Radon Transform and Dynamic Stochastic Resonance based Technique for Line Detection from Noisy Images

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***Abstract*—The Radon transform is an important transform to detect line feature from the noisy image. Radon transform can transform two-dimensional images (with noisy or disturbed lines) into a domain of possible parameters of line, where each line in the image will give a peak position at the corresponding parameters of the line. It has led to many line detection applications within image processing, computer vision, earthquake engineering *etc*. When the lines are subjected to very high background noises, Radon transform alone is not so effective. Here, in this paper, we propose dynamic stochastic resonance (DSR) based Radon transform for weak line extraction. The DSR is an iterative process that tunes the coefficient of Radon transform so that we may get the enhanced lines of the image. We compare our proposed method with the results of the Gaussian low pass filter. The proposed technique adopts local adaptive processing, and it significantly enhances the line feature of an image. Experimental results are also given to show the effectiveness of the proposed method.**

***Index Terms*—Radon Transform, Dynamic Stochastic Resonance, Noise, Denoising.**

I. Introduction

The extraction of the lines of an image is one of the basic need in the image processing application where line features are closely related to the edges of an image. The edge is defined as a boundary between two regions with relatively distinct gray levels. Guido *et al.* [1] introduce an algorithm to detect a line which is based on weighted minimum mean square error. This algorithm involves the use of matrices and uses a set of matrix operations such as transpose and multiplications. Marco *et al.* [2] developed a set of algorithms to detect a line based on a general formulation of a combinatorial optimization problem. In the algorithm, a lot of expensive operations like multiplication, power and exponent functions are used. However, algorithms perform faster. Guru *et al.* [3] proposed an algorithm based on small eigenvalue analysis to extract straight line segments in an image. The algorithm depends on scanning the input image from the top left corner to the bottom down the corner with a moving mask matrix using the small eigenvalue analysis and pixel connectivity. Also, this algorithm is improved by Lee *et al.* [4] and it consists of two significant steps. The first step is labeling the edge of the image. The second step is applying the principal components analysis for each labeled edge. However, Kazuhito *et al.* [5] proposed a high-speed line detection method using the Hough transform in the local area, which consists of thinning and thresholding as a major operation. There are many existing techniques adopted to improve the performance of the Hough transform. Dahyot *et al.* [6] introduced the concept of statistical Hough transform, where he extended the formulation to continuous kernel estimates. Furthermore, Dahyot *et al.* discussed the robustness and insensitive of the estimated density to noise and the choice of the origin of the spatial coordinates [6].

Vongioi *et al.,* [7] proposed the improved Hough transform which can implement the automatic search and fill peak points. However, this algorithm has a high demand for image preprocessing; otherwise, it causes the mass error accumulation. Kyrki *et al.* [8] discussed an improved line detection algorithm combining the local and the global detection which improved the speed effectively and minimizes memory occupation. But this algorithm had weak robustness [9].

Here, we take a noisy line image, and after subtracting mean value from image pixels, we apply Radon transform and then convert it into the frequency domain using DCT. After that, we apply the DSR iterative equation. This DSR system enhances the energy of the noisy line of the image. It also suppresses the noise because the parameter of the DSR system has been calculated with the condition of maximization of SNR.

Organization of the paper: The rest of the conceptual part is arranged as follows. Section II deals with the basic mathematical framework. Implementation, results and discussions are presented in Section III. We make concluding remarks in Section IV.

II. Preliminary Mathematics

Here, in this section, we discuss the basic mathematics of dynamic stochastic resonance (DSR) and Radon transform.

A. *Dynamic Stochastic Resonance*

The concept of SR was first proposed to explain the recurrence of the ice age by Benzi et al. [10]. The mechanism of SR is based on the addition of a moderate amount of noise. It was traditionally believed that the presence of noise could only make a system worse. However, recent studies have shown that in non-linear systems, noise can induce more ordered regimes and increase the signal-to-noise ratio (SNR), and noise can be used to play a productive role in enhancing the weak input signal [11]–[13]. A concept of dynamic stochastic resonance (DSR) that uses noise to improve the performance of a system has been used for different image and signal processing applications such as image enhancement [14], [15], edge detection [16], de-noising [17], encryption [18],and watermarking [19]–[22]. The SR mechanism shows that at lower noise intensities the weak signal is unable to cross the threshold, thus giving a very low SNR, for large noise intensities the output is dominated by noise, also leading to a low signal to noise ratio.

The bistable model is conventionally used by the physicists to explain DSR phenomenon. A classic one-dimensional non-linear dynamic system that exhibits SR is modeled with the help of Langevin equation of motion which is given in Eq. (1).

$$m\frac{d^2x(t)}{dt^2}+\gamma\frac{dx(t)}{dt}=-\frac{dU(x)}{dx}+\sqrt{D}\xi(t). \tag{1}$$

Eq. (1) describes the motion of a particle of mass m moving in the presence of friction (γ). The restoring force is expressed as the gradient of bistable potential function $U(x)$. D is the noise variance and $\xi(t)$ is an additive zero mean stochastic fluctuation (noise). If the double well system is heavily damped, the inertial term $m\frac{d^2x(t)}{dt^2}$ can be neglected. $U(x)$ is a bistable quartic potential given in Eq. (2).

$$U(x)=-a\frac{x^2}{2}+b\frac{x^4}{4}, \tag{2}$$

where, a and b are positive bistable double-well parameters. The $x_m=\pm\sqrt{\frac{a}{b}}$ are separated by a barrier of height and $\Delta U=\frac{a^2}{4b}$, when the $\xi(t)$ is zero. Eq. (1) can be normalized by the constant term γ. Now, the addition of a periodic input signal B $\sin(\omega t)$ to the bistable system makes it time-dependent whose dynamics are governed by Eq. (3).

$$\frac{dx(t)}{dt}=-\frac{dU(x)}{dx}+B\sin(\omega t)+\sqrt{D}\xi(t), \tag{3}$$

where B and ω are the amplitude and frequency of the periodic signal respectively.

The most common quantifier of SR is SNR whose expression for DSR as derived in [23] is given below.

$$SNR=\left[\frac{4a}{\sqrt{2}\left(\sigma_0\sigma_1\right)^2}\right]exp\left(\frac{-a}{2\sigma_0^2}\right), \tag{4}$$

where σ_1 is the standard deviation of the noise in the SR-based system and σ_0 is the internal noise standard deviation of the original bistable system. In [20], these double well parameters a and b have been obtained by maximization of the SNR expression of DSR. This gives $a=2\sigma_0^2$ for obtaining optimum SNR. To ensure that the low contrast signal is subthreshold, a condition for the value of parameter b has been derived. The value of b is obtained as

$$b<\frac{4a^3}{27}. \tag{5}$$

For computational implementation of DSR in digital images, Eq. 3 is discretized into Eq. (6).

$$x\left(n+1\right)=x(n)+\Delta t\left[\left(ax(n)-bx^3(n)\right)+Input\right]. \tag{6}$$

Note that Input=$B\sin\left(\omega t\right)+\sqrt{D}\xi(t)$ denotes the sequence of the input signal and noise with the initial condition being $x(0)=0$. The final stochastic simulation is obtained after the certain number of iterations.

B. Radon Transform

The Radon transform computes the line integrals from multiple sources along parallel paths in a certain direction. $g(q,\rho)$ is the line integral of the image intensity $f(x,y)$ along a line l and that is the distance from the origin, and it is the angle from X-axis to the normal direction of the line.

$$g(\theta,\rho)=\int f(x,y)dl. \tag{7}$$

Radon transform is defined in continuous domain as follows.

$$R(\theta,\rho)=\int\int f(x,y)\delta(\rho-x\cos\theta-y\sin\theta)dxdy, \tag{8}$$

where $f(x,y)$ is the grayscale image and $\delta(x)$ is given as follows.

$$\delta(x)=\begin{cases}1, k=0,\\0, k\neq 0.\end{cases} \tag{9}$$

We can say that Radon transform detects the line by integrating image pixels in all possible direction in an image. Let the integral of the image function $f(x,y)$ along the line $\rho=x\cos\theta+y\sin\theta$ is denoted by $R(\theta,\rho)$. Each line will give the peak in the transform domain which corresponds to a line that is brighter than the background. Thus, line detection in the image is simplified as the detection of peaks and through in transform domain.

III. Implementation, Results & discussion

The steps of line detection are as in Algorithm 1. The experiments are carried out to evaluate the performance of proposed technique of line detection. We have taken an image and then we add different salt & pepper noise for making the original noisy image. For the simulation, all experiments are conducted on MATLAB 2014b with 4GB RAM and Intel(R) Core(TM)i3-4130 CPU 3.40 GHz. The parameters of the DSR are taken as follows. $m=10^{-15}$, $a=2m((std(R'(\theta)))^2)$, $t=0.002$, and $b=(4(a^3)/27)$.

A. Quantitative Parameters

We consider noise variance (NV), noise mean value (NMV) and noise standard deviation (NSD). Furthermore, we consider mean square difference (MSD) as other crucial parameter.

1) Noise mean value (NMV), noise variance (NV) and noise standard variance (NSV) can be mathematically expressed as follows.
   a) $NMV=\frac{\sum_{i,j}I_{out}(i,j)}{I\times J}$,
   b) $NV=\frac{\sum_{i,j}(I_{out}(i,j)-NMV)^2}{I\times J}$,
   c) $NSD=\sqrt{NV}$,

   where I_{out} is the output image with dimension $I\times J$. A lower value of variance gives a clearer image although it depends on the intensity.
2) Mean square difference (MSD) indicates the average difference of the pixels throughout the image where I_{out}

Algorithm 1 Proposed DSR based Line Detection

procedure GRAY SCALE LINE IMAGE IS TAKEN AND ADD THE NOISE INTO THE IMAGE(X)

Mean value of image pixels is calculated, and this value is subtracted by every pixel of the image so that we can make the image rotation invariant.

$$f^n(x', y') = f(x, y) - mean(f(x, y)), \qquad (10)$$

where $f(x, y)$ is the gray scale image and $f^n(x, y)$ is the processed image after subtracting mean value from original.

Radon transform is applied on the processed image $f^n(x, y)$ for different θ angles given by following equation,

$$R'(\theta) = \int_{-\infty}^{+\infty} f^n(x' \cos\theta - y' \sin\theta, x' \sin\theta + y' \cos\theta) dy'. \qquad (11)$$

Here, $R'(\theta)$ is the Radon transform coefficients.

Discrete cosine transform (DCT) is calculated after Radon transform for converting image pixel into frequency domain

$$R'_{dct}(\theta) = DCT(R'(\theta)) \qquad (12)$$

Now applying the DSR iterative equation on DCT coefficient for enhancing the energy of line pixel and reducing the noise effect by DSR based double well system. Putting Radon transform in Eq. 6, we can write the equation as follows.

$$R_{dsr}(n+1) = R_{dsr}(n) + (\Delta t) \times \left[aR_{dsr}(n) - bR^3_{dsr}(n) + R'_{dct}(\theta)\right] \qquad (13)$$

This is an iterative process. Here, a and b are double good parameter selection of these parameters given above and n is the number of iteration and we consider $n = 25$. $R_{dsr}(0) = 0$ is considered.

end procedure

is the output image and I_{in} is the original image with salt & pepper noise. It can be defined as follows.

$$MSD = \frac{\sum_{i,j} \left(I_{out}(i, j) - I_{in}(i, j)\right)^2}{I \times J}$$

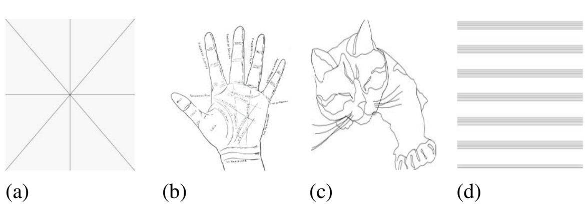

Fig. 1: Four different input images with distinct edges information. (a) Test-1, (b) Test-2, (c) Test-3, (d) Test-4.

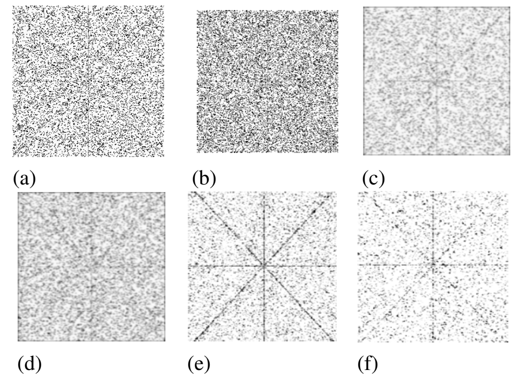

Fig. 2: Images (a) & (b) represent the noisy input images, however, (c)–(f) are the output images. (a) var=0.3, (b) var=0.5, (c) output of LPF for (a) image, (d) output of LPF for (b) image, (e) output of **Proposed DSR** for (a) image, (f) output of **Proposed DSR** for (b) image.

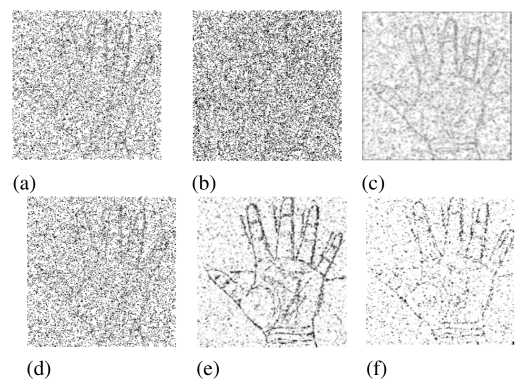

Fig. 3: Images (a) & (b) represent the noisy input images, however, (c)–(f) are the output images. (a) var=0.3, (b) var=0.5, (c) output of LPF for (a) image, (d) output of LPF for (b) image, (e) output of **Proposed DSR** for (a) image, (f) output of **Proposed DSR** for (b) image.

B. Discussion

Silent feature of our proposed line detection method discuss here. In Fig. 2 and Fig. 3, we have shown the outcomes of the proposed method and [24] when the input images are noisy images. The Fig. 2(e) and Fig. 2(f) are the proposed output for Fig. 2(a) and Fig. 2(b) noisy inputs. It can be seen that our proposed method can detect and extract the lines clearly with compare to [24]. Similarly, we can discuss for the Test-2 images, as shown in Fig. 3.

TABLE I: different assessment parameters results on noise (variance)= 0.30

	Gaussian LPF [24]				**Proposed DSR based method**			
Image	MSD	NMV	NV	NSD	MSD	NMV	NV	NSD
Test-1	167.13	201.25	0.002	0.0490	205.36	195.93	0.0005	0.0235
Test-2	164.91	205.21	0.002	0.0433	206.37	201.05	0.0005	0.0225
Test-3	173.68	202.01	0.001	0.9335	215.62	198.03	0.0006	0.0250
Test-4	167.02	201.10	0.002	0.0457	216.92	193.33	0.0010	0.0315

The quantitative parameters such as MSD, NMV, NV, and NSD can be studied in Table I. Our proposed method has much better capability compared to others to detect the lines in a noisy environment. The features and discussion of the benefits of DSR have been provided below.

1) Mechanism of Dynamic Stochastic Resonance for Line Detection from noisy image: The basic mechanism of DSR is to improve the performance of traditional line detector. The DSR iteration tunes the DCT based Radon transform coefficients. It increases the energy of line pixels present in the image. Our technique is to detect line from heavy background noise images. Noise may occur in digital images for several reasons. The signal is usually corrupted by noise which is generally salt & Pepper noise or maybe additive Gaussian noise. In this noise scenario, the Radon transform is unable to detect lines from the image. Therefore, in our technique, we used the combination of Radon transform and dynamic stochastic resonance. The coefficients are tuned in such a way that the energy of the information part increases and the effect of noise is suppressed. For this, we used an adaptive process for selection of parameters of DSR, which is calculated by maximization of SNR. DSR iterative equation given in Eq. 6 tunes the DCT coefficients of Radon transform.

IV. Conlusion

We have seen from results that proposed line detection algorithm works well in case of heavy background noise. The results of the proposed algorithm have been compared with conventional Gaussian Low pass filter (LPF) on real-life images with noise. It is to be noted that on all the real-life images considered; the proposed algorithm produced fairly good results on different noise intensity values.

In the current line detection scenario, lines can be considered to be a weak signal as it is statistically invisible when the image is subjected to noise. The image apart from the lines can be considered as noise. Here, stochastic fluctuation (noise) can be given to the transformed coefficients of the noisy image so that its distribution varies in such a way that at some optimum noise density, the Radon coefficient can jump from the noisy state to the enhanced state. The results suggested that the Radon coefficients get enhanced and would be easily detected the line features of the image. The experimental results in Table I, Fig. 2 and Fig. 3 show that our method is very effective in-line detection for many images.

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