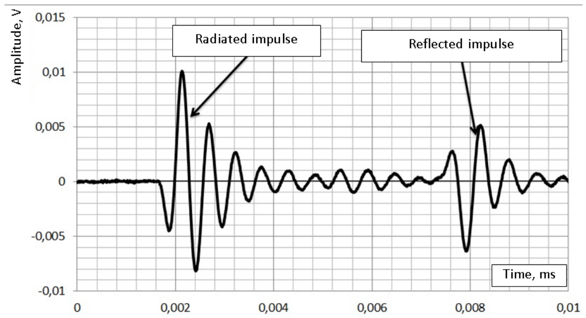

Fig. 5. Echo signal in the waveguide when a short sinusoidal pulse is applied to the piston

Fig. 6 shows the spectra of the forward (from the emitter) and reverse (reflected from the end) pulse. The analysis shows that the spectrum of the reverse pulse is wider and smaller in amplitude. Assuming that the medium (the waveguide was filled with distilled water) is homogeneous and isotropic, possible changes in the pulse propagation speed and its shape are possible only from the boundary conditions on the waveguide walls. It is obvious that the effect of finite impedance value manifests itself in the dispersion rate and the change in pulse shape. Of practical interest is the type of function describing the frequency dependence. Within the framework of the research described in this paper, this was done fairly approximately.

In the original equation (13), an unknown function - the impedance of the internal walls of the waveguide determines not only the distortion of the pulse shape, but also determines the speed of wave propagation. We estimate the speed of wave propagation in an experimental waveguide in comparison with the speed in free space. The total length of the cylindrical waveguide used in the experiment was, with a radiator installed at one end and a free "water-air" boundary at the other $l_1 = 6.5$ [m]. The hydrophone for fixing the acoustic pulse in the waveguide was installed $l_2 = 0.9$ [m] at a distance from the emitter. Thus, the direct signal from the emitter to the moment of its fixation on the hydrophone passed the distance $L_{pr} = 1.2$ [m]. The return signal passed the distance $l_1 = 6.5$ [m] reflected from the water-air boundary with a change in phase and came back to the hydrophone from the opposite side. The total distance passed by the reverse pulse was $L_{rev} = 6.5+5.3 = 11.8$ [m]. The forward and reverse pulses were recorded in real time, which allowed us to determine the correction to the speed of propagation of the pulse in the waveguide in comparison with the speed of sound in a boundless medium. Within the accuracy of the hydrophone installation, the speed of the propagating quasi-plane wave was experimentally equal to $c_{z0} \approx 1420–1440$ [m/c], which is approximately 3% less than the speed of the wave in free space. Numerical calculation on a finite element model also showed a deceleration of the propagating pulse in a waveguide with an "elastic" impedance on the wall surface. An error of 3% does not allow us to compare the calculated data with the experimental data with a sufficient degree of accuracy. However, the expression (12) allows us to approximate the value of the impedance of the pipe surface.

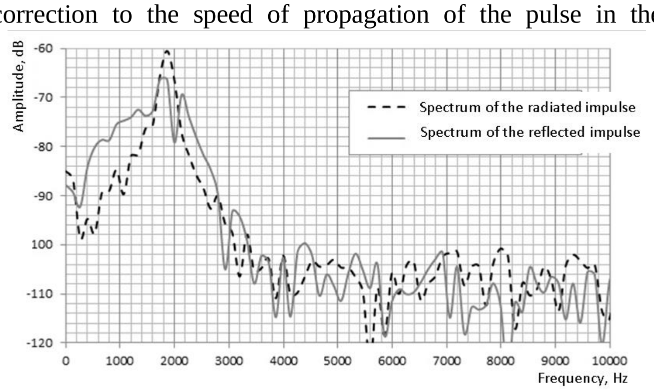

Fig. 6. The spectra of the forward and return signal

$$z_t = i\,\frac{\rho c_0^2}{2R\omega}\,\frac{c_{z0}}{c_0 - c_{z0}} \quad . \tag{14}$$

This formula is very convenient when using tone signals as a pulse. Since in general, the impedance and speed are frequency-dependent, which is manifested in the dispersion of the speed of sound and distortion of the original wide-field pulse. This paper does not present the results of studies using tone signals. In this regard, we assume that the changes in the spectrum are insignificant for forward and reverse pulses and $c_{z0} = const$. Equation (13) in this case allows you to move from the integral form of the record to a purely linear form (which does not take into account the variance). In this case, the impedance of the waveguide surface is represented by a purely elastic characteristic $z_t \approx i/\omega K_t$, where K_t – a certain constant value characterizes the malleability of the walls. From (12) we obtain an expression for the characteristic of the impedance of the inner surface of the waveguide in the form:

$$K_t \approx \left(\frac{\rho c_0^2}{2R}\,\frac{c_{z0}}{c_0 - c_{z0}}\right)^{-1} \quad . \tag{15}$$

For the waveguide under consideration, the estimate of the impedance value under the assumption is:

$$z_t \approx 1.5 \cdot 10^{11} \cdot i/\omega \text{[Pa·s/m]}.$$

At the end of the analysis, the analysis of the wave front propagating in a waveguide with impedance walls was performed. It is shown (fig. 7–8) calculations of the front of the propagating pulse in the waveguide in the approximation of "absolutely" rigid walls (fig. 7) and "nonabsolute" rigid walls (fig. 8) in the same time frame ($t_{control} = 1.25$ [mc]).

In the case of finite impedance on the inner surface of the waveguide, the front of the propagating pulse becomes quasi-plane. An example of numerical calculation of the propagating pulse front for a waveguide with different wall impedance is shown in fig. 7 and 8. The wave Front (10) depends on the frequency and impedance characteristic. However, for a hydroacoustic waveguide with rigid walls, the wave front is almost flat (deviations do not exceed 1–2%).

The analysis of the obtained results shows that the pulse propagation speed in a waveguide with "absolutely" rigid walls is higher than in a waveguide with finite impedance. At the same time, the frequency of the isosurfaces shows that the shape of the pulse changes during propagation (the pulse is blurred). This effect indicates the velocity dispersion determined by the wall surface impedance.

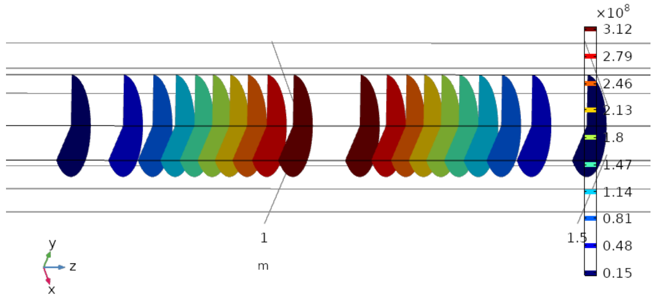

Fig. 7. The wave front (ISO surfaces of equal pressure) in a waveguide with "absolutely" rigid walls

V. Calculation of Energy Dissipation in a Waveguide

The above assumption for the medium about the immutability of entropy does not provide for accounting for the mechanisms of dissipative losses in the calculation model. However, this condition must be violated in the boundary layer. Change in entropy means the presence of irreversible processes in the system due to the work of dissipative forces that lead to the transition of vibrational energy to the internal (thermal) energy of the system. This automatically leads to a transition from the Helmholtz equation (1) to a more General type of wave equation for the hydrodynamics of compressible media – the Navier-Stokes equation. According to the theory of hydrodynamics [2], the total energy of the pulse at time *t*:

$$W(t)=W_0-W(\tilde{S},t). \quad (16)$$

The flow of sound energy (acoustic power) through the section of the acoustic tube can be estimated as:

$$\hat{P}(t)=\frac{dW(\tilde{S},t)}{dt}=\frac{dW}{d\tilde{S}}\frac{d\tilde{S}}{dt}=T_0\dot{S}'\neq 0. \quad (17)$$

Changes in energy are determined by the presence of viscous and heat losses

$$\dot{S}'T_0=\int_V \frac{\chi(\nabla T')^2}{T_0}dV+\int_V\left(\frac{\partial\tilde{v}_i}{\partial x_k}+\frac{\partial\tilde{v}_k}{\partial x_i}\right)^2 dV. \quad (18)$$

The sound pressure and the vibrational velocity vector (10) make it possible to estimate the amount of absorbed energy in the case of a waveguide boundary with "non-ideal" acoustically rigid walls. In general, this solution for a cylindrical waveguide is quite cumbersome. For a qualitative evaluation of the result,

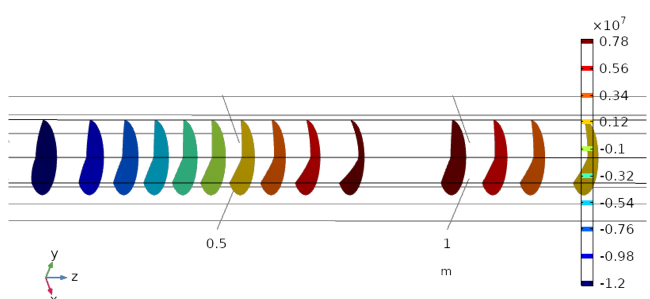

Fig. 8. The wave front (ISO surfaces of equal pressure) in a waveguide with "non- absolutely " rigid walls

you can use the expressions obtained for a wave propagating in a waveguide with rigid walls. In the work of L.D. Landau [2] the expression for the fraction of absorbed energy per unit distance for a wave propagating in a waveguide with rigid walls is obtained as:

$$\frac{\Delta W}{W}=\frac{\sqrt{\omega}}{\sqrt{2}Rc_0}\left[\sqrt{\eta}+\left(\frac{c_p}{c_v}-1\right)\sqrt{\chi}\right]. \quad (19)$$

Similar in physical sense reasoning in the problem of wave damping in a waveguide due to viscous friction forces in the near-surface layer was used in his works by S.N. Rzhevkin. For a cylindrical waveguide of radius *R*, an expression for the damping coefficient is given in [3]:

$$\gamma=k_0\phi\left|\frac{\sqrt{2}}{\sqrt{1+\sqrt{1+4\phi^2}}}\right|, \quad (20)$$

where $\phi=\frac{1}{2\omega\rho_0 R}\sqrt{2\rho_0\upsilon\omega}$ is the formula obtained earlier in the works of Stokes and Helmholtz.

For media with high thermal conductivity in the works of S.N. Rzhevkin [3], an additional correction to the absorption coefficient was obtained, which slightly increases the coefficient value. However, this and other possible additives do not significantly affect the final result of the formula (20).

The calculated values of the attenuation coefficient calculated using formulas (19) and (20) have some differences between them. The calculated values of the attenuation coefficient for wave propagation in the tube according to the formulas from the works of Rzhevkin S.N. and Landau L.D. 0.1÷0.18 [dB/m] were at frequencies 5000–6000 [Hz]. At the same time, measurements in the hydroacoustic waveguide showed a value (table 1) of approximately 0.22÷0.24 [dB/m]. The formulas (19–20) are obtained in the approximation of absolutely rigid walls and do not take into account exactly the mechanism of energy dissipation for the considered waveguide with real wall impedance. However, for the considered waveguide, it is possible to obtain an estimate with a high degree of accuracy.

Conclusion

In this paper, studies of wave processes in a cylindrical waveguide are performed. The propagation and attenuation of a plane wave in a waveguide with "imperfectly" rigid walls is considered. Analytical and numerical analysis of the change in the wave front is performed taking into account the final impedance of the waveguide walls. The estimation of wave absorption due to thermoacoustic energy conversion in boundary acoustic layers in the approximation of "absolutely" rigid walls is made.

Experimental results of energy dissipation for a real waveguide and calculated results of energy dissipation for a waveguide with absolutely rigid walls are shown. The calculated data correspond fairly accurately to the experimental results for the waveguide. However, the calculated data were lower than the experimental values. Taking into account the impedance on the inner wall of the waveguide will allow us to

Supplement the calculation model and increase the accuracy of calculating the dissipative losses of the acoustic system of the waveguide.

References

[1] Legusha F.F., Maslov V.L., Lebedev G.A., Chizhov V.Y. Energy dissipation on solid spherical particle in the field of travelling acoustic wave. X International conference navy and shipbuilding nowadays. Saint-Petersburg, 2019.

[2] Ландау Л.Д., Лифшиц Е.М. Гидродинамика. Главная редакция физико-математической литературы. Москва, 1988.

[3] Ржевкин С.Н. Курс лекций по теории звука. Издательство Московского университета. Москва, 1960.

[4] Исакович М.А. Общая акустика. Издательство «Наука». Москва 1973.