



The Development of Arithmetic Operations on Multiple Byte Integers

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Abstract

Big integers are very essential in many applications. Cryptography is one of these applications. In this study, the objective is to create a multiple byte integer type, with its arithmetic operations defined. The operations are: addition, subtraction, multiplication, division and modular exponentiation are overloaded, to work on this multiple byte integer type.

The creation of the multiple byte integer is done by using doubly linked lists, a well known technique in data structure. The reason is that doubly linked lists enable us to create integer of unlimited size. That is, you do not have to pre-specify the size of the arrays storing these integers.

This is done by dynamically allocating the memory to store the digits constructing the integers.

The operations on these integers are defined using the simple and straight forward techniques, learnt in school.

The results obtained are satisfactory and reliable. The type could be extended to help define multiple byte floating point numbers.

In this work, an improvement has been made to the work of BH Flowers.

Introduction:

The computer consists of bits and bytes. The important thing in computer is its word size which can be data or some computer operation, represented by an integer for everything. The largest integers we have been able to work with have been confined to short, an int or a long. In C++ language, for example, these are one byte, two bytes and four bytes long, respectively. For all types, advanced arithmetical operations are made standard for only that domain of type, otherwise the compiler just ignore the function output, no overflow message.

In principle, however, there is no reason to confine an integer to a specific number of bytes: the concept of a list allows us to work with any number of bytes, dynamically determined according to the transient needs of the program.

Integers limit in size made files vulnerable to attacks, even thought they were encrypted, because the size of the private key is limited. Also in real life, the size of amounts of large integers is limited to just more than 4 millions.

Multiple Byte Integers

The lists we have discussed so far have been singly linked: each node contains a single pointer to the succeeding node in the list. Multiple byte integers, however, are not efficiently dealt with by means of single linkages: we shall find it necessary to scan a list both forwards and backwards, and this is most easily done if each node contains a pointer to the previous node as well as a pointer to the succeeding node. Moreover, it is no longer desirable to work with pointers to the elements of a fixed data array: since we do not know in advance the number of bytes required by an integer, it would be most inconvenient to work with fixed arrays; we need to work with the bytes themselves.

An 8-bit byte, unsigned, can accommodate the integers 0, 1, ..., 255. If we set $B = 256$, we may represent any non-negative integer N as a polynomial in B :

$$N = a_m B^m + \dots + a_1 B + a_0,$$

$$\text{Or } N = (a_m, \dots, a_1, a_0)B,$$

Where, in the second formula, we have recognized B as the radix, and the integer coefficients a_i , which must obey the restriction $0 < a_i < B$, as the digits of the integer to the base B. There may in principle be any number digits. If B were equal to 10, we should have the usual decimal representation, but we shall take B to be 256.

If we define a digit to be a node containing a byte b and two pointers, a multiple byte integer (which we shall call mult) may be represented by a list of digits. In diagrammatic form as shown in figure 3.1.[1]:

Computer Operations with Integers:

Before computers were invented mathematicians did computations either by hand or by using mechanical devices. Either way, they were only able to work with integers of limited size. Many number theoretic computations, such as factoring and primality testing, require computations with integers with as many as 50 or even 100 digits. In this section we will study some of the basic algorithms for doing computer arithmetic.

It had been mentioned that computers internally represent numbers using bits, or binary digits. Computers have a built-in limit on the size of integers that can be used in machine arithmetic. This upper limit is called the **word size**, which we denote by **w**. The word size is usually a power of 2, such as 2^{35} , although sometimes the word size is a power of 10[5].

To do arithmetic with integers larger than the word size, it is necessary to devote more than one word to each integer. To store an integer $n > w$, we express n in base w notation, and for each digit of this expansion we use one computer word. For instance, if the word size is 2^{35} , using ten computer words we can store integers as large as $2^{350} - 1$, since integers less than 2^{350} have no more than ten digits in their base 2^{35} expansions. Also note that to find the base 2^{35} expansion of an integer, we need only group together blocks of 35 bits[5].

The first step in discussing computer arithmetic with large integers is to describe how the basic arithmetic operations are methodically performed.

We will describe for performing addition, subtraction, and multiplication of two n-digit integers $a = (a_{n-1} a_{n-2} \dots a_1 a_0)_r$ and $b = (b_{n-1} b_{n-2} \dots b_1 b_0)_r$. The algorithms described are used both for binary arithmetic with integers less than the word size of a computer, and for multiple precision arithmetic with integers larger than the word size w, using w as the base[5].

Addition of Multiple Byte Integers:

The addition of two numbers can be represented as the addition of two lists (polynomials) as shown now:

Let

$$n = a_k b^k + a_{k-1} b^{k-1} + \dots + a_1 b^1 + a_0$$

$$m = c_k b^k + c_{k-1} b^{k-1} + \dots + c_1 b^1 + c_0$$

Then

$$n + m = (a_k + c_k) b^k + (a_{k-1} + c_{k-1}) b^{k-1} + \dots + (a_1 + c_1) b^1 + (a_0 + c_0) [1]$$

They don't have to be equal in length; i.e., n has k while m has j, and $k \neq j$. But the radix is the same, for all arithmetical operations done.

However, it is often easier to visualize the addition of two large numbers as the digit-by-digit addition at each position. When we add the digits at a particular position, the largest resulting value is $2b - 2$ [5].

We first discuss the algorithm for addition. When we add a and b, we obtain the sum

$$a + b = \sum_{j=0}^{n-1} a_j r^j + \sum_{j=0}^{n-1} b_j r^j = \sum_{j=0}^{n-1} (a_j + b_j) r^j,$$

The two number lists must be traversed from right to left; i.e., from the smallest weight, adding the two corresponding nodes and the carry digit, what is greater than the radix is then added to the next nodes sum. Every addition result is inserted into a node of the resultant number list. Finally if there is also a carry, a node is again inserted and attached at the front of the list representing the sum of the two number lists.

This is precisely what happens in the definition of the operator(+).

mult& mult::add(mult ms1, mult ms2)

```
{
    int a;           // The value of adding the two digits.
    int carry = 0;   // The carry of if the sum of the two
                    // digits is greater than the radix, initially is zero.
    mult ms3;       // The product list of addition.
    multscan thisptr(ms1, NEXT); // Traverse from
    // right to left, the first is the most smallest weight
    dgtp pr1;
    multscan thatptr(ms2, NEXT);
    dgtp pr2;
    while(((pr1 = thisptr()) != 0) && ((pr2 = thatptr()) != 0))
        // While both are equal in length.
    {
        a = int(pr1 -> dgt) + int(pr2 -> dgt) + carry;
        if(((pr1 -> dgt) + (pr2 -> dgt) + carry) >= radix)
```

```

        {
            carry = a / radix;
            a = a % radix;
        } else carry = 0;
    ms3.append(a);    // [1]
}
while((pr1 = thisptr()) != 0) // While the first list is greater
{
    // in length than the second list.
    a = int(pr1 -> dgt) + carry;
    if(a >= radix)
        {
            carry = a / radix;
            a = a % radix;
        } else carry = 0;
    ms3.append(a);
}
while((pr2 = thatptr()) != 0) // While the second list is greater
{
    // in length than the first list.
    a = int(pr2 -> dgt) + carry;
    if(a >= radix)
        {
            carry = a / radix;
            a = a % radix;
        } else carry = 0;
    ms3.append(a);
}
if(carry >= 1) ms3.append(carry); // If the last digit
// addition carry is not zero, an attached node.
// Is appended to the product list.
if((ms1.sgn == -1) && (ms2.sgn == +1)) // Checking if the
{
    // two lists, doesn't have the same sign.
    ms1.sgn = ms2.sgn = +1; // Changing the signs first.

```

```

    ms3 = ms2 - ms1; // A SWAP is done with a
        subtraction, but the sign will be negative.
    ms1.sgn = -1; // The product will have a negative
        sign.
}
if((ms1.sgn == +1) && (ms2.sgn == -1)) // Checking if the
{
    // two lists, doesn't have the same sign.
    ms1.sgn = ms2.sgn = +1; // Changing the signs first.
    ms3 = ms1 - ms2; // A subtraction is done here.
    ms2.sgn = -1; // The product will have a negative
        sign.
}
rs = ms3; // I need two variables, because
sometimes in an addition, I also perform a subtraction.
printmult(ms3); // One is local and the other is global
to return the result.
return rs;
}

```

The inline operator does the inline + operator between two multiple-Byte integers, where m1 is the first number list, m2 is the second number list, pa is a temporary resultant number list.

```

inline mult operator + (mult m1, mult m2) {
    return (pa.add(m1, m2)); }

```

The lists in number1 and number2 are traversed from right to left using list <Byte int> reverse iterators that are moved through the lists synchronously by using the increment operator. The carry digit and the blocks at each position of the iterator are added, the new carry digit and sum block are calculated, and the sum block inserted at the front of the list in sum using list's insert() operator.

Sign Test for Addition:

- a. If the sign of the first number list is positive and the sign of the second number list is positive, a normal addition will be done with the resultant sign, will be positive.

- b. If the sign of the first number list is negative and the sign of the second number list is positive, a subtraction operation is done using the subtraction operator, after treating that both signs are positive and the first number list will be the second number list for the subtraction operation; i.e., reversing numbers.
- c. If the sign of the first number list is positive and the sign of the second number list is negative, a subtraction operation is done after treating that both signs are positive.
- d. The length of numbers can be tested here as addition is no good as subtraction for the subtraction has a fixed rule the state:
 - (i). If first's sign is negative and second's sign is negative for subtraction, the resultant sign will be negative and the operation will be subtraction.
 - (ii). If first's sign is negative and second's sign is positive for subtraction, the resultant sign will be negative and the operation will be addition.
 - (iii). If first's sign is positive and second's sign is negative for subtraction, the resultant sign will be positive and the operation will be addition.
 - (iv). If first's sign is positive and second's sign is positive for subtraction, the resultant operation will depend on the length of each of the number lists size. If the second length is greater, the resultant sign will be negative otherwise it will be positive.

Subtraction of Multiple Byte Integers:

The subtraction of two numbers is very similar to addition and can be easily implemented as byte-by-byte subtraction. However, when subtracting two digits, we often encounter the situation where the first digit is smaller than the second. In this case, we have to borrow 1 from the digit in the next position. This is actually done by subtracting 1 from the digit in the next higher position and adding the value of the base to the current digit[5].

We consider

$$a - b = \sum_{j=0}^{n-1} a_j r^j - \sum_{j=0}^{n-1} b_j r^j = \sum_{j=0}^{n-1} (a_j - b_j) r^j,$$

$$a - b = (d_{n-1} d_{n-2} \dots d_1 d_0)_r.$$

The two number lists must be traversed from right to left; i.e., from the smallest weight, subtracting the two corresponding nodes and the borrow digit if the result is negative, will be taken a radix in size from the exceeding first number digit; i.e., will be subtracted from the next subtraction of the other two corresponding digits.

```
mult& mult::sub(mult mt1, mult mt2)
```

```
{
```

```
    int borrow = 0;    // The borrow if the first list
                      // digit was smaller than the second list digit, initially is
                      // zero.
```

```
    int at;           // The value of subtracting the two digits.
```

```
    mult mt3, buf;    // The buf is needed when the
                      // second list length is greater than the first list.
```

```
    multscan thisptr(mt1, NEXT); // Traverse from
                                  // right to left, the first is the most smallest weight
```

```
    dgtp pt1;
```

```
    multscan anotherptr(mt2, NEXT);
```

```
    dgtp pt2;
```

```
    while(((pt1 = thisptr()) != 0) && ((pt2 = anotherptr()) != 0))
```

```
    {
```

```
        at = int(pt1 -> dgt) - int(pt2 -> dgt) - borrow;
```

```
        if((pt1 -> dgt) < (pt2 -> dgt))
```

```
        {
```

```
            at = at + radix;
```

```
            borrow = 1;
```

```
        } else borrow = 0;
```

```
        mt3.append(at);
```

```
    }
```

```

if(mt1.numdgt > mt2.numdgt)
{
    at = int(pt1 -> dgt) - borrow;

    if(at == 0)
    {
        at = at + radix;
        borrow = 1;
    } else borrow = 0;

    if(at <= 0) borrow = 1;

    mt3.append(at);
}

while((pt1 = thisptr()) != 0) // While the first list is
greater in length than the second list.
{
    at = int(pt1 -> dgt) - borrow;

    if(at <= 0) borrow = 1;

    mt3.append(at);
}

mt3.sgn = +1;

if((mt1.sgn == -1) && (mt2.sgn == -1)) mt3.sgn = -1;

// The product will have a negative sign.

if((mt1.sgn == -1) && (mt2.sgn == +1))
{
    mt1.sgn = mt2.sgn = +1;

    mt3 = mt1 + mt2;

    mt3.sgn = -1;

    mt3 = rs;
}

```

```

if((mt1.sgn == +1) && (mt2.sgn == -1)) // Checking if the
{
    // two lists, doesn't have the same sign.

    mt1.sgn = mt2.sgn = +1; // Changing the signs
first.

    mt3 = mt1 + mt2; // Addition is done here.

    mt3.sgn = +1;

    // The product will have a positive sign.

    mt3 = rs;
}

if(mt1.numdgt < mt2.numdgt) // If the second list length is
{ // greater than the first list. SWAP

    mt3 = mt2 - mt1;

    mt3.sgn = -1;

    // The product will have a negative sign.

    mt3 = rs;
}

rs = mt3;

printmult (mt3);

return rs;
}

```

Sign Test for Subtraction:

- If the sign of the first number list is negative and the sign of the second number list is positive, an addition operation is done after treating both signs as positive. The resultant sign will be set to negative.
- If both signs were negative, then the sign of the resultant is also negative.
- If the sign of the first number list is positive and the sign of the second number list is negative, an addition operation is done after treating both signs as positive. The sign of the resultant is set to positive.

- d. If the length of the first number list is less than that of the first, then a message will appear and the numbers will be swapped; i.e., the first will be the second for the subtraction operation. The resultant will have a negative sign.

Multiplication of Multiple Byte Integers:

The simplest way to implement multiplication is by using repeated addition. To compute $a * b$, we set the result to 0, then we repeatedly add a to the result for b number of times[5].

. But before discussing multiplication, we describe shifting. To multiply $(a_{n-1} \dots a_1 a_0)_r$ by r^m , we need only shift the expansion left m places, appending the expansion with m zero digits.

$$ab = a \sum_{j=0}^{n-1} b_j r^j = \sum_{j=0}^{n-1} (a b_j) r^j.$$

We multiplying two integers with base r expansion, we use the familiar method of multiplying decimal integers by hand[5].

Again the two number lists were traversed from right to left, taking the first digit of the second number list and multiply it with each digit of the first number list. Each multiplication if greater than the radix, a carry digit will be added to the next proceeding nodes' result. A node will be created when the last digit of the first number list is reach and there was a carry, at the front of the list. The next multiplication of the next digit of the second number list will be done in the same way except that a zero digit will be inserted first at the most right node of the result. The next other ones, every time an increment is done; i.e., two zero's will be inserted in the most right nodes of the result, and so one, using an iterator. Every resultant of each multiplication of a digit of the second number list is added to the one before it, and the sum will be the final resultant of the multiplication operation. The addition operator will be used here, but without the sign test. Both signs will be treated as positive.

The Sign Test results only in the final resultant sign, and if the sign of the two numbers lists were different, then the resultant sign will be negative, otherwise it is positive.

```

mult& mult::mul(mult m1, mult m2)
{
    int carrym;
    long unsigned int x, count = 0;
    long int ssum;
    mult mmp, mb;    // mb to store each last
                    // addition of the multiplications done.
    multscan anotherptr(m2, NEXT);
    // Traverse from right to left, the first is the
    // most smallest weight
    dgtp p2;
    while((p2 = anotherptr()) != 0) // For every digit of the
    {
        // second list, it multiple all the first list.
        carrym = 0;
        mult ma;
        // Each time, a new initiated variable list is needed.
        for(x = 0; x < count; x++) ma.append(0);
        // Shifting the first leading zeros.
        multscan otherptr(m1, NEXT);
        dgtp p1;
        while((p1 = otherptr()) != 0)
        {
            ssum = int(p1 -> dgt) * int(p2 -> dgt) + carrym;
            if(ssum >= radix)
            {
                carrym = ssum / radix;
                ssum = ssum % radix;
            } else carrym = 0;
            ma.append(ssum);
        }
        if(carrym >= 1) ma.append(carrym); // Attaching
        // the last carry to the product list.
    }
}

```

```

if(count == 0) mmp = ma; // Initial addition to the multiplication.

else // Else adding every last multiplications to the next one.
    {
        mmp = ma + mb;

        mmp = rs;

    }

count++; // The shifting of the leading zeros is
incremented.

mb = mmp; // The value of storing additions, is
updated.

printmult(mmp);

}

mmp.sgn = +1;

if((m1.sgn == -1) && (m2.sgn == +1)) mmp.sgn = -1;
// Only the sign will be negative.

if((m1.sgn == +1) && (m2.sgn == -1)) mmp.sgn = -1;

if((m1.sgn == -1) && (m2.sgn == -1)) mmp.sgn = +1;

rs = mmp;

printmult(mmp);

return rs;

}

```

Division of Integers:

Again the two number lists must be traversed from right to left, subtracting the two corresponding digits. Here I'm not using the subtraction operator because I need a flag, which after each subtraction if there was a negative result, the flag is set to positive, indicating the end of the operation. The second number list is then subtracted from the resultant of the last subtraction until the flag is positive. The number of these subtractions while the flag is negative is the final resultant, been converted to a list itself; i.e., each time this iterator which count these subtraction is reach the radix size, a node of the final resultant is created and the modulo of the division of this

iterator is inserted into a new node, creating the final division number list. The iterator was declared as long as to be able to present as large as possible for a really big integers as it can be for the resultant.

```

mult& mult::dvd(mult mr1, mult mr2)
{
    int borrow = 0, cs, d2, d1;

    long unsigned int n = 0;

    int ar, ps = 0;

    mult mr3, mdval, buff = mr1;

if(mr2.numdgt > buff.numdgt) // If the second list
    { // length was greater than the first list.
        fp = 1;

// This is a flag to show that the result will contain
floating points.

cout << "\n Float division not integers \n";

        mr3 = mdval = 0; // The result will be zero.

        ps = 1;

// To terminate this operation, and not to go to next
loop.

        n = 0;

    }

while(ps == 0)

    { // Traverse from right to left, the first is the
most smallest weight

        Borrow = cs = 0;

        multscan thisptr(mr1, NEXT);

        dgtp ptr1;

        multscan anotherptr(mr2, NEXT);

        dgtp ptr2;

// I didn't use the subtraction operator because I needed
a flag.

```



```

while(((ptr1 = thisptr()) != 0) && ((ptr2 = anotherptr()) != 0))
{
    d2 = int(ptr2 -> dgt);
    d1 = int(ptr1 -> dgt);
    ar = d1 - d2 - borrow;
    borrow = 0;
    if((ptr1 -> next != 0) && (ar <= 0))
    {
        ar = ar + radix;
        if(ptr1 -> next == 0) ar = 0;
        borrow = 1;
    }
    if((ptr1 -> next == 0) && (ar < 0)) ar = 0;
    ptr1 -> dgt = ar;
    if((ar < 0) && (mr1.numdgt == mr2.numdgt) && (ptr1 -> next == 0)) ps = 1;
    // The flag is on if the result is negative.
}

while((ptr1 = thisptr()) != 0) // When the first list is greater
than the second.           {
    cs++;
    ar = int(ptr1 -> dgt) - borrow;
    borrow = 0;
    if((ptr1 -> next != 0) && (ar <= 0))
    {
        ar = ar + radix;
        borrow = 1;
    }
    if((ar <= 0) && (ptr1 -> next == 0)) ar = 0;
    ptr1 -> dgt = ar;
    if((ar <= 0) && (ptr1 -> next == 0)) mr1.numdgt = mr1.numdgt - 1;
}

```

```

n = n + 1;
if(mr1.numdgt == 0) ps = 1;
if(mr1.numdgt < mr2.numdgt) ps = 1;
// Counting the number of subtractions.
if((mr1.numdgt == mr2.numdgt) && (d2 > ar)) ps = 1;
// Counting the number of subtractions.
if((mr1.numdgt == mr2.numdgt) && (d2 > ar)) ps = 1;
// Floating Point.
if((buff.numdgt == mr2.numdgt) && (d2 > d1))
{
    ps = 1;
    n = 0;
}
if(ar <= 0) mr1.numdgt = mr1.numdgt - 1;
// If last digit is less than zero,
if(mr1.numdgt == 0) ps = 1; // then the length of it
decreases. if(mr1.numdgt < mr2.numdgt) ps = 1;
// Floating Point result...
}
mdval = mr1;
mdval.sgn = +1; // The sign of the
modulo is always positive.
pr = mdval;
mr1 = buff;
while(n > 0) // Converting the
decimal number of the counting into a
multiple Byte List.
{
    ar = n % radix;
    n = n / radix;
    mr3.append(ar);
}
mr3.sgn = +1;

```

```

if((mr1.sgn == -1) && (mr2.sgn == +1)) mr3.sgn = -1;

// Only the sign changes.

if((mr1.sgn == +1) && (mr2.sgn == -1)) mr3.sgn = -1;

if((mr1.sgn == -1) && (mr2.sgn == -1)) mr3.sgn = +1;

    cout << "\n The module : \n ";

//if(fp == 1) mdval = 0;

printmult (mdval);

cout << "\n The divisor : \n ";

rs = mr3;

if(fp == 1) // A suggested loop for Floating
Point operation.
    {

        cout << " This is a fraction : 0.";

        for(n = 0; n < fc; n++) cout << "0";

    }

printmult(mr3);

if(fp == 1) cout << " Sorry but this fraction can not be
returned. This opens up a new research. \n";

return rs;

}

```

A test is done first to see if the second number length of nodes is greater than the first. If it is so, a message will appear, telling that there will be a Floating-Point result, which is another research to be done. If both numbers' lengths are equal, traverse from left to right, here is the advantage or 'use' of the doubly-linked list structure reason of being there, to check if the second number is greater than the first; again there is the message of a Floating-Point research.

The Sign Test results only in the final resultant sign, and if the signs of the two numbers' lists were different, then the resultant sign will be negative, otherwise it is positive. Just like multiplication.

Exponential Operation:

Here the first number list is multiplied by itself, using the multiplication operator, according to the second number digits times. The second number is traverse from right to left, each digit is multiplied by the radix raised to its weight, and this result number control the number of iteration of the iteration of the multiplication of the first number list. This number if it is greater than the maximum of integers for the mathematic function then there will be a problem as these functions were built on integers, log or pow, which raise the radix to its coefficient.

```

mult& mult::expo(mult me1, mult me2)
{
    long unsigned int x, count = 0;

    mult me3;

    me3 = me1;

    multscan thatptr(me2, NEXT); // Traverse
from right to left, the first is the most smallest weight

    dgtp pe2;

    while((pe2 = thatptr()) != 0)
    {
        for(x = 0; x < ((int(pe2 -> dgt) *
(exp((count) * (log(radix)))) - 1); x++))

            me3 = me3 * me1;

            count++;

    }

    if(me2.sgn == -1)

    {

        me3 = 0;

        cout << "This is a fractional, beyond
this research \n";

    }

    cout << "\n The exponential : \n ";

    rs = me3;

    printmult(me3);

    return rs;

}

```

Warning: The second number digit if it is zero and length one, is trivially known that any number, raised to zero is one. No need to do it here.

The second number sign can not be negative as Floating-Point numbers are outside this scope of research.

Comparison Operation:

Here, the lengths of the two numbers are compared, if their signs are the same. The larger is the big one.

```
mult& mult::cmp(mult mc1, mult mc2)
```

```
{
    int pr = 0, d2, d1;

    mult mc3;

    multscan (mc2, LAST);    // Traverse from left to
    // right, the first is the most greatest weight

    dgtp p2; // Here shows why it was a doubly-linked list

    multscan otherptr(mc1, LAST);

    dgtp p1;

    if((mc1.sgn == +1) && (mc2.sgn == -1))
    {
        pr = ptrgt = 1;    // A flag so as not to perform the next loop.
        mc3 = mc1;    // The greatest will be the first list.
    }

    if((mc1.sgn == -1) && (mc2.sgn == +1))
    {
        pr = ptrlt = 1; // A flag of the less
        // than is set on.    mc3 = mc2;
        // The greatest will be the second list.
    }

    if((mc1.numdgt < mc2.numdgt) && (mc1.sgn == mc2.sgn))
    {
        pr = ptrlt = 1;

        mc3 = mc2;
    }
}
```

```
if((mc1.numdgt > mc2.numdgt) && (mc1.sgn ==
mc2.sgn))
{
    pr = ptrgt = 1;

    mc3 = mc1;
}

if((mc1.numdgt == mc2.numdgt) && (mc1.sgn ==
mc2.sgn))
{
    while(((p1 = otherptr()) != 0) && ((p2 = anotherptr()) !=
0))
    {
        d2 = int(p2 -> dgt);
        d1 = int(p1 -> dgt);

        cout << "D1 " << d1 << "
        D2 " << d2 << "\n";

        if(d2 > d1)
        {
            pr = ptrlt = 1;

            mc3 = mc2;

            cout << "M2 >
M1 \n";    }

        else if(d2 < d1)
        {
            pr = ptrgt = 1;

            mc3 = mc1;

            cout << "M1 >
M2 \n";    }
    }

    if(pr == 0)
    {
        cout << "They are Equal 'But' : ";
    }
}
```

```

if(mc1.sgn == mc2.sgn) mc3 = 0;

        ptreq = 1; // A flag of the equal than is set
on.

        }

cout << "\n The Greater is : \n ";

if(mc3.sgn == +1) ch = '+';

        else ch = '-';

        rs = mc3;

        printmult(mc3);

        return rs;

        }

```

If one is negative, then it is trivially that the other number is the bigger whatever is the length is of its nodes.

If the lengths are equal, same sign, here the number lists are traversed from left to right, digits are compared, and the first one which is bigger than the other, then this one is the bigger. If there are all the same, then these numbers are equal, totally.

Binary conversion:

Multiple byte integers is usually based on 28 or the radix is 256. So to convert this number to binary, each digit is converted to 8 digits of binary.

Results:

These are some examples done by my program:

Addition:

{+, 56, 78, 98, 245, 160} + {+, 45, 3, 68} = {+, 101, 81, 166, 245, 160}

In decimal = + 435161593248.

{+, 34, 120, 36, 86} + {-, 100, 200} = {+, 190, 175, 35, 86} \equiv + 3199148886.

{-, 34, 25, 78, 90, 200} + {+, 200, 220, 98} = {-, 90, 60, 235, 89, 200} \equiv - 387569113544. {-, 120, 38, 98} + {-, 36, 200} = {+, 156, 238, 98} \equiv + 10284642.

Subtraction:

{+, 200, 39, 87, 65, 55} - {+, 48, 98, 120} = {+, 152, 197, 222, 64, 55} \equiv + 656154705975. {-, 46, 24, 33, 78, 200} - {+, 47, 75, 98} = {-, 93, 99, 131, 78, 200} \equiv - 401101508296.

{+, 40, 39, 28, 36, 20} - {-, 200, 180} = {+, 240, 219, 28, 36, 20} \equiv + 1034468205588.

{-, 230, 48, 170, 78} - {-, 49, 160, 120, 35, 84} = {-, 75, 111, 206, 212, 83} \equiv - 323998372947.

Multiplication:

{+, 160, 147, 222} * {+, 250, 59} = {+, 64, 10, 98, 37, 52} \equiv + 275052111156.

{+, 20, 49, 120, 223} * {-, 98, 78, 57} = {-, 168, 225, 106, 24, 47, 6, 50}

{-, 206, 187, 135} * {+, 24, 69, 47} = {-,

80, 33, 102, 203, 146, 51} \equiv - 88104388760115.

{-, 201, 167, 96, 58} * {-, 222, 189, 49} =

{+, 78, 229, 41, 25, 204, 87, 11}

Division & Modulus:

{+, 222, 190, 211} / {+, 208, 199} = {+, 16, 1} \equiv + 4097.

{+, 222, 190, 211} % {+, 208, 199} = {+, 221, 113, 0} \equiv + 14512384.

Exponential:

{+, 87, 75, 129, 89, 221} ^ {+, 4} = {+, 33, 96, 31, 76, 246, 132, 214, 67, 181, 98, 89, 82, 83,

145, 234, 146, 69, 220, 24, 143} \equiv + 190540291944837396024051783373730745 330450568591.

Comparison:

{+, 39, 67, 128, 226} < {-, 87, 75, 129, 89, 221} = Greater {+, 39, 67, 128, 226}

< False.

Related Work:**BH Flowers's work:**

Double linked list were used to store the digits, used to define the integers. The power of C++ is used to define functions to construct list. A decimalizing constructor is made that convert any stored list in decimal number. This number is returned as strings as the limit of integers were exceeded. There are also forward and backward constructors, traversing of these lists.

Future Work:

- (i). Arithmetic operators on multiple byte integer.
- (ii). Multiple byte float point numbers and they arithmetic operations.

Conclusion:

The sign character is not showing in the final result of the decimal string. The Boolean type BH Flowers made, did not work for me. If the first digit of the string is zero, the decimal will be zero. The error message function contains errors on the standard.h header file.

Chew Keong Tan's Work:

The implementation of asymmetrical cryptographic schemes often requires the use of numbers that are many times larger than the integer data types that are supported natively by the compiler. In this article, an introduction was given to the implementation of arithmetic operations involving large integers. No attempt was tried to give a full coverage of this topic since it is both complex and lengthy. For a more detailed treatment, the reader is referred to the listed [references](#) of his.

The source code that accompanies his article implements the `BigInteger` class supporting large integer arithmetic operations. Overloaded operators includes `+`, `-`, `*`, `/`, `%`, `>>`, `<<`, `==`, `!=`, `>`, `<`, `>=`, `<=`, `&`, `|`, `^`, `++`, `--` and `~`. Other additional features such as modular exponential, modular inverse, pseudoprime generation and probabilistic primality testing are also supported.

Features:

- a. Arithmetic operations involving large signed integers in 2's complement representation.
- b. Prime number tests using Fermat's Little Theorem, Rabin Miller's method and Solovay Strassen's method.
- c. Modular exponential with Barrett reduction.
- d. Modular inverse.
- e. Random Pseudoprime generation.
- f. Random Coprime generation.
- g. Greatest common divisor.

Future Work:

- (i). Faster implementation of arithmetic operations.
- (ii). More robust primality testing methods.
- (iii). Faster pseudo prime generation

Conclusion : In his article, he has provided a short introduction to the topic of large integer arithmetic. Then he has looked at how large integer addition,

subtraction and multiplication can be implemented. Also he examined the problem of primality testing and introduced the concept of primality testing based on Fermat's Little Theorem. His implementation of `BigInteger` class can be downloaded from his page and provides the overloading of most arithmetic operators. He has pointed out the limitations of his implementations of primality testing and is working towards more robust primality testing methods and faster implementation of arithmetic operators.

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