# An Optimization Model of a Retailer and a Manufacturer in a Green Supply Chain 

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# An Optimization Model of a Retailer and a Manufacturer in a Green Supply Chain 

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#### Abstract

In recent years, the development of a green supply chain model that takes sustainability into consideration has become an urgent issue in corporate management and policy operation. For example, in the fashion industry, the proper circulation of not only used items purchased by consumers, but also unsold items generated in retail stores is one of the most important issues. In this study, we examine a supply chain model for the appropriate circulation of unsold new products from retail outlets. Specifically, we considered a supply chain model in which a retailer's inventory is divided into two states under stochastic demand fluctuations: new and old items, and the unsold old items are collected and reproduced by a manufacturer. By formulating this model as a Markov decision process, the optimal decisions regarding the retailer's ordering policy and the retail price and the manufacturer's wholesale price are obtained. Optimal investment and appropriate institutional design to reduce $\mathrm{CO}_{2}$ emissions generated in a supply chain are also considered. Specifically, we examine the decision of green investment in manufacturing technology to reduce $\mathrm{CO}_{2}$ emissions when a manufacturer produce items. And we examine the design of a carbon tax system to control $\mathrm{CO}_{2}$ emissions. Sensitivity analysis on the carbon tax system shows that raising the carbon tax rate increases the optimal retail price, the optimal wholesale price, and the optimal green investment.


Keywords: Supply chain management, Green supply chain, Markov decision process, Green investment, Carbon tax

## 1 Introduction

Recently, the development of a supply chain (SC) model that takes sustainability into consideration from the perspective of environmental conservation has become an urgent issue for companies and governments.

In [1], optimal production, ordering, and green investment decisions in a SC model under uncertainty about the occurrence of defective items and demand fluctuations are obtained by formulating a Markov decision process (MDP) model. Here, a manufacturer and a retailer receive penalty costs for $\mathrm{CO}_{2}$ emissions. the manufacturer makes green investment in production technology to reduce $\mathrm{CO}_{2}$ emissions. However, the study only covers the traditional forward chain model.
The study also examines the SC that produces apparel items. In the fashion industry,
"Sustainable Fashion" initiatives are flourishing. In considering the SC model recommended by Sustainable Fashion, we would like to note, for example, the current situation in which many unsold items from retail stores are sold out as clearance items. This is because, while it reduces items disposal in stores, it also encourages consumers to make unnecessary purchases, resulting in overconsumption. In fact, in January 2022, France's circular economy law (Loi Anti-Gaspillage pour une Économie Circulaire) banned fashion companies from disposing of unsold new clothing by incineration or landfill [2].

For example, there is the buyback model, in which a seller makes a supplier buy back unsold items. There are also many studies on a closed-loop SC as the SC model that recycles resources through reproduction [3]. Note, however, that this model does not involve unsold items in stores, but rather reproduction through the collection of used items.

Other effective approaches to unsold items in the SC is to optimize the retail price of items. [4] discusses the case of Zara, one of the fast fashion retailers. According to the study, the implementation of the system of price optimization of unsold items and the associated revenue forecasting in stores worldwide increased sales at Zara by about $6 \%$ ( $\$ 90$ million). In addition, [5] divides the inventory of items into two states, new items (just delivered) and old items (unsold and imminently disposed of), and find the optimal ordering policy with price differentiation between the two items.
In this study, a retailer's inventory was divided into two states one for new items and the other for old items under stochastic demand fluctuations. Then we consider a green supply chain (GSC) in which unsold old items from a retailer's inventory are collected, and a manufacturer reproduces items using collected items. Given a wholesale price, the retailer's inventory condition is formulated as a MDP model, and the optimal decisions on order quantity, retail price, wholesale price, and green investment are obtained to maximize the expected reward of a retailer and manufacturer, respectively. We also perform a sensitivity analysis on an appropriate carbon tax system to reduce $\mathrm{CO}_{2}$ emissions generated in the production process.

## 2 Model Description and Formulation

The model is an individual optimization model in which a retailer and a manufacturer aim to maximize their respective finite period total expected rewards in $(T+1)$ periods.

Given the manufacturer's wholesale price $w$, the optimal order quantity $a_{t}$ and retail price $r$ in each period that maximizes the total expected reward are determined using MDP model for a retailer's inventory state. Based on the probability distributions for $a_{t}$ and collected items $y_{t}$ at that time, a manufacturer determines the optimal $w$ and green investment $G$ that maximizes its own total expected reward.

### 2.1 Description of Supply Chain Model

The model diagram and the flow in the period are shown in Fig. 1 and Fig. 2.
Fig. 1 shows that the model is a decentralized SC consisting of a manufacturer and a retailer. The following assumptions are considered.

- The model transitions in discrete time. The number of periods is $T+1$ (finite).
- The quantity demanded for items sold by a retailer in each period is defined by the random variable $D_{t}$ denoted by $d_{t}$ and follows the probability distribution $P\left(D_{t}=d_{t}\right)$.
- A retailer cannot hold more than three periods of inventory of delivered items, and any unsold items after two periods of delivery is collected by a manufacturer as raw material for remanufacturing in the next period. The standard quantity for collection is set, and any unsold items that exceed this quantity are not collected but disposed of.
- For a retailer's inventory, old items are purchased in preference to new items.
- A retailer cannot place orders in excess of the maximum inventory capacity.
- A manufacturer produces one unit of item for each unit of raw material.
- A manufacturer determines the green investment in manufacturing technology before the start of the sales period.
- A manufacturer is responsible for the $\mathrm{CO}_{2}$ emissions generated in production. The regulatory standard for $\mathrm{CO}_{2}$ emissions is set and a manufacturer is charged the penalty cost of the carbon tax per unit emission $[\mathrm{kg}]$ for the excess $\mathrm{CO}_{2}$ emissions.


Fig. 1. Schematic diagram of the model.


Fig. 2. Flow during the period.

Based on Fig. 2, the flow during the period is described below.
0. Prior to the start of the period, a manufacturer determines the wholesale price of the finished item and the amount of green investment. In response, a retailer determines the retail price.

1. At the beginning of the period, a retailer determines the order quantity based on the beginning of period inventory state.
2. Upon receiving an order from a retailer, a manufacturer produces items (new production or reproduction). If the order cannot be fulfilled only by reproduction using collected items, a manufacturer new-produce the shortage. If the order is sufficient only for reproduction, all excess collected items are disposed of by a manufacturer and will not be used for reproduction at the next period.
3. During the sales term, items in retailer's inventory are purchased by consumers.
4. At the end of the sales term, the old items at the end of period are removed from a retailer's inventory. A manufacturer collects it as raw materials for reproduction at the next period, and any excess is disposed of by a retailer.
5. At the end of the period, items ordered at the beginning of the period is delivered to the retailer.
6. The inventory state at the end of the period when all actions have been completed is used as the inventory state at the beginning of the next period, and the process returns to 1 . Repeat this for $(T+1)$ periods.

In addition, the following settings are given.

- Initial inventory is zero. Therefore, in period 0 , no items are sold, and no unsold items are collected.
- In period $T$, a retailer does not place orders and a manufacturer does not produce.
- In period $(T-1)$ and $T$, no unsold items are collected.

In period T , a retailer disposes of all inventory at the end of the period.

### 2.2 Formulation of Supply Chain Model

The parameters and mathematical model are shown below.
$r$ : Retail price (retailer's decision variable)
$a_{t}$ : Order quantity in period $t$ (retailer's decision variable)
$w$ : Wholesale price (manufacturer's decision variable)
$G$ : Green investment (manufacturer's decision variable)
$P\left(D_{t}=d_{t}\right):$ Demand distribution
$d_{\text {max }}$ : Maximum demand
$T$ : Final period $(t=0, \ldots, T)$
$I_{\max }$ : Maximum inventory capacity
$I_{0, t}$ : Inventory quantity of old items at the beginning of period $t$
$I_{1, t}$ : Inventory quantity of new items at the beginning of period $t$
$I_{0, t}^{\prime}$ : Inventory quantity of old items at the end of period $t$
$I_{1, t}^{\prime}$ : Inventory quantity of new items at the end of period $t$
$y_{t}$ : Quantity of collected items used for reproduction in period $t$
$y_{\text {max }}$ : Standard quantity of collected items
$h, b$ : Inventory holding/disposal cost per unit
$c^{n}, c^{r}$ : New/re-production cost per unit
$P_{t}^{n}, P_{t}^{r}:$ New/re-production quantity in $t$ period
$p_{1}$ : Raw material price for new production
$p_{2}$ : Purchase price of collected items
$e^{n}, e^{r}: \mathrm{CO}_{2}$ emission per unit from new/re-production $[\mathrm{kg}]$
$L_{M}$ : Regulatory standard for $\mathrm{CO}_{2}$ emissions $[\mathrm{kg}]$
$t_{M}$ : Carbon tax rate [ $/ \mathrm{kgCO}_{2}$ ]
$V_{0}\left(s_{0}\right)$ : Retailer's finite period total expected reward (retailer's objective function) $R_{M}(w, G)$ : Manufacturer's finite period total expected reward (manufacturer's objective function)

## The Retailer Model

Retailer value function : $V_{t}\left(s_{t}\right)$

$$
\begin{equation*}
V_{t}\left(s_{t}\right)=\max _{a_{t} \in A\left(s_{t}\right)}\left\{r_{R}\left(s_{t}, a_{t}\right)+\sum_{s_{t} \in S} P\left(s_{t+1} \mid s_{t}, a_{t}\right) V_{t+1}\left(s_{t+1}\right)\right\} \tag{1}
\end{equation*}
$$

Retailer's expected reward per unit period : $r_{R}\left(s_{t}, a_{t}\right)$

$$
\begin{array}{r}
r_{R}\left(s_{t}, a_{t}\right)=R\left(I_{0, t}, I_{1, t}\right)+C\left(I_{0, t}, I_{1, t}\right)-\left\{H\left(I_{0, t}, I_{1, t}\right)+B\left(I_{0, t}, I_{1, t}\right)+w a_{t}\right\}  \tag{2}\\
t \in\{1, \ldots, T-1\} .
\end{array}
$$

$R\left(I_{0, t}, I_{1, t}\right)$ and $C\left(I_{0, t}, I_{1, t}\right)$ are the expected sales from selling items and sending unsold items for collection in period $t$, respectively. $H\left(I_{0, t}, I_{1, t}\right)$ and $B\left(I_{0, t}, I_{1, t}\right)$ are the expected costs for holding inventory and discarding unsold items in period $t$, respectively. In addition, $w a_{t}$ is the purchase cost of orders in period $t$.

In particular, note that

$$
\begin{align*}
& r_{R}\left(s_{0}, a_{0}\right)=-w a_{0} \\
& r_{R}\left(s_{T}, a_{T}\right)=R\left(I_{0, T}, I_{1, T}\right)-B_{T}\left(I_{0, T}, I_{1, T}\right) \tag{3}
\end{align*}
$$

No sale and recovery are made in period 0 to set $s_{0}=(0,0) . B_{T}\left(I_{0, T}, I_{1, T}\right)$ is the disposal cost in the last period $T$. Only in the end of period $T$, all new and old items in inventory are disposed of.

Transition probability from state $s_{t}$ to $s_{t+1}$ :

$$
P\left(s_{t+1} \mid s_{t}, a_{t}\right)=\left\{\begin{array}{cc}
\sum_{d_{t}=0}^{I_{0, t}} P\left(D_{t}=d_{t}\right) & \text { if } I_{0,(t+1)}=I_{1, t}, I_{0,(t+1)}=a_{t}  \tag{4}\\
P\left(D_{t}=I_{0, t}+I_{1, t}-I_{0,(t+1)}\right) \\
& \text { if } 0<I_{0,(t+1)}<I_{1, t}, I_{0,(t+1)}=a_{t} \\
\sum_{d_{t}=I_{0, t}+I_{1, t}}^{d_{\max }} P\left(D_{t}=d_{t}\right) & \text { if } I_{0,(t+1)}=0, I_{0,(t+1)}=a_{t} \\
0 & \text { otherwise. }
\end{array}\right.
$$

## The Manufacturer Model

Manufacturer's finite period total expected reward :

$$
\begin{equation*}
R_{M}(w, G)=\sum_{t=0}^{T} \sum_{a_{t}=0}^{I_{\max }}\left\{\check{\pi}_{t}^{*}\left(a_{t}^{*}\right) r_{M}\left(w, G \mid a_{t}^{*}\right)-\sum_{y_{t}=0}^{y_{\max }} \hat{\pi}_{t}^{*}\left(a_{t}^{*}, y_{t}\right) c_{M}\left(w, G \mid a_{t}^{*}, y_{t}\right)\right\} \tag{5}
\end{equation*}
$$

where,

$$
\begin{equation*}
r_{M}\left(w, G \mid a_{t}^{*}\right)=w a_{t} . \tag{6}
\end{equation*}
$$

$$
\begin{equation*}
c_{M}\left(w, G \mid a_{t}^{*}, y_{t}\right)=p_{1} P_{t}^{n}+p_{2} P_{t}^{r}+c^{n} P_{t}^{n}+c^{r} P_{t}^{r}+b \max \left(y_{t}-a_{t}, 0\right)+T_{t}(G) . \tag{7}
\end{equation*}
$$

Here, $w a_{t}$ is the amount of sales in period $t$, and $p_{1} P_{t}^{n}$ is the cost of newly produced raw materials in period $t . p_{2} P_{t}^{r}$ is the purchase cost of collected items in period $t . c^{n} P_{t}^{n}$ and $c^{r} P_{t}^{r}$ are the new production and reproduction cost in period $t$, respectively. The value of $b \max \left(y_{t}-a_{t}, 0\right)$ is the disposal cost of collected items surplus to reproduction in period $t$ and $T_{t}$ is the carbon tax in period $t$.

In particular, $T_{t}(G)$ is given in the following.

$$
\begin{equation*}
T_{t}(G)=\max \left\{(1-H(G))\left(e^{n} P_{t}^{n}+e^{r} P_{t}^{r}\right)-L_{M}, 0\right\} t_{M} \tag{8}
\end{equation*}
$$

$H(G)$ is the function of the percentage of $\mathrm{CO}_{2}$ emissions reduced by green investment [6].

$$
\begin{equation*}
H(G)=\theta(1-\exp (-m G)),(0<\theta<1, m \geq 0) \tag{9}
\end{equation*}
$$

In addition, $\hat{\pi}_{t}^{*}\left(a_{t}^{*}, y_{t}\right)$ and $\check{\pi}_{t}^{*}\left(a_{t}^{*}\right)$ are the joint distributions of the optimal order quantity $a_{t}^{*}$ and the quantity of collected items $y_{t}$ and the distributions of $a_{t}^{*}$, respectively. These can be derived by computing the probability distributions in the following order according to Fig. 3.

Let $1(x=A)$ be the indicator function for $x=A$.

$$
1(x=A)= \begin{cases}1 & \text { if } x=A  \tag{10}\\ 0 & \text { otherwise }\end{cases}
$$



Fig. 3. State transition diagram.
The distribution $\pi_{t}^{*}\left(s_{t}\right)$ of a retailer's inventory $s_{t}$ at the beginning of period $t$ under the optimal policy is obtained from the following equation.

$$
\begin{align*}
\pi_{0}^{*}\left(s_{0}\right) & = \begin{cases}1 & \text { if } s_{0}=(0,0) \\
0 & \text { otherwise }\end{cases}  \tag{11}\\
\pi_{t+1}^{*}\left(s_{t+1}\right) & =\sum_{s_{t} \in S} P\left(s_{t+1} \mid s_{t}, a_{t}^{*}\right) \pi_{t}^{*}\left(s_{t}\right),(t=0, \ldots, T-1) .
\end{align*}
$$

Using $\pi_{t}^{*}\left(s_{t}\right)$, we obtain the joint distribution $\bar{\pi}_{t}^{*}\left(s^{\prime}{ }_{t}, a_{t}^{*}\right)$ of the inventory $s^{\prime}{ }_{t}$ at the end of period $t$ and optimal order quantity $a_{t}^{*}$ in period $t$ under the optimal policy from the following equation.

$$
\begin{equation*}
\bar{\pi}_{t}^{*}\left(s^{\prime}{ }_{t}, a_{t}^{*}\right)=\sum_{s_{t} \in S} \bar{P}\left(s^{\prime}{ }_{t} \mid s_{t}\right) \pi_{t}^{*}\left(s_{t}\right) 1\left(a_{t}^{*}=a_{t}^{*}\left(s_{t}\right)\right) \tag{12}
\end{equation*}
$$

where $\bar{P}\left(s^{\prime}{ }_{t} \mid s_{t}\right)$ is the transition probability from the inventory at the beginning and end of period $t$.

$$
\bar{P}\left(s^{\prime}{ }_{t} \mid s_{t}\right)= \begin{cases}P\left(D_{t}=I_{0, t}-I_{0, t}^{\prime}\right) & \text { if } 0 \leq I_{0, t}^{\prime} \leq I_{0, t}, I_{1, t}^{\prime}=I_{1, t}  \tag{13}\\ P\left(D_{t}=I_{0, t}+I_{1, t}-I_{1, t}^{\prime}\right) & \text { if } I_{0, t}^{\prime}=0,0<I_{1, t}^{\prime}<I_{1, t} \\ \sum_{d_{t}=I_{0, t}+I_{1, t}}^{d_{\max }} P\left(D_{t}=d_{t}\right) & \text { if } I_{0, t}^{\prime}=0, I_{1, t}^{\prime}=0 \\ 0 & \text { otherwise. }\end{cases}
$$

Using $\bar{\pi}_{t}^{*}\left(s^{\prime}{ }_{t}, a_{t}^{*}\right)$, we obtain $\hat{\pi}_{t}^{*}\left(a_{t}^{*}, y_{t}\right)$.

$$
\begin{gather*}
\hat{\pi}_{0}^{*}\left(a_{0}^{*}, y_{0}\right)= \begin{cases}1 & \text { if } a_{0}^{*}=a_{0}^{*}(0,0), y_{0}=0 \\
0 & \text { otherwise }\end{cases} \\
\hat{\pi}_{t}^{*}\left(a_{t}^{*}, y_{t}\right)=\sum_{s_{t} \in S} \sum_{a_{t}^{*}=0}^{I_{\max }} \bar{\pi}_{t}^{*}\left(s^{\prime}{ }_{t}, a_{t}^{*}\right) 1\left(a_{t+1}^{*}=a_{t+1}^{*}\left(s^{\prime}{ }_{t}, a_{t}^{*}\right), y_{t+1}=s^{\prime}{ }_{t}\right)  \tag{14}\\
,(t=0, \ldots, T-1) .
\end{gather*}
$$

Furthermore, using $\hat{\pi}_{t}^{*}\left(a_{t}^{*}, y_{t}\right)$, we also obtain the distribution $\check{\pi}_{t}^{*}\left(a_{t}^{*}\right)$.

$$
\begin{equation*}
\check{\pi}_{t}^{*}\left(a_{t}^{*}\right)=\sum_{y_{t}=0}^{y_{\max }} \hat{\pi}_{t}^{*}\left(a_{t}^{*}, y_{t}\right) . \tag{15}
\end{equation*}
$$

## 3 Numerical Experiment and Sensitivity Analysis

The parameters were set as follows.

$$
\begin{gathered}
T=10, d_{\max }=10, I_{\max }=20, p_{1}=450, p_{2}=0.15 \mathrm{w}, \\
h=50, b=200, c^{n}=150, c^{r}=100, y_{\max }=5, L_{M}=25, \\
e^{n}=30, e^{r}=20, \theta=0.8, m=0.0003, t_{M}=7.5
\end{gathered}
$$

The demand distribution is assumed to follow a binomial distribution given the condition that an increase in the retail price lowers the average demand as follows ( $q$ is the probability of occurrence of the event.)

$$
\begin{gather*}
P\left(D_{t}=d_{t}\right)=\binom{d_{\max }}{d_{t}} q^{d_{t}}(1-q)^{d_{\max }-d_{t}}  \tag{16}\\
\text { where, } d_{\max } q=d_{\max }-0.002 r, \quad\left(w \leq r \leq \frac{d_{\max }}{0.002}\right)
\end{gather*}
$$

### 3.1 Numerical Experiment to Find the Optimal Ordering Policy

The following are the results of numerical experiments in which the retailer's optimal ordering policy is obtained without considering $r, w$, and $G$.

Table 1. Comparison of optimal ordering policies by $r(w=1000, G=0)$.


The average order quantity is highest in period 0 , stabilizes approximately from period 1 to 8 , and decreases from period 9 to 10 . The larger $r$ is, the smaller the average demand becomes, and the smaller the order quantity in each period.

The ratio of the quantity of items reproduced to the total quantity of items produced was less than $20 \%$ in all cases. This is because under the optimal ordering policy, collected items are not generated very often.

### 3.2 Numerical Experiment to Find Optimal Wholesale and Retail Price

Next, the results of numerical experiments to find the optimal $r$ and $w$ under the optimal ordering policy without considering $G$ are given.

Table 2 shows the retailer's optimal $r^{*}, V_{0}\left(s_{0}\right)$ and $R_{M}(w, G)$ for each $w$.

Fig. 4 shows that for $G=0, R_{M}(w, G)=40797$ is the maximum when $w=2325$, under which $r^{*}=3710$, and $V_{0}\left(s_{0}\right)=32433$.

Now, let us consider the reason why $R_{M}(w, G)$ repeatedly increases and decreases as $w$ increases. Fig. 5 shows the relationship between the retail price $r^{*}$ and $R_{M}(w, G)$ for each $w$. As shown, $r^{*}$ increases in $w$.

Table2. The optimal $r^{*}$ for each $w$ and the values of $V_{0}\left(s_{0}\right)$ and $R_{M}(w, G)$ at that time $(G=0)$.

| $w$ | $r^{*}$ | $R m(w, G)$ | $V_{0}(s o)$ |
| :---: | :---: | :---: | :---: |
| 2100 | 3600 | 38464.269 | 38446.433 |
| 2125 | 3600 | 39169.505 | 37742.270 |
| 2150 | 3605 | 39811.879 | 37038.461 |
| 2175 | 3605 | 40515.815 | 36335.620 |
| 2200 | 3675 | 38883.410 | 35670.310 |
| 2225 | 3680 | 39111.622 | 35013.098 |
| 2250 | 3700 | 39243.759 | 34361.797 |
| 2275 | 3700 | 39888.904 | 33717.562 |
| 2300 | 3700 | 40518.772 | 33073.332 |
| 2325 | 3710 | 40796.641 | 32432.588 |
| 2350 | 3760 | 39597.786 | 31817.601 |
| 2375 | 3810 | 38318.327 | 31223.896 |
| 2400 | 3810 | 38898.845 | 30644.242 |
| 2425 | 3815 | 39296.058 | 30066.317 |
| 2450 | 3815 | 39873.589 | 29489.664 |
| 2475 | 3920 | 36547.910 | 28964.028 |
| 2500 | 3925 | 36808.583 | 28449.293 |



Wholesale price w
Fig. 4. Variation of $R_{M}(w, G)$ for each $\left(w, r^{*}\right),(G=0)$.


Fig. 5. Comparison of optimal $r^{*}$ and $R_{M}(w, G)$ for each $w$.
The shading in Fig. 5 shows that $R_{M}(w, G)$ decreases when $r^{*}$ rises rapidly. On the other hand, $R_{M}(w, G)$ increases at other times when $r^{*}$ rises slowly. This is because a rapid rise in $r$ decreases average demand of consumers, thereby reducing the order quantity by a retailer and a manufacturer's wholesale sales.

Consider the reason why the optimal $r^{*}$ rises rapidly. As can be seen in Fig. 6, there is the multimodality in the variation of $V_{0}\left(s_{0}\right)$. This means that a small change in w may cause $r^{*}$ to move to the top of different peaks where $V_{0}\left(s_{0}\right)$ is maximum.
Furthermore, the multimodality is due to the discreteness of order quantity $a_{t}$. For example, in Fig. 6(a), the optimal order quantity in period $0 a_{0}^{*}$ is 4,3 , and 3 for $r=$ 3815, 3875 and 3920, respectively. $a_{0}^{*}$ changes from 4 to 3 exactly at $r=3875$. This may have changed the distribution of inventory states in each period, affecting the value of $V_{0}\left(s_{0}\right)$.


Fig. 6. Comparison of the optimal $r^{*}$ for which $V_{0}\left(s_{0}\right)$ is maximum.

### 3.3 Numerical Experiment to Find Optimal Green Investment

Finally, the results of numerical experiments to find the optimal $G$ are shown below. Fig. 7 shows that $G^{*}=975$ is the optimal decision under $r^{*}=2325$ and $w^{*}=3710$. Fig. 8 shows that the penalty cost in the form of carbon tax on the environmental factor of $\mathrm{CO}_{2}$ emissions encourages a manufacturer to make the optimal green investment to maximize its own reward (minimize the sum of green investment and carbon tax cost), resulting in a reduction of about $29.5 \%$ in $\mathrm{CO}_{2}$ emissions (from $G=0$ to $G^{*}=975$ ).


Fig. 7. Variation of $R_{M}(w, G)$ for each $G, \quad(r=2325, w=3710)$.


Fig. 8. Carbon Tax Cost and $\mathrm{CO}_{2}$ Emissions by $G$, $\left(t_{M}=7.5\right)$.

### 3.4 Sensitivity Analysis on Carbon Tax System

We compared the optimal decisions for each carbon tax rate, referring to [7]. The following are the results of numerical experiments for $t_{M}=2.5,7.5$, and 12.5 .

Fig. 9 and Fig. 10 show the optimal decisions for each carbon tax rate (Fig. 7 shows the case $t_{M}=7.5$ ). Fig. 10 also shows that $R_{M}(w, G)$ does not increase with green investment when $t_{M}=2.5$. This is because the cost of carbon tax is higher than the amount of investment when the carbon tax rate is very low.


Fig. 9. Comparison of change in $R_{M}(w, G)$ for each $t_{M},(G=0)$.


Fig. 10. $G$ that maximizes $R_{M}(w, G)$ for each $t_{M}$.

## 4 Conclusion

In this study, in a GSC in which a manufacturer reproduces using a retailers' unsold items and a manufacturer makes green investment in production technology to reduce $\mathrm{CO}_{2}$ emissions, the optimal decisions of a retailer and a manufacturer are obtained, and the optimal decisions for each carbon tax rate and the expected total rewards at that time are discussed. Numerical experiments show that the optimal retail price, wholesale
price, and green investment that maximize the total rewards of both a retailer and a manufacturer in a GSC with a carbon tax system increase with each carbon tax rate.
Future work may include conducting the analysis in an extended model with a single manufacturer and multiple retailers, or a three-party SC consisting of a manufacturer, retailer, and collector, and differentiating the price of new and old items.

To solve the problem of $R_{M}(w, G)$ repeatedly increasing and decreasing as $w$ increases, it may be necessary to change the retailer's order quantity, which was set as a discrete quantity in this study, to a continuous quantity, or to set the maximum stock quantity and demand quantity to larger values. However, this may cause new problems such as increased computation time and memory shortage.

## References

1. Karim, R., Nakade, K.: Modelling a One Manufacturer Supply Chain System Considering Environmental Sustainability and Disruption. International Journal of Systems Science: Operations \& Logistics, vol.8, No.4, pp.297-320 (2021)
2. Consumer Affairs Agency in Japan Homepage, Case Study: Initiatives in the French Fashion Industry,
https://www.caa.go.jp/policies/policy/consumer_research/white_paper/2022/white_paper_example_13.html, last accessed 2023/09/26
3. Kusukawa, E., Paku, Y.: Optimal Operation and Supply Chain Coordination in a ClosedLoop Supply Chain with Loss-Averse Attitude. Asian Journal of Management Science and Applications, vol.3, No.3, pp.252-278 (2018)
4. Caro, F., Gallien, J.: Clearance Pricing Optimization for a Fast-Fashion Retailer. Operations Research, vol.60, No.6, pp.1404-1422 (2012)
5. Nakade, K., Ikeuchi, K.: Optimal Ordering and Pricing on Clearance Goods, " International Journal of Industrial Engineering, vol.23, No.3, pp. 155-165 (2016)
6. Datta, T. K.: Effect of Green Technology Investment on a Production-Inventory System with Carbon Tax. Advances in Operations Research, 12 pages (2017), doi:10.1155/2017/4834839
7. Ministry of the Environment in Japan Homepage, Recent Developments in Carbon Tax and Border Adjustment Measures, https://www.env.go.jp/content/900499203.pdf, last accessed 2023/09/26
