The Characteristics of Neutrosophic Pi-Generated Regular-Closed Sets in Neutrosophic Topological Spaces

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Abstract
The aim of this paper is to introduce and investigate Neutrosophic pi-generated regular-closed sets in Neutrosophic topological space (NTS). And study the connection between Neutrosophic π gr-closed sets and other Neutrosophic set classes is demonstrated. Furthermore, a new concept is researched and discussed of (NTS) known as a Neutrosophic $\pi gr - T_{1/2} -$ space. The goal of investigating the properties of these new notions of Neutrosophic open sets using examples, counter examples, and some of their fundamental results.

Keywords: Neutrosophic sets theory, Neutrosophic regular generated-closed set, Neutrosophic pi-generated $\alpha -$ closed, Neutrosophic pi-generated regular$-T_{1/2} -$ space.
Subject Classification: 06D72, 03E72.

1. Introduction

Topology is a traditional subject, with many different types of topological spaces introduced during the recent years as a generalization. Using L.A. Zadeh's[1] fuzzy sets, C.L. Chang[2] presented and created fuzzy topological space. Using Atanassov's[3] Intuitionistic fuzzy set, Coker[4] proposed the concepts of Intuitionistic fuzzy topological spaces. Salama [5]et al. introduced neutrosophic topological spaces (NTS). D. Andrijevic [6] proposed b-open sets in topological space in 1996, while R.Dhavaseelan[8] and SaiedJafari proposed Neutrosophic generalized closed sets in 1997. Smarandache [7] introduced neutrality, or the degree of indeterminacy, as a separate concept in 1998. He also based the Neutrosophic set on three topological spaces with Neutrosophic components (T- Truth, F -Falsehood ,I- Indeterminacy). Neutrosophic set it is non- classical set such as soft sets [9-14], fuzzy sets [15-21], nano sets [22], permutation sets [23-28], and others ([29,30]). The focus of this study is to present Neutrosophic π gr-closed sets and explore the relationship between them and other neutrosophic sets. in (NTS). Moreover, new class of (NTS) is researched and discussed in this work it is called Neutrosophic π gr-$T_{1/2}$ -space. Employing examples, counter examples, and some of their basic premises to investigate the features of these novel conceptions of Neutrosophic open sets.

2. Preliminaries:

We'll get the basic information from the sources [31-38] in this section.

Definition 2.1:

Assume that $\Psi \neq \varphi$, then $K = \{\langle \varepsilon, \gamma_K(\varepsilon),\ \rho_K(\varepsilon),\ r_K(\varepsilon)\rangle : \varepsilon \in \Psi\}$ is reported to be neutrosophic set (NS) , where γ_K, ρ_K , r_K are three fuzzy sets. Also, if $H = \{\langle \varepsilon, \gamma_H(\varepsilon),\ \rho_H(\varepsilon),\ r_H(\varepsilon)\rangle : \varepsilon \in \Psi\}$ is (NS). Then;

(1) $K \mathrel{\tilde{\subseteq}} H$ if and only if $\gamma_K(\varepsilon) \leq \gamma_H(\varepsilon)$, $\rho_K(\varepsilon) \geq \rho_H(\varepsilon)$ and $r_K(\varepsilon) \geq r_H(\varepsilon)$,

(2) $K \prod H = \{\langle \varepsilon,\ \min\{\gamma_K(\varepsilon), \gamma_H(\varepsilon)\},\ \max\{\rho_K(\varepsilon), \rho_H(\varepsilon)\}, \max\{r_K(\varepsilon), r_H(\varepsilon)\}\rangle : \varepsilon \in \Psi\}$,

(3)$K^c = \{\langle \varepsilon, r_K(\varepsilon), 1 - \rho_K(\varepsilon), \gamma_K(\varepsilon)\rangle : \varepsilon \in \Psi\}$,

(4) $K \coprod H = \{\langle \varepsilon, \max\{\gamma_K(\varepsilon), \gamma_H(\varepsilon)\}, \min\{\rho_H(\varepsilon), \rho_H(\varepsilon)\}, \min\{r_K(\varepsilon), r_H(\varepsilon)\ \}\rangle : \varepsilon \in \Psi\}$.

(5) if $\grave{\varepsilon} \in \Psi$, we say $f = \{\langle \varepsilon, \gamma_{\grave{\varepsilon}}(\varepsilon),\ \rho_{\grave{\varepsilon}}(\varepsilon),\ r_{\grave{\varepsilon}}(\varepsilon)\rangle : \varepsilon \in \Psi\}$ is a neutrosophic singleton set if $\gamma_{\grave{\varepsilon}}(\varepsilon) \neq 0$, when $\varepsilon = \grave{\varepsilon}$ and $\gamma_{\grave{\varepsilon}}(\varepsilon) = 0, \rho_{\grave{\varepsilon}}(\varepsilon) =\ r_{\grave{\varepsilon}}(\varepsilon) = 1$, when $\varepsilon \neq \grave{\varepsilon}$. Also, if f belong to K, we denote for that by $f \mathrel{\tilde{\in}} K$.

Definition 2.2:

Assume $\tau = \{f_j \big| j \in \Delta\}$ be a collection of neutrosophic sets (NSs) of Ψ. We say (Ψ, τ) is a neutrosophic topological space (NTS) if τ satisfies:

(1) $0_N = \{\langle \varepsilon, (0,1,1)\rangle : \varepsilon \in \Psi\} \in \tau$ and $1_N = \{\langle \varepsilon, (1,0,0)\rangle : \varepsilon \in \Psi\} \in \tau$.

(2) $f_m \prod f_k \in \tau, \forall\, f_m, f_k \in \tau$,

(3) $\coprod_{j \in \nabla} f_j \in \tau$ for any $\nabla \subseteq \Delta$. Moreover, if $f_j \in \tau$ we have f_j is neutrosophic open set (NOS) while ${f_j}^c$ is known as neutrosophic closed set (NCS) .

Definition 2.3:

Assume f is (NS), then

(1) $Ncl(f) = \prod\{f_j | f_j\ is\ (NCS)\ and\ \ f \mathrel{\tilde{\subseteq}} f_j\}$ and $Nint(f) = \coprod\{f_j | f_j\ is\ (NOS)\ and\ \ f_j \mathrel{\tilde{\subseteq}} f\}$, are neutrosophic closure and neutrosophic interior of f, respectively.

(2) $Nrcl(f) = \prod\{f_j | f_j\ is\ (NRCS)\ and\ \ f \mathrel{\tilde{\subseteq}} f_j\}$ and $Nrint(f) = \coprod\{f_j | f_j\ is\ (NROS)\ and\ \ f_j \mathrel{\tilde{\subseteq}} f\}$ are neutrosophic regular closure and neutrosophic regular interior of f, respectively.

(2) $N\alpha cl(f) = \prod\{f_j | f_j\ is\ (N\alpha CS)\ and\ \ f \mathrel{\tilde{\subseteq}} f_j\}$ and $N\alpha int(f) = \coprod\{f_j | f_j\ is\ (N\alpha OS)\ and\ \ f_j \mathrel{\tilde{\subseteq}} f\}$ are neutrosophic $\alpha -$ closure and neutrosophic $\alpha -$ interior of f, respectively.

(2) $Npcl(f) = \prod\{f_j | f_j\ is\ (NpCS)\ and\ \ f \mathrel{\tilde{\subseteq}} f_j\}$ and $Npint(f) = \coprod\{f_j | f_j\ is\ (NpOS)\ and\ \ f_j \mathrel{\tilde{\subseteq}} f\}$

are neutrosophic pre-closure and neutrosophic pre-interior of f, respectively.

(2) $Nscl(f) = \prod\{f_j | f_j\ is\ (NSCS)\ and\ \ f \tilde{\subseteq} f_j\}$ and $Nsint(f) = \coprod\{f_j | f_j\ is\ (NSOS)\ and\ \ f_j \tilde{\subseteq} f\}$ are neutrosophic regular closure and neutrosophic regular interior of f, respectively.

(2) $Nbcl(f) = \prod\{f_j | f_j\ is\ (NbCS)\ and\ \ f \tilde{\subseteq} f_j\}$ and $Nbint(f) = \coprod\{f_j | f_j\ is\ (NbOS)\ and\ \ f_j \tilde{\subseteq} f\}$ are neutrosophic regular closure and neutrosophic regular interior of f, respectively.

Definition 2.4:
Let K be a (NS) in (NTS). Then it is a neutrosophic π-open set ($N\pi OS$) if K =$\coprod\{H / H$ is (NROS) in (NTS)}

Definition:2.5

Let (Ψ,τ) be a (NTS) and K be neutrosophic set (NS) of Ψ . Then K is called

(i) a neutrosophic rg -closed set if $Ncl(K) \tilde{\subseteq} L$ whenever $K \tilde{\subseteq} L$ and L is ($N\pi OS$).

(ii) a neutrosophic π^* g-closed if $Ncl(N\mathrm{int}(K)) \tilde{\subseteq} L$ whenever $K \tilde{\subseteq} L$ and L is ($N\pi OS$).

(iii) a neutrosophic π gα-closed ($N\pi G\alpha CS$) if $N\alpha cl(K) \tilde{\subseteq} L$ whenever $K \tilde{\subseteq} L$ and L is ($N\pi OS$).

(iv) a neutrosophic π gp-closed ($N\pi GPCS$) if $Npcl(K) \tilde{\subseteq} L$ whenever $K \tilde{\subseteq} L$ and L is ($N\pi OS$).

(v) a neutrosophic π g $b-$closed ($N\pi GbCS$) if $Nbcl(K) \tilde{\subseteq} L$ whenever $K \tilde{\subseteq} L$ and L is ($N\pi OS$).

(vi) a neutrosophic π gs-closed ($N\pi GSCS$) if $Nscl(K) \tilde{\subseteq} L$ whenever $K \tilde{\subseteq} L$ and L is ($N\pi OS$).

3. Neutrosophic Pi-Generated Regular-Closed Sets ($N\pi GRCS$):

In the beginning define ($N\pi GRCS$) in (NTS), and then look at the link between ($N\pi GRCS$) and other (NSs) in (NTS).

Definition:3.1

Assume (Ψ,τ) is (NTS) and K is a (NS) of Ψ . We say K is a neutrosophic π gr-closed set ($N\pi GRCS$) in Ψ if $Nrcl(K) \tilde{\subseteq} L$ whenever $K \tilde{\subseteq} L$, where L is ($N\pi OS$) in Ψ.The family of all ($N\pi GRCSs$) of Ψ is denoted by $N\pi$GRC(Ψ,τ) .

Result :3.2

Any (NRCS) is($N\pi GRCS$), but not the other way around.

Example:3.3

Let $\Psi = \{l,j,n,m\}$ and H_i $(1 \le i \le 6)$ be (NSs), where
H_1 =$\{\langle l,(0.5,1,0.2)\rangle, \langle j,(0,0,1)\rangle, \langle n,(0,0.2,1)\rangle, \langle m,(0,1,0)\rangle\}$,

H_2 $=\{\langle l,(0,0,1)\rangle,\langle j,(0.3,1,0.4)\rangle,\langle n,(0,1,0)\rangle,\langle m,(0.6,0.3,1)\rangle\}$,
H_3 $=\{\langle l,(0.5,0,0.2)\rangle,\langle j,(0.3,0,0.4)\rangle,\langle n,(0,0.2,0)\rangle,\langle m,(0.6,0.3,0)\rangle\}$,
H_4 $=\{\langle l,(0,0,1)\rangle,\langle j,(0.3,1,0)\rangle,\langle n,(0,1,0)\rangle,\langle m,(0.6,0.3,1)\rangle\}$,
H_5 $=\{\langle l,(1,0,0)\rangle,\langle j,(1,0,0.4)\rangle,\langle n,(1,0,0)\rangle,\langle m,(1,0,0)\rangle\}$,
H_6 $=\{\langle l,(0.5,0,0.2)\rangle,\langle j,(0.3,0,0)\rangle,\langle n,(0,0.2,0)\rangle,\langle m,(0.6,0.3,0)\rangle\}$.
Now, let τ $=\{1_N,0_N,H_1,H_2,\dots,H_6\}$, then (Ψ,τ) is a (NTS). Here the (NS) H_5 is $(N\pi GRCS)$ but not (NRCS).

Remark:3.4

The notion of (NCS) and $(N\pi GRCS)$ are independent.

Example:3.5

By Example (3.3), we have the following:

(i) The (NS) $W=\{\langle l,(0,1,1)\rangle,\langle j,(0.4,1,1)\rangle,\langle n,(0,1,1)\rangle,\langle m,(0,1,1)\rangle\}=H_5^c$ of Ψ is (NCS) but not $(N\pi GRCS)$ in Ψ.

(ii) We have $K=\{\langle l,(1,0,0)\rangle,\langle j,(1,0,0.4)\rangle,\langle n,(1,0,0)\rangle,\langle m,(1,0,0)\rangle\}$ is $(N\pi GRCS)$ but not (NCS) in Ψ.

Remark: 3.6

The notion of (NGCS) and $(N\pi GRCS)$ are independent.

Example:3.7

See Example (3.3), we have the following:

(i) The (NS) $W=\{\langle l,(0,1,1)\rangle,\langle j,(0.4,1,1)\rangle,\langle n,(0,1,1)\rangle,\langle m,(0,1,1)\rangle\}$ of Ψ is (NGCS) but not $(N\pi GRCS)$ in Ψ.

(ii) We have $K=\{\langle l,(1,0,0)\rangle,\langle j,(1,0,0.4)\rangle,\langle n,(1,0,0)\rangle,\langle m,(1,0,0)\rangle\}$ is $(N\pi GRCS)$ but not (NGCS) in Ψ.

Theorem : 3.8

Any $(N\pi GRCS)$ is $(N\pi G\alpha CS)$, $(N\pi GPCS)$, $(N\pi GbCS)$, $(N\pi GSCS)$, $(N\pi GCS)$ and $(N\pi^* GCS)$ but not the other way around.

Proof: Straight forward.

Example : 3.9

See Example (3.3), (i)The (NS) $W=\{\langle l,(0,1,1)\rangle,\langle j,(0.4,1,1)\rangle,\langle n,(0,1,1)\rangle,\langle m,(0,1,1)\rangle\}$ of Ψ is $(N\pi G\alpha CS)$ and $(N\pi GCS)$ but not $(N\pi GRCS)$.

ii)The (NS) $F=\{\langle l,(0,0.2,1)\rangle,\langle j,(0,1,0.4)\rangle,\langle n,(0,1,1)\rangle,\langle m,(0,0.3,1)\rangle\}$ of a (NTS) Ψ is $(N\pi GbCS)$, $(N\pi GPCS)$ and $(N\pi GSCS)$ but not $(N\pi GRCS)$.

iii) We have $G = \{\langle l, (0,1,0.3)\rangle, \langle j, (0,0,1)\rangle, \langle n, (0,0.3,1)\rangle, \langle m, (0,1,0.2)\rangle\}$ of Ψ is $(N\pi^{*}GCS)$ but not $(N\pi GRCS)$.

Theorem :3.10

Any $(N\pi GRCS)$ is $(NRGCS)$.

Proof: Straight forward

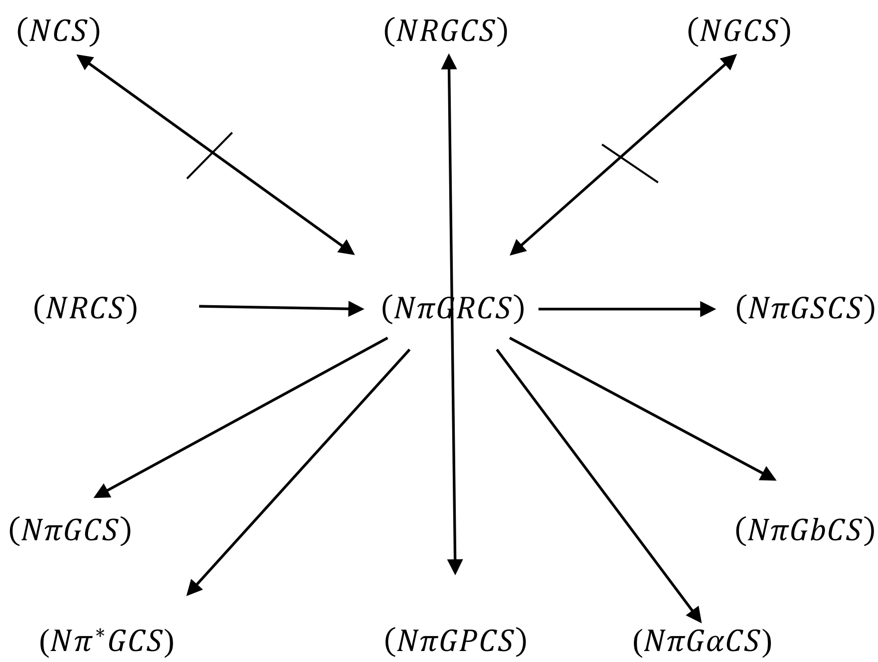
However, not the other way around for Theorem (3.10), see the following example.

Example:3.11

See Example (3.3), the (NS) $W = \{\langle l, (0,1,1)\rangle, \langle j, (0.4,1,1)\rangle, \langle n, (0,1,1)\rangle, \langle m, (0,1,1)\rangle\}$ of Ψ is $(NRGCS)$ but not $(N\pi GRCS)$ in Ψ.

Remark:3.12

The following graphic depicts the relationship between $(N\pi GRCS)$ and others (NSs):



Remark:3.13

If each of H and D is $(N\pi GRCS)$, then $H \coprod D$ is also $(N\pi GRCS)$.

Remark:3.14

If each of H and D is $(N\pi GRCS)$, then $H \prod D$ is not necessary to be $(N\pi GRCS)$.

Example:3.15

By example (3.3), The (NSs) I $=\{\langle l,(0.5,0,0)\rangle,\langle j,(0.4,1,0)\rangle,\langle n,(0.7,0,0)\rangle,\langle m,(0.6,0,0)\rangle\}$, and $J=\langle l,(0,0,1)\rangle,\langle j,(0.3,0,0.4)\rangle,\langle n,(0,1,0)\rangle,\langle m,(0.6,0.3,1)\rangle\}$ are neutrosophic π gr-closed sets in Ψ but their intersection H_2 $=\{\langle l,(0,0,1)\rangle,\langle j,(0.3,1,0.4)\rangle,\langle n,(0,1,0)\rangle,\langle m,(0.6,0.3,1)\rangle\}$ is not $(N\pi GRCS)$ in Ψ.

Theorem:3.16

If K is $(N\pi OS)$ and $(N\pi GRCS)$, then it is $(NRCS)$.

Proof: Let K be $(N\pi OS)$ and $(N\pi GRCS)$. Thus $Nrcl(K)\tilde{\subseteq} K$. But $K\tilde{\subseteq} Nrcl(K)$. Hence $Nrcl(K)= K$ and then K is $(NRCS)$.

Corollary :3.17

If K is $(N\pi OS)$ and $(N\pi GRCS)$, then it is (NCS).

Proof: By (Theorem 3.16) we have K is $(NRCS)$ and hence K is (NCS) in Ψ.

Theorem:3.18

If K is $(N\pi GRCS)$ of a (NTS) Ψ and $K\tilde{\subseteq} M \tilde{\subseteq} Nrcl(K)$. Then B is also $(N\pi GRCS)$ of ψ.

Proof: Let K be a $(N\pi GRCS)$ in Ψ and $M\tilde{\subseteq} T$, where T is $(N\pi OS)$. Because $K\tilde{\subseteq} M\tilde{\subseteq} T$ and K is $(N\pi GRCS)$, thus $Nrcl(K)\tilde{\subseteq} T$. Given $M\tilde{\subseteq} Nrcl(K)$. Therefore, $Nrcl(M)\tilde{\subseteq} Nrcl(K)\tilde{\subseteq} T$. So $Nrcl(M)\tilde{\subseteq} T$. Then M is $(N\pi GRCS)$.

Theorem:3.19

If K is $(N\pi GRCS)$, then $Nrcl(K)-K$ has no non-empty $(N\pi CS)$.

Proof: Assume F is a non-empty $(N\pi CS)$ with $F\tilde{\subseteq} Nrcl(K)-K$. The above implies $F\tilde{\subseteq}\Psi-K$. Since K is$(N\pi GRCS)$, $\Psi-K$ is $(N\pi GROS)$. Since F is $(N\pi CS)$, $\Psi-F$ is $(N\pi OS)$. Since $Nrcl(K)\tilde{\subseteq}\Psi-F$, $F\tilde{\subseteq}\Psi-Nrcl(K)$. Thus $F\tilde{\subseteq}\Phi$, which is a contradiction. The above implies $F=\Phi$ and hence $Nrcl(K)-K$ does not contain non-empty $(N\pi CS)$.

Corollary:3.20

Let K be a $(N\pi GRCS)$. Then K is $(NRCS)$ iff $Nrcl(K)-K$ is $(N\pi CS)$.

Proof : Let K be $(N\pi GRCS)$. Then $Nrcl(K)=K$ and $Nrcl(K)-K=\Phi$, which is $(N\pi CS)$. On the other hand, let us suppose that $Nrcl(K)-K$ is $(N\pi CS)$. Then by theorem 3.19, $Nrcl(K)-K=\Phi$. The above implies $Nrcl(K)=K$. Hence K is $(NRCS)$.

4. Neutrosophic Pi-Generated Regular-Open Sets $(N\pi GROS)$:

In this paragraph, we will define and discuss the conception of $(N\pi GROS)$ in (NTS).

Definition: 4.1

A (NS) K is called a neutrosophic π gr-open set $(N\pi GROS)$ in a (NTS) (Ψ,τ), if the relative complement K^c is $(N\pi GRCS)$ in (Ψ,τ) and the family of all $(N\pi GROSs)$ in a (NTS) (Ψ,τ) is denoted by $N\pi GRO((\Psi,\tau))$

Remark: 4.2

Any (NS) K of Ψ satisfies $Nrcl(\Psi - K)=\Psi - Nr\text{int}(K)$.

Theorem: 4.3

K is $(N\pi GROS)$ in (NTS) Ψ iff $F \tilde{\subseteq} Nr\text{int}(K)$ whenever F is $(N\pi CS)$ and $F \tilde{\subseteq} K$.

Proof: Let K be $(N\pi GROS)$ and F be $(N\pi CS)$ with $F \tilde{\subseteq} K$. Then $\Psi - K \tilde{\subseteq} \Psi - F$. where $\Psi - F$ is $(N\pi OS)$. Since K is $(N\pi GROS)$, $\Psi - K$ is $(N\pi GRCS)$. Then $Nrcl(\Psi - K) \tilde{\subseteq} \Psi - F$. Since $Nrcl(\Psi - K) = \Psi - Nr\text{int}(K) \Rightarrow \Psi - Nr\text{int}(K) \tilde{\subseteq} \Psi - F$. Hence $F \tilde{\subseteq} Nr\text{int}(K)$.

Conversely, let F be $(N\pi CS)$ and $F \tilde{\subseteq} K$ implies $F \tilde{\subseteq} Nr\text{int}(K)$. Let $\Psi - K \tilde{\subseteq} U$, where $\Psi - U$ is $(N\pi CS)$. By hypothesis, $\Psi - U \tilde{\subseteq} Nr\text{int}(K)$. Hence $\Psi - Nr\text{int}(K) \tilde{\subseteq} U$. since $Nrcl(\Psi - K)= \Psi - Nr\text{int}(K)$.The above implies $rcl^S(\Psi - K) \tilde{\subseteq} U$, whenever $\Psi - K$ is $(N\pi OS)$. Then $\Psi - K$ is $(N\pi GROS)$ in Ψ.

Theorem: 4.4

If $Nr\text{int}(K) \tilde{\subseteq} B \tilde{\subseteq} K$, and K is $(N\pi GROS)$, then B is $(N\pi GROS)$.

Proof: Given $Nr\text{int}(K) \tilde{\subseteq} B \tilde{\subseteq} K$. Then $\Psi - K \tilde{\subseteq} \Psi - B \tilde{\subseteq} Nrcl(\Psi - K)$. Since K is $(N\pi GROS)$, $\Psi - K$ is $(N\pi GRCS)$. Then $\Psi - B$ is also $(N\pi GRCS)$. Hence B is $(N\pi GROS)$.

Remark: 4.5

Let K be (NS) of (NTS) Ψ, then $Nr\mathrm{int}\,(Nrcl(K)-K)=\Phi$.

Theorem: 4.6

If $K \tilde{\subseteq} \Psi$ is $(N\pi GRCS)$, then $Nrcl(K)-K$ is $(N\pi GROS)$.

Proof: Let K be $(N\pi GRCS)$ and T be a $(N\pi CS)$ with $T \tilde{\subseteq} Nrcl\ (K)-K$. Therefore $T=\Phi$. So, $T \tilde{\subseteq}$ $(Nr\mathrm{int}\,(K)-K)$. Hence $Nrcl\ (K)-K$ is $(N\pi GROS)$.

Theorem: 4.7

If each of H and D is $(N\pi GROS)$, then $H \prod D$ is also $(N\pi GROS)$.

Proof: Straight forward.

Remark:4.8

If each of H and D is $(N\pi GROS)$, then $H \coprod D$ is not necessary to be $(N\pi GROS)$.

Example: 4.9

Let $C=\{\langle l,(0.5,0.2,1)\rangle,\langle j,(0.4,0,0.3)\rangle,\langle n,(0,0.1,1)\rangle,\langle m,(0,1,1)\rangle\}$ and $B=\{\langle l,(0.5,0.2,0.2)\rangle,\langle j,(0,0,0.3)\rangle,\langle n,(0,0.1,1)\rangle,\langle m,(1,1,0)\rangle\}$ are two $(N\pi GROSs)$. Then $C \coprod B = D = \{\langle l,(0.5,0.2,0.2)\rangle,\langle j,(0.4,0,0.3)\rangle,\langle n,(0,0.1,1)\rangle,\langle m,(1,1,0)\rangle\}$ is not $(N\pi GROS)$ in (Ψ,τ).

5. NEUTROSOPHIC π GR-$T_{1/2}$-SPACE$\left(\boldsymbol{N\pi GR T_{1/2}-S}\right)$:

Let us introduce and study the notion of $\left(N\pi GR T_{1/2}-S\right)$.

Definition: 5.1

A (NTS)(Ψ,τ) is a neutrosophic $\pi-$generated regular-$T_{1/2}$-space $\left(N\pi GR T_{1/2}-S\right)$ if every $(N\pi GRCS)$ is $(NRCS)$.

Theorem: 5.2

For a (NTS)(Ψ,τ), the following conditions are equivalent.

(i) The (NTS)(Ψ,τ) is $\left(N\pi GR T_{1/2}-S\right)$.

(ii) Any singleton of Ψ is either $(N\pi CS)$ or $(NROS)$.

Proof:

(i) $\Rightarrow$(ii): Let L be a neutrosophic singleton set in Ψ and let L be not $(N\pi CS)$. Then Ψ - L is not $(N\pi OS)$ and hence Ψ - L is trivially $(N\pi GRCS)$. Since in a $\left(N\pi GR T_{1/2}-S\right)$, every $(N\pi GRCS)$ is $(NROS)$.Then Ψ - L is $(NRCS)$. Hence L is $(NROS)$.

(ii) $\Rightarrow$(i) : Assume that any singleton of a (NTS) Ψ is either $(N\pi CS)$ or $(NROS)$. Let L be a $(N\pi GRCS)$ in Ψ. Obviously, $L \tilde{\subseteq} Nrcl(L)$. To prove $Nrcl(L) \tilde{\subseteq} L$, let $D \tilde{\in} Nrcl(L)$, where D is singleton set, we want to show $D \tilde{\in} L$. Now, we have two cases (since D is either $(N\pi CS)$ or $(NROS)$).

Case (i): when D is $(N\pi CS)$, let D be not belong to L. Hence $D \tilde{\subseteq}$ $Nrcl(L)$-L, which is a contradiction to the fact that $Nrcl(L)$ - L has not any non-empty subset and it is $(N\pi CS)$. Thus, $D \tilde{\in} L$. So $Nrcl(L) \tilde{\subseteq} L$. Then L is $(NRCS)$ and hence every L $(N\pi GROS)$ is $(NRCS)$. Hence the (NTS)Ψ is $\left(N\pi GRT_{1/2} - S\right)$.

Case(ii): when D is $(NROS)$. in Ψ, we have $D \prod L \neq \Phi$. (since $D \in Nrcl(L)$). Hence $D \tilde{\in} L$. Therefore, $Nrcl(L) \tilde{\subseteq} L$. Then $Nrcl(L) = L$, thus L is $(NRCS)$ and hence Ψ is$\left(N\pi GRT_{1/2} - S\right)$.

Theorem: 5.3

(i) $SRO((\Psi,\tau)) \tilde{\subseteq} S\pi RGO((\Psi,\tau))$

(ii) A (NTS)(Ψ,τ) is $\left(N\pi GRT_{1/2} - S\right)$ iff $NRO(\Psi,\tau) = N\pi GRO(\Psi,\tau)$

Proof:

(i) Let K be $(NROS)$. Then $\Psi - K$ is $(NRCS)$ and so $(N\pi GRCS)$. Hence K is $(N\pi GROS)$ and hence $NRO(\Psi,\tau) \tilde{\subseteq} N\pi GRO(\Psi,\tau)$

(ii) Necessity: assume (Ψ,τ) is $\left(N\pi GRT_{1/2} - S\right)$ and $K \in N\pi GRO(\Psi,\tau)$. Then $\Psi - K$ is $(N\pi GRCS)$. $\Psi - K$ is $(NRCS)$ [Since (Ψ,τ) is $\left(N\pi GRT_{1/2} - S\right)$]. The above implies K is $(NROS)$ in Ψ. Hence $N\pi GRO(\Psi,\tau) = NRO(\Psi,\tau)$.

Sufficiency: Let $N\pi GRO(\Psi,\tau)$=$NRO(\Psi,\tau)$ and let K be $(N\pi GRCS)$. Then $\Psi - K$ is $(N\pi GROS)$. Thus $\Psi - K \in NRO(\Psi,\tau)$ and hence K is (NROS).

4. Conclusion

The concepts of Neutrosophic pi-generated regular-closed sets and Neutrosophic $\pi gr - T_{1/2} -$ space, both of which are fundamental results for further research on Neutrosophic topological spaces, are introduced in this work, with the goal of investigating the properties of these new notions of Neutrosophic open sets using examples, counter examples, and some of their fundamental results. I believe that the discoveries in this paper will aid and encourage additional research into Neutrosophic soft topological spaces in order to develop a generic framework for their applications in compactness, connectedness, separation axioms, and other areas.

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