Modeling Ideal k and c Value for Lower Knee Prosthetic Leg Using Spring-Damper Model with MATLAB

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January 12, 2024

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***Abstract*— Prosthetic leg are designed to allow people who have lost their leg due to disability or amputation to regain mobility. To achieve this functionality, prosthetic legs must be designed to imitate the behavior and flexibility of real leg. In order to imitate the flexibility of a real leg, a prosthetic leg needs to be modeled with a spring and damper system. This paper aims to design an optimal lower knee prosthetic leg by determining the spring constant and damping coefficient values that can achieve ideal critical damping response with a damping ratio of 1.0 for various body mass and specific activity usage. The method used includes the formulation of system dynamics from a spring and damper model which is simulated using MATLAB. The results of ideal value of spring constant and damping coefficient are presented in the form of graphs, system performance metrics, and system response analysis.**

Keywords*— Prosthetic Leg, Spring-Damper, MATLAB, Body Mass

I. Introduction

A prosthetic leg is an artificial leg technology that is made to replace a lost leg. Prosthetic legs are designed to allow people who have lost their legs due to disability or amputation to regain their mobility[1]. Therefore, prosthetic legs need to be designed in such a way as to imitate the behavior of a real leg. In order to imitate a real leg[2], a prosthetic leg must requires the ability to absorb external forces generated by the user's activities.

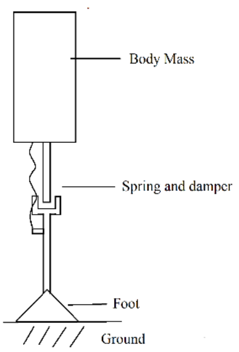

Fig. 1. Lower Knee Prosthestic Leg

Human leg are a complex part of the body that is necessary for mobility and daily activities [3], so a modeling system is needed that can simulate its flexibility. Fig 1 is a model of lower knee prosthetic leg with a spring and damper system. This spring mass damper system functions as flexibility of motion on the prosthetic leg to mimic the real leg [4].To imitate the behavior of real leg, it is necessary to model a spring and damper system that is able to absorb the forces acting on the system and the reaction forces generated during activities. The reaction force generated by the system against the user's mass can be expressed by Newton's second law where the force exerted by an object will be proportional to the mass of the object multiplied by the acceleration of the object ($F = m\ a$), which can be used to predict the movement of the model under the conditions when the forces applied on it are unbalanced [5]. The force acting on a system can be expressed using Newton's law of gravity where the force acting on a system is proportional to the mass of the object multiplied by the magnitude of the gravitational force ($F = mg$), this is used to get a realistic value of the force applied based on the user's body weight to the spring and damper system [6] . The behavior of the spring is also measured based on Hooke's law where the elasticity of the spring and the increase in its length are proportional to the force acting on the object ($F = -kx$) , this formula can be applied to see the reaction of the spring system to the applied force [7].

The problem is if the value of spring and damper systems on prothestic leg are not the same as the forces acting on them, this will result in a system not being optimal which can disrupt the user's mobility and comfort during activities [8]. This study aims to design an optimal lower knee prosthetic leg by determining the spring constant and damping coefficient values that can achieve ideal critical damping response with a damping ratio of 1.0 for various body mass and specific activity usage.

II. Method

The dynamic response of the prosthetic leg is modeled using spring damper modeling. With the main elements namely body mass (m), spring constant (k) and damping coefficient (c). Body mass (m) is considered as the independent variable. Spring constant (k), damping coefficient (c), damping ratio, Applied Force to system (F) are the dependent variables. Human activities frequency (f_{human}), Applied Force duration, Body mass scaling factor based on activities as a control variable during the experimental simulation.

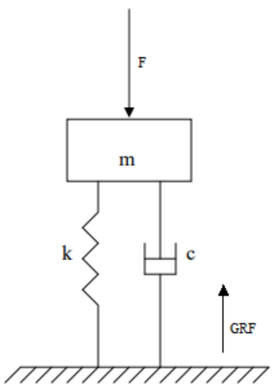

Fig. 2. MSD System – Forced Response

Experimental simulations were carried out using MATLAB with mathematical modeling calculations of the system. Fig 2 is the mass spring damper system model for lower knee prothestic leg. The system is modeled to operate under a forced response[9], indicating that an external force is applied, and the dynamic behavior of the system is simulated accordingly. The differential equation of the system can be written as follow :

$$m\ddot{x} + c\dot{x} + kx = Fx(t) \quad (1)$$

From the equation (1) can be changed using laplace transform as below :

$$(ms^2 + cs + k)X(s) = FY(s) \quad (2)$$

Then, we can determine the transfer function from equation (2) as below :

$$H(s) = \frac{X(s)}{Y(s)} = \frac{F}{ms^2 + cs + k} \quad (3)$$

Based on body mass m, the applied force to the system can be determine as :

$$F = (\,scaling\ factor \times m) \times g \quad (4)$$

In Equation (4), F represent the stance force applied to system, m is body mass, and g is the gravity force = 9.81 m/s^2. Scaling factor is used to adjust the magnitude of the applied force. Corresponding to different activities, signify the scaling factors applied to the ground reaction force during the simulated scenario. On flat terrain for walking the scaling factor is adjusted to 1.4 [10], for running is adjusted to 2.5 [11] , and for jumping is set to 5.5.

To determine the spring consant (k) , first using Hooke's law and Newton's law formula :

$$F = -kx \quad (5)$$

$$F = m\,a \quad (6)$$

In equation (5) and (6) where F is the force, k is spring constant, x is the displacement, m is mass and a is acceleration. Combining equation (5) and (6) into :

$$F = -kx = ma \quad (7)$$

Then substitute a in equation (7) with acceleration of simple harmonic motion :

$$a = -\omega^2 x \quad (8)$$

Where $\omega = 2\pi f$ in equation (8), then we get :

$$-kx = -(2\pi f)^2\, mx \quad (9)$$

From equation (9) we get the value for spring constant :

$$k = (2\pi f_{human})^2 \times m \quad (10)$$

In Equation (10), human natural frequencies are influenced by different activities. For the activity walking is set to 1.8 Hz , for running is set to 2.5 Hz, and for jumping is set to 3.0 Hz [12]. These adjusted natural frequencies reflect the biomechanical characteristics inherent to each different physical activity, thereby contributing to the dynamic behavior of the lower knee prosthetic leg model.

To determine the damping coefficient (c) using the formula:

$$c = 2\sqrt{k\ m} \quad (11)$$

Using k and c value from equation (10) and (11), we can determine the damping ratio as :

$$\zeta = \frac{c}{2\sqrt{k\ m}} \quad (12)$$

To model the motion of prosthetic leg to an external force using differential equation as below :

$$m\ddot{y} + 2\,\zeta\omega_{leg}\dot{y} + \omega_{leg}{}^2 y = \frac{f}{m}\sin(2\omega_{leg}t) \quad (13)$$

Equation (13) is used to simulate the movement of the prosthetic leg, which determines the response of the spring-damper system to the displacement, velocity and acceleration of the leg to the external force acting using Runge-Kutta method [9]. The natural frequency of the prothestic leg ω_{leg} can be determine using formula :

$$\omega_{leg} = \sqrt{\frac{k}{m}} \quad (14)$$

Where Natural frequency of prosthetic leg ω_{leg} on equation(14) represents the magnitude of oscillations at which a prosthetic leg tends to vibrate when exposed to an external force, k is spring constant and m is body mass.

III. Simulation

In this simulation, user body mass are used, that is 30kg, 55kg, and 100 kg. The simulated movement of the prosthetic leg follows a stance duration, by touching the ground and then lifting to see the system response. when touching the ground an external force will work then when lifted no force will work.. Stance duration is adjusted for different activities, 0.6s for walking, 0.3 s for running, and 0.1 s for jumping. A reaction time range from 0 to 1 seconds was used to observe the impact of the response to the prosthetic leg with the applied force.

Based on the independent variable used, calculation results for dependent variables were obtained to obtain the ideal value of the spring damper system for various activities and an estimate of the force acting on the prosthetic leg during these activities was obtained, as in the following table:

Table 1 .result of calculated F, k, c and ζ for walk

Activity	Variable	Unit	Value		
			30 kg	55 kg	100 kg
Walk	Applied Force	N	412.02	755.37	1373.40

	Spring constant (k)	N/m	3837.30	7035.05	12791.01
	Damping coefficient (c)	Ns/m	678.58	1244.07	2261.95
	Damping Ratio (zeta)	-	1.00	1.00	1.00

Table 1 contains variable values obtained from calculations for walking activities. The dependent variable values for the spring constant and damping coefficient for each body mass are obtained. With the given calculated k and c value, the force generated by the spring will be well damped so that it can reach the ideal damping ratio $\zeta = 1.0$. This value is then used to simulate the system response.

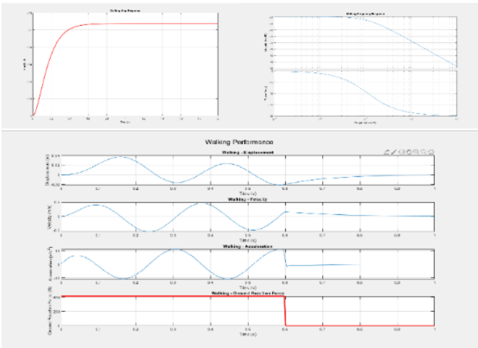

Fig. 3. system response for 30kg walk

Fig.3 is a simulation of walking activity for m = 30kg. The results obtained were k = 3837.30 N/m and c = 678.58 Ns/m. You can see the system response to the external force applied F = 412.02 N as in the table below,

Table 2. Step response properties for 30kg walk

Parameter	Value
Rise Time	0.2971 s
Settling Time	0.5159 s
Settling Minimum	0.0972
Settling Maximum	0.1074
Overshoot	0.00%
Undershoot	-0.00%
Peak Amplitude	0.1074
Max Displacement	0.0367 m
Displacement Velocity	0.4589 m/s
Displacement Acceleration	11.1786 m/s^2

Table 2 is the system response from prosthetic leg simulation results obtained for m = 30kg for walking activities. The step response results show that it is critically damped without overshoot. Settling time for the system to return to its initial equilibrium point is 0.5 s after a reaction force is applied. The maximum mass displacement to the system due to the reaction force applied is 3.7cm with a maximum displacement velocity of 0.46m/s and continues to decrease in unit time. Maximum displacement acceleration of 11.18 m/s^2 .

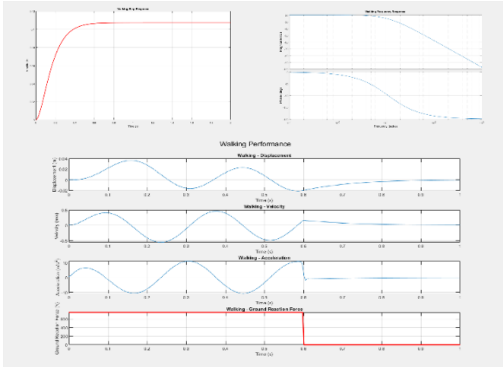

Fig. 4. system response for 55kg walk

Fig.4 is a simulation of walking activity for m = 55kg. The results obtained were k = 7035.05 N/m and c = 1244.07 Ns/m. You can see the system response to the external force applied F = 755.37N as in the table below,

Table 3. Step response properties for 55kg walk

Parameter	Value
Rise Time	0.2971 s
Settling Time	0.5159 s
Settling Minimum	0.0972
Settling Maximum	0.1074
Overshoot	0.00%
Undershoot	-0.00%
Peak Amplitude	0.1074
Max Displacement	0.0367 m
Displacement Velocity	0.4589 m/s
Displacement Acceleration	11.1786 m/s^2

Table 3 is the system response from prosthetic leg simulation results obtained for m = 55kg for walking activities. The step response results show that it is critically damped without overshoot. Settling time for the system to return to its initial equilibrium point is 0.5 s after a reaction force is applied. The maximum mass displacement to the system due to the reaction force applied is 3.7cm with a maximum displacement velocity of 0.46m/s and continues to decrease in unit time. Maximum displacement acceleration of 11.18 m/s^2.

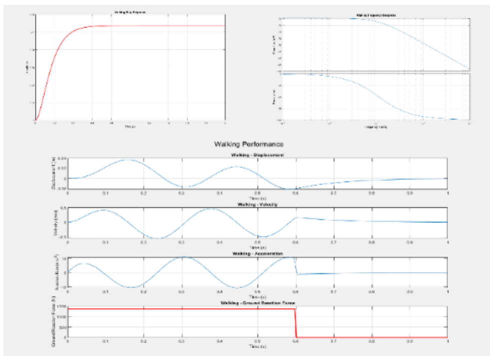

Fig. 5. system response for 100kg walk

Fig.5 is a simulation of walking activity for m = 100kg. The results obtained were k = 12791.01 N/m and c = 2261.95 Ns/m. You can see the system response to the external force applied F = 1373.40N as in the table below,

Table 4. Step response properties for 100kg walk

Parameter	Value
Rise Time	0.2971 s
Settling Time	0.5159 s
Settling Minimum	0.0972
Settling Maximum	0.1074
Overshoot	0.00%
Undershoot	-0.00%
Peak Amplitude	0.1074
Max Displacement	0.0367 m
Displacement Velocity	0.4589 m/s
Displacement Acceleration	11.1786 m/s^2

Table 4 is the system response from prosthetic leg simulation results obtained for m = 100kg for walking activities. The step response results show that it is critically damped without overshoot. Settling time for the system to return to its initial equilibrium point is 0.5 s after a reaction force is applied. The maximum mass displacement to the system due to the reaction force applied is 3.7cm with a maximum displacement velocity of 0.46m/s and continues to decrease in unit time. Maximum displacement acceleration of 11.18 m/s^2 .

Table 5 .result of calculated F, k, c and ζ for run

Activity	Variable	Unit	Value		
			30 kg	55 kg	100 kg
Run	Applied Force	N	735.75	1348.88	2452.50
	Spring constant (k)	N/m	7402.2	13570.71	24674.01
	Damping coefficient (c)	Ns/m	942.48	1727.88	3141.59
	Damping Ratio (zeta)	-	1.00	1.00	1.00

Table 5 contains variable values obtained from calculations for run activities. The scale between calculated k and c value for each body mass reach the ideal damping ratio $\zeta = 1.0$. The value is then used to simulate the system response.

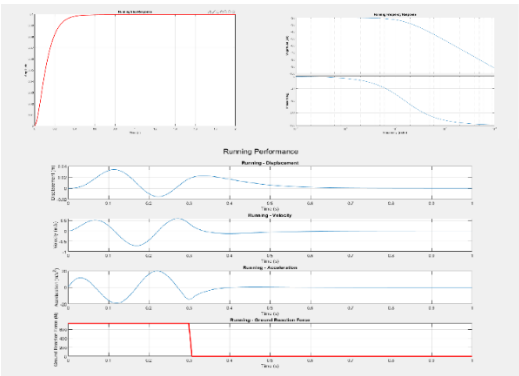

Fig. 6. system response for 30kg run

Fig.6 is a simulation of run activity for m = 30kg. The results obtained were k = 7402.2 N/m and c = 942.48Ns/m. You can see the system response to the external force applied F = 735.75 N as in the table below,

Table 6. Step response properties for 30kg run

Parameter	Value
Rise Time	0.2140 s
Settling Time	0.3715 s
Settling Minimum	0.0897
Settling Maximum	0.0994
Overshoot	0.00%
Undershoot	-0.00%
Peak Amplitude	0.0994
Max Displacement	0.0340 m
Displacement Velocity	0.5903 m/s
Displacement Acceleration	19.9677 m/s^2

Table 6 is the system response from prosthetic leg simulation results obtained for m = 30kg for run activities. The step response results show that it is critically damped without overshoot. Settling time for the system to return to its initial equilibrium point is 0.37 s after a reaction force is applied. The maximum mass displacement to the system due to the reaction force applied is 3.4cm with a maximum displacement velocity of 0.59m/s and continues to decrease in unit time. Maximum displacement acceleration of 19.97 m/s^2.

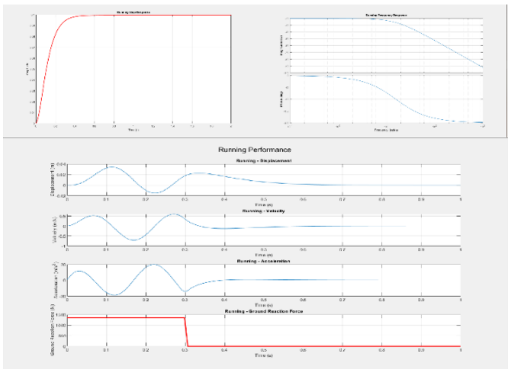

Fig. 7. system response for 55kg run

Fig.7 is a simulation of run activity for m = 55kg. The results obtained were k = 13570.71 N/m and c = 1727.88 Ns/m. You can see the system response to the external force applied F = 1348.88N as in the table below,

Table 7. Step response properties for 55kg run

Parameter	Value
Rise Time	0.2140 s
Settling Time	0.3715 s
Settling Minimum	0.0897
Settling Maximum	0.0994
Overshoot	0.00%
Undershoot	-0.00%
Peak Amplitude	0.0994
Max Displacement	0.0340 m
Displacement Velocity	0.5903 m/s
Displacement Acceleration	19.9677 m/s^2

Table 7 is the system response from prosthetic leg simulation results obtained for m = 55kg for run activities. The step response results show that it is critically damped without overshoot. Settling time for the system to return to its initial equilibrium point is 0.37 s after a reaction force is applied. The maximum mass displacement to the system due to the reaction force applied is 3.4cm with a maximum displacement velocity of 0.59m/s and continues to decrease in unit time .Maximum displacement acceleration of 19.97 m/s^2.

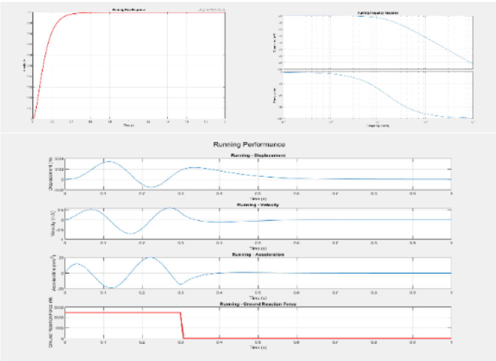

Fig. 8. system response for 100kg run

Fig.8 is a simulation of run activity for m = 100kg. The results obtained were k = 24674.01 N/m and c = 3141.59 Ns/m. You can see the system response to the external force applied F = 2452.50N as in the table below,

Table 8. Step response properties for 100kg run

Parameter	Value
Rise Time	0.2140 s
Settling Time	0.3715 s
Settling Minimum	0.0897
Settling Maximum	0.0994
Overshoot	0.00%
Undershoot	-0.00%
Peak Amplitude	0.0994
Max Displacement	0.0340 m
Displacement Velocity	0.5903 m/s
Displacement Acceleration	19.9677 m/s^2

Table 8 is the system response from prosthetic leg simulation results obtained for m = 100kg for run activities. The step response results show that it is critically damped without overshoot. Settling time for the system to return to its initial equilibrium point is 0.37 s after a reaction force is applied. The maximum mass displacement to the system due to the reaction force applied is 3.4cm with a maximum displacement velocity of 0.59m/s and continues to decrease in unit time . Maximum displacement acceleration of 19.97 m/s^2.

Table 9 .result of calculated F, k, c and ζ for jump

Activity	Variable	Unit	Value		
			30 kg	55 kg	100 kg
Jump	Applied Force	N	1618.65	2967.53	5395.50
	Spring constant (k)	N/m	10659.17	19541.82	35530.58
	Damping coefficient (c)	Ns/m	1130.97	2073.45	3769.91
	Damping Ratio (zeta)	-	1.00	1.00	1.00

Table 9 contains variable values obtained from calculations for jump activities. The scale between calculated k and c value for each body mass reach the ideal damping ratio $\zeta = 1.0$. The value is then used to simulate the system response.

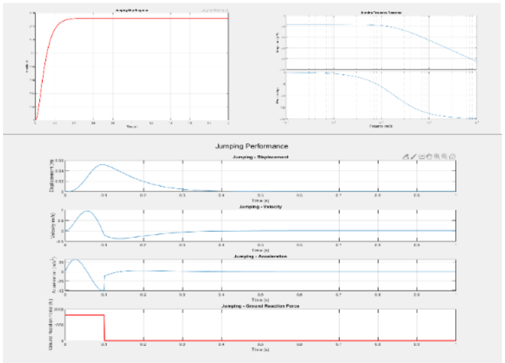

Fig. 9. system response for 30kg jump

Fig.9 is a simulation of jump activity for m = 30kg. The results obtained were k = 10659.17 N/m and c = 1130.97Ns/m. You can see the system response to the external force applied F = 1618.65 N as in the table below,

Table 10. Step response properties for 30kg jump

Parameter	Value
Rise Time	0.1785 s
Settling Time	0.3095 s
Settling Minimum	0.1375
Settling Maximum	0.1519
Overshoot	0.00%
Undershoot	-0.00%
Peak Amplitude	0.1519
Max Displacement	0.0519 m
Displacement Velocity	0.9557 m/s
Displacement Acceleration	25.9478 m/s^2

Table 10 is the system response from prosthetic leg simulation results obtained for m = 30kg for jump activities. The step response results show that it is critically damped without overshoot. Settling time for the system to return to its initial equilibrium point is 0.31 s after a reaction force is applied. The maximum mass displacement to the system due to the reaction force applied is 5.1cm with a maximum displacement velocity of 0.96m/s and continues to decrease in unit time. Maximum displacement acceleration of 25.95 m/s^2.

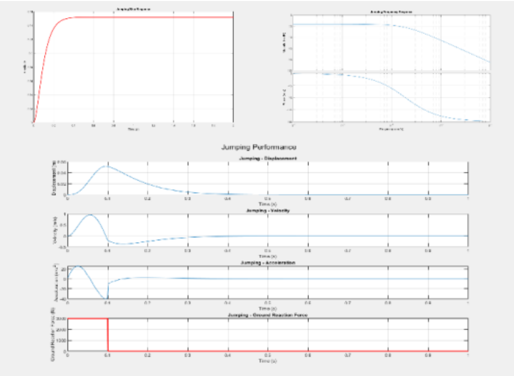

Fig. 10. system response for 55kg jump

Fig.10 is a simulation of jump activity for m = 55kg. The results obtained were k = 19541.82 N/m and c = 2073.45 Ns/m. You can see the system response to the external force applied F = 2967.53N as in the table below,

Table 11. Step response properties for 55kg jump

Parameter	Value
Rise Time	0.1785 s
Settling Time	0.3095 s
Settling Minimum	0.1375
Settling Maximum	0.1519
Overshoot	0.00%
Undershoot	-0.00%
Peak Amplitude	0.1519
Max Displacement	0.0519 m
Displacement Velocity	0.9557 m/s
Displacement Acceleration	25.9478 m/s^2

Table 11 is the system response from prosthetic leg simulation results obtained for m = 55kg for jump activities. The step response results show that it is critically damped without overshoot. Settling time for the system to return to its initial equilibrium point is 0.31 s after a reaction force is applied. The maximum mass displacement to the system due to the reaction force applied is 5.1cm with a maximum displacement velocity of 0.96m/s and continues to decrease in unit time . Maximum displacement acceleration of 25.95 m/s^2.

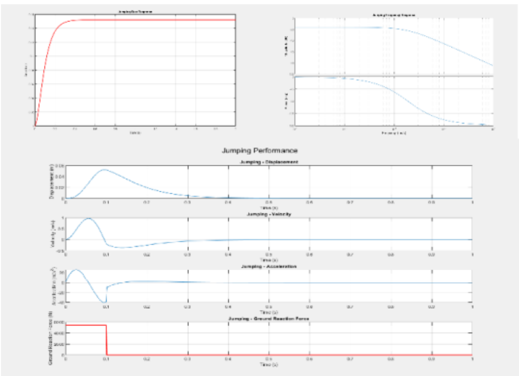

Fig. 11. system response for 100kg jump

Fig.11 is a simulation of jump activity for m = 100kg. The results obtained were k = 35530.58 N/m and c = 3769.91Ns/m. You can see the system response to the external force applied F = 5395.50N as in the table below,

Table 12. Step response properties for 100kg jump

Parameter	Value
Rise Time	0.1785 s
Settling Time	0.3095 s
Settling Minimum	0.1375
Settling Maximum	0.1519
Overshoot	0.00%
Undershoot	-0.00%
Peak Amplitude	0.1519
Max Displacement	0.0519 m
Displacement Velocity	0.9557 m/s
Displacement Acceleration	25.9478 m/s^2

Table 12 is the system response from prosthetic leg simulation results obtained for m = 100kg for jump activities. The step response results show that it is critically damped without overshoot. Settling time for the system to return to its initial equilibrium point is 0.31 s after a reaction force is applied. The maximum mass displacement to the system due to the reaction force applied is 5.1cm with a maximum displacement velocity of 0.96m/s and continues to decrease in unit time. Maximum displacement acceleration of 25.95 m/s^2.

IV. Discussion

Based on table 1 for walking activities, it shows that the k and c values required for each body mass are smaller than the k and c values required for running activities in table 5 and jumping activities in table 9. In table 5 for running activities for each user mass the value of k and c required is greater than the walking activity in table 1, but smaller than the jumping activity in table 9. For the jumping activity in table 9, the k and c values required for the system to remain stable for each user mass are greater than the walking activity in table 1 and running activity in table 5.

From that experimental data, to obtain a stable spring and damper system, spring constant and damping coefficient values are needed that are appropriate to the force received. The force applied to the prosthetic leg is based on user's body mass and the activity they are performing. The greater the mass of the user and the heavier the activity carried out, the greater the force exerted on the spring and damper system. So to achieve stability in the system by adjusting the k and c values to achieve an ideal damping ratio ζ of 1.0.

However, the system response results obtained are still limited to the activity frequency which is kept constant during the simulation and is assumed to be in flat terrain conditions. So improvements for future research can be made by considering variations in activity, frequency of movement, and more realistic terrain conditions.

V. Conclusion

Based on the experimental simulations carried out, it can be concluded that the user's body weight and activity will influence the ideal spring constant and damping coefficient values so that the prosthetic leg is more stable. The user's body weight and activity exert different forces acting on the spring and damper system. Light body weight and small activities such as walking will exert a force that is not too large on the system so that the required spring constant and damping coefficient values are not too large for the system to be stable. However, heavier body weight and activities such as running and jumping will produce a greater force acting on the system, so that the spring constant and damping coefficient will be greater so that the prosthetic leg remains stable. With system stability based on ideal spring constant and damping coefficient values, the system will be able to achieve a faster return time to the equilibrium point, short mass displacement from the system to the given reaction force, low displacement velocity, and high displacement acceleration.

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