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# How to Derive and Implement a Minimalistic RC Model from Thermodynamics for the Control of Thermal Parameters for Assuring Thermal Comfort in Buildings

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**Abstract**—Heating, ventilation, and air conditioning (HVAC) systems of buildings account for a major part of global energy demand and HVAC optimization offers significant potential to improve energy efficiency. As a promising optimization technology, Model Predictive Control (MPC) can reduce the energy demand while maintaining thermal comfort in buildings, but it also requires a thermal building model. Most existing models are too complex for a reliable parameter identification from measurements or too simple to represent thermal comfort. In this paper, we derive and implement a minimalistic thermal building model that can be applied to (i) parameter identification from measurements (grey-box modeling), and (ii) control of thermal parameters for assuring thermal comfort. We derive our grey-box model from the laws of thermodynamics, heat transfer, and the electro-thermal RC analogy. As a novelty, we present not only the detailed theoretical derivation but also the open-source code for applying the identification to various buildings. The proposed minimalistic model ensures a reliable parameter estimation and requires only a few measurements of temperatures, heating, and global radiation. We identify and validate our model with measurements from a research building under real-world conditions.

**Index Terms**—building control, thermal comfort, operative temperature, thermal building model, grey box model, model identification

## I. INTRODUCTION

Buildings are responsible for approximately 40% of energy demand [1], [2], which creates a significant potential for energy savings and load shifting. Almost half of this energy is consumed by heating, ventilation and air conditioning (HVAC) systems [3]. Advanced control strategies of HVAC, e.g. by model predictive control (MPC), provide great possibilities for energy savings [4], the integration of renewable energy [5] or grid stabilization by Demand Response (DR) [6]–[8]. For instance, DR can help to shift and store the fluctuating renewable energies of future energy systems [9], [10].

Despite the goal to reduce and shift energy consumption, the building occupants' preferences and requirements on the indoor thermal environment must be taken into account. On one side, the consideration of occupants' individual preferences is crucial and it leads to higher satisfaction and well-being [11]. On the other side, the occupants' preferences shape their energy consumption-related behavior and partially drive the occupant-building interactions. These actions have a direct impact on the buildings' energy consumption and should therefore be considered in control. This bidirectional impact could be summarized as (i) occupant-building

interactions in terms of manual control (e.g. windows openings or setpoint adjustments) and (ii) the impact of the thermal conditions on occupants' well-being and perceived thermal conditions. In that context, the manual setpoint adjustments have been in the focus of several existing studies [12], [13]. The thermal conditions that pose a significant driver for the occupants' actions have been in the focus of the thermal comfort research during the past decades [14]–[16]. Wagner et al. [15] analyzed the correlation between the occupants' satisfaction and measured thermal conditions. Langevin et al. [14] showed that thermal conditions are one of the key drivers for occupants' adaptive actions. Frontczak and Wargocki [16] pointed out that the thermal conditions have the greatest impact on humans' perceived comfort. In summary, the thermal conditions have an undoubted impact on the occupants' actions and therefore, holistic building energy consumption optimization is required for ensuring a high-quality indoor thermal environment.

In the context of MPC for assuring thermal comfort and DR, a major drawback for the widespread application of MPC is the necessity of suitable thermal building models. These models should be easily applicable to a variety of different buildings and computationally efficient. Typically, thermal buildings models rely on physical equations and material parameters (white box model), on a combination of a physical structure and measurement data (grey box model), or on data-driven technologies (black box model). White box models are complex, require excessive modeling effort, and are computationally unfriendly for the MPC application. Although black-box models are easily applicable as they need no prior knowledge of the system, they might require large data sets and lack reliability. They need to be estimated individually for each building and are therefore not yet available immediately after the buildings go into operation. Grey box models, however, offer a simple physical structure and make use of parameter identification from measurements. In the context of thermal building models, grey box models usually apply the electro-thermal analogy of heat transfer [17], [18] and are hence called RC models.

These RC models have gained significant importance for MPC in buildings because they can address the limitations of white and black box models thanks to their following benefits: (i) reliability outside identification range, (ii) requiring few data for model identification, (iii) high adaptability for MPC (thanks to continuity, linearity, and

differentiability), and (iv) representability for most buildings [17].

#### A. State of the Art

RC models have been investigated since the 1980s [19]–[25] and the first models applied two resistors and one capacitor (2RIC) to model the building envelope. After discovering the model performance drawbacks of these oversimplified models, Gouda et al. proposed more complex models in 2002 [26]. In 2018, Koeln et al. [27] remind: “*However, a series of simulation tests from [3]<sup>1</sup> that compared low order models to a 21R20C benchmark model demonstrated that a slightly higher-order model, 3R2C, provided significant model accuracy improvements over the 2RIC model with a tolerable increase in computational effort*”.

In general, Koeln et al. [27] separate the thermal building model technologies into two different categories: (i) lumped and (ii) constructive approaches. The lumped approach utilizes only a few thermal elements, thermal resistors, and capacitors, to represent an entire building. These elements do not necessarily need to represent single physical elements such as walls, windows, roofs, or floors; they can be equivalent parameters to multiple thermal elements and physical effects. The parameter of lumped models are usually identified by measurement data, also referred to as grey-box modeling [28]. For example, Park et al. [29] represent an entire building with only one resistor and one capacitor (1R1C). Harb et al. [30] developed three grey-box models and concluded that a 4R2C model performs most appropriately. Attoue et al. [31] examined the necessary model order depending on the heating power and conducted a sensitivity analysis on each parameter; model orders between two and three yielded the best results. Although the lumped approach is often applied on one-zone thermal building models that represent the entire building, this approach is not limited to only one zone and can be used for each zone of a multi-zone model [32].

In contrast, the constructive approach is often applied for more complex models. “*The constructive approach systematically builds up a zone model based on the individual building element models*” [27]. Each thermal element, such as walls, roof, floors, or windows, are individually modeled by resistors and capacitors. This approach, combined with multiple thermal zones, yields high complexity and is less applicable for parameter identification (grey box modeling) or MPC. Instead, it is applied to derive complex white-box models. Those are capable of delivering the most accurate results [33], although it is time-consuming and difficult to gather the necessary data.

In summary, we conclude that data-driven lumped approach models offer a sufficient balance between modeling effort and model performance for MPC.

#### B. Contribution to the Field

Despite the high availability of RC models and thermal comfort studies in the literature, there is no – to the best knowledge of the authors – straightforward explanation of RC models and their derivation via conservation of energy, including thermodynamic assumptions in internal energy, enthalpy, and heat transfer. Furthermore, little literature on building control with a focus on thermal comfort is available as Park et al. [34] state: “*We find that building control focuses predominantly on energy savings rather than incorporating results from thermal comfort, especially when it comes to occupant satisfaction. We identify potential research directions in terms of bridging the two fields*”.

<sup>1</sup>Remark, that [3] is [26] in the present paper.

To incorporate results from thermal comfort with building control, we answer the following **Research Questions**:

- 1) How can we derive a minimalistic RC model for the control of thermal parameters required for thermal comfort?
- 2) How can we create a link between the theoretical derivation and the practical application of the model?

## II. THEORETICAL FOUNDATIONS AND DERIVATION OF THERMAL BUILDING MODEL

We describe the foundations of heat transfer and thermodynamics that support an understanding of the RC modeling technology and thermal parameters required for thermal comfort in buildings.

### A. Heat Transfer

As a foundation, we explain the basics of heat transfer by conduction, convection, and radiation, similarly to literature [35], [36].

1) *Conduction*: Heat conduction describes the heat flow within a body, which spontaneously occurs from warm to cold and in absence of an external driving energy source. In a planar wall, one-dimensional heat transfer  $\Phi_{a \rightarrow b \text{conduction}}$  from temperature  $T_a$  to  $T_b$  can be simplified by Fourier’s Law in Eq. (1) [36]:

$$\Phi_{a \rightarrow b \text{conduction}} = \frac{kA}{L}(T_a - T_b) \quad (1)$$

$k$  - thermal conductivity,  $A$  - plane area,  $L$  - plane thickness.

2) *Convection*: Convective heat transfer is a superposition of heat conduction and movement of fluids. In contrast to only conduction, the convective heat transfer is additionally driven by fluid velocity, e.g. wind, considered by the heat transfer coefficient [36]:

$$\Phi_{a \rightarrow b \text{convect}} = hA(T_a - T_b) \quad (2)$$

$h$  - heat transfer coefficient,  $A$  - contact area.

3) *Radiation*: Thermal radiation is a form of heat transfer where a heated surface transmits energy in all directions at the speed of light. The radiation results from the thermal motion of particles. In contrast to conduction and convection, the heat transfer depends on the temperatures in the fourth-order. Eq. (3) presents the radiative heat transfer over a distance between two grey bodies in sight [36]:

$$\Phi_{a \rightarrow b \text{radiate}} = \frac{A_a \sigma (T_a^4 - T_b^4)}{\frac{1}{F} + \frac{1 - \epsilon_a}{\epsilon_a} + \frac{A_a(1 - \epsilon_b)}{A_b \epsilon_b}} \quad (3)$$

$\epsilon$  - emissivity of the surfaces,  $A$  - surfaces,  $\sigma$  - Stefan–Boltzmann constant,  $F$  - view factor between two surfaces  $a$  and  $b$ .

4) *Summary and Ohm’s Law*: While all types of heat transfer are driven by temperature differences, the main differences are that conduction occurs without an external driving energy source, convection is strongly dependent on a fluid’s velocity and radiation transmits heat over distances between bodies in sight.

The different types of heat transfer could be superpositioned and simplified by Ohm’s Law, where equivalent heat flows only depend on temperature differences and resistors. On the analogy of Ohm’s law, Eq. (4) simplifies heat flows  $\Phi_{a \rightarrow b}$ , e.g. from Eq. (1), (2), (3), with:

$$\Phi_{a \rightarrow b} = \frac{T_a - T_b}{R_{a,b}} \quad (4)$$

where  $R_{a,b}$  is the thermal resistance between the nodes  $a$  and  $b$ . For example, in the case of only convection:  $R_{a,b} = \frac{1}{hA}$  (Cf. Eq. (2)). This thermal resistance approach is an exact approximation for convection and conduction, while for radiation it is only a linear approximation.

## B. Thermal Comfort in Buildings

Modeling thermal comfort in buildings involves all three thermal heat transfer effects (radiation, convection, and conduction, Cf. Sec. II-A), as illustrated in the thermal environment in Fig. 1. The thermal environment characterizes the heat transfer between humans and buildings. Radiating heat transfer results from the surrounding surfaces and their mean radiant temperature. Convective heat transfer depends on the air temperature and the air velocity. Conduction occurs in direct contact between bodies.

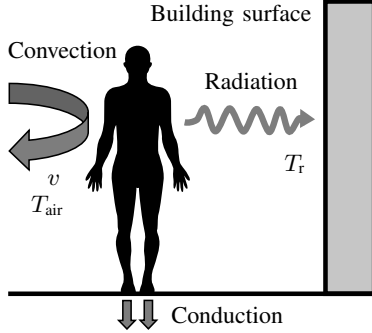


Fig. 1: Thermal environment and heat transfer effects for thermal comfort in buildings (inspired by [37], svg icons from svgrepo.com)

Thermal comfort of humans in buildings depends on the thermal environment and on personal variables, such as clothing, activity, or metabolic rate [38]. The building control system can usually affect parameters of the thermal environment, e.g. air temperature  $T_{\text{air}}$ , radiant temperature  $T_r$ , or air velocity  $v$ .

Combining the thermal parameters and effects into a single index could equivalently characterize the warmth of an environment, resulting in the operative Temperature  $T_{\text{op}}$  in Eq. (5) [38]:

$$T_{\text{op}} = \begin{cases} 0.56T_{\text{air}} + 0.44T_r & \text{for } v \leq 0.1 \text{ m s}^{-1}, \\ \frac{0.44T_r + 0.56(5 - \sqrt{10}v)(5 - T_{\text{air}})}{0.44 + 0.56\sqrt{10}v} & \text{for } v > 0.1 \text{ m s}^{-1}. \end{cases} \quad (5)$$

In that context, Gaffoor et al. [39] state: “*Indoor operative temperature is found to have the most significant influence on occupant’s thermal comfort [...]. While clothing, air velocity, and relative humidity affect thermal sensation, they have a weak correlation with TSV [Thermal Sensation Votes] and their influence is much weaker or statistically insignificant than that of operative temperature*”.

## C. Derivation of Thermal Building Model

A thermal building model is based on heat transfer (Cf. Sec. II-A) and thermodynamic equations. The thermal building model must satisfy energy conservation by the first law of thermodynamics. Neglecting changes in kinetic and potential energy, this law yields Eq. (6) [40]. The derivative of internal energy  $\frac{dU}{dt}$  depends on the sum of heat flows  $\Phi$ , work flows  $\dot{W}$ , and enthalpy flows  $\dot{H}$ :

$$\underbrace{\frac{dU}{dt}}_{\text{derivation of internal energy}} = \underbrace{\sum \Phi}_{\text{heat flows}} + \underbrace{\sum \dot{W}}_{\text{work flows}} + \underbrace{\sum \dot{H}}_{\text{enthalpy flows}}. \quad (6)$$

1) *Internal energy*: The internal energy  $U$  is the energy within a thermodynamic system, such as a thermal building element. From the fundamental thermodynamic relation  $dU = TdS - pdV$  with constant volume yields the inner energy derivative  $\frac{dU}{dt}$ :

$$\frac{dU}{dt} = C \cdot \frac{dT}{dt}. \quad (7)$$

2) *Heat flows*: Two types of heat flows occur in RC models: (i) heat flows due to internal heat generation (e.g. heating system) or external heat sources (e.g. the sun), and (ii) heat flows driven by a temperature difference. (i) Heat flows due to internal heat generation or external sources are widely called “heat gains”. A typical example is that the power used within a system will be partially converted into heat, e.g. by lights or appliances such as a printer. (ii) Heat flows driven by a temperature difference  $\Delta T_{a,b}$  are characterized by the thermal resistance  $R$  according to the electro-thermal analogy (Cf. Eq. (4)). This simplifies conduction, convection, and radiation between two nodes (Cf. Sec. II-A) into only one equivalent heat flow [35], [41]:

$$\Phi_{a \rightarrow b \text{ heat flows}} = \frac{\Delta T_{a,b}}{R_{a,b}} = \frac{T_a - T_b}{R_{a,b}}. \quad (8)$$

3) *Enthalpy flows*: Enthalpy flows describe by mass flow  $\dot{m}$  transported energy, e.g. due to ventilation. The difference between in- and outflowing enthalpy  $\Delta \dot{H}_{a,b}$  is equivalent to a heat flow  $\Phi_{a \rightarrow b}$  according to Eq. (9),

$$\Phi_{a \rightarrow b \text{ enth flows}} = \Delta \dot{H}_{a,b} = \dot{m} \cdot c \cdot (T_a - T_b) = \frac{T_a - T_b}{R_{a,b}} \quad (9)$$

where  $c$  is the specific heat capacity. Enthalpy flows occur in buildings due to ventilation and infiltration of air. Under application of an RC analogy, we derive the equivalent heat transfer resistor  $R_{a,b} = \frac{1}{\dot{m} \cdot c}$ .

4) *Work flows*: Work can be divided into “flow work”  $p \cdot v$  and other forms of work across the boundaries of the control volume [40]. Flow work is included in the enthalpy [40] and we neglect any other form of work:  $\dot{W} = 0$ .

5) *Summary – Lumped Capacitance Model*: From the energy conservation in Eq. (6), with the internal energy from Eq. (7), the heat flows and enthalpy flows in RC analogy in Eq. (8) and (9), we obtain the temperature differential equation for each node  $i$ ,

$$C_i \underbrace{\frac{dT_i}{dt}}_{\text{derivation of internal energy}} = \underbrace{\sum_{j=1}^m \Phi_{i,j}}_{\text{heat gains}} + \underbrace{\sum_{k=1}^n \frac{\Delta T_{i,k}}{R_{i,k}}}_{\text{heat transfer between nodes}} \quad (10)$$

where the resistor  $R_{i,k}$  combines several thermodynamic and heat transfer effects: enthalpy flows, conduction, convection, and radiation. In addition to heat transfer between nodes, heat gains occur, which are typically a result of the heating system, electrical appliances, and solar radiation. The heat flows of the lumped capacitance model occur between thermal elements, such as the wall or the air. Each wall can be further separated into multiple thermal elements.

The number of nodes  $z$  inside a wall determines the accuracy of the wall’s temperature profile. Fig. 2 presents a general wall model with  $z$  nodes. For  $z = 1$ , the wall temperature is homogeneous; the assumption of a homogeneous temperature applies for fast conductive heat transfer inside the wall in relation to the convective heat transfer on the wall’s surfaces [36]. A homogeneous wall temperature is rarely the case for thermal building models, as the evaluation in Sec. I-A indicates. Typically,  $z = 2$  yields appropriate model performance inside the wall [26].

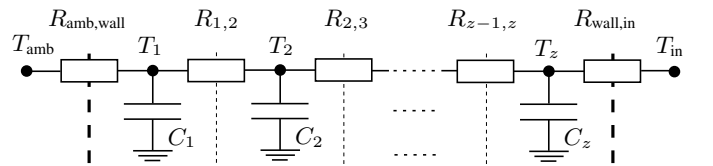


Fig. 2: Lumped parameter wall for  $z$  layers.

### III. RESULTING MODEL FOR CONTROL

Inspired by the state of the art grey box models [30], we illustrate a lumped thermal building model for control applications that is easily applicable to a variety of buildings in Fig. 3. After a general explanation of the model, we describe in the following how this model enables the control of thermal parameters required for thermal comfort.

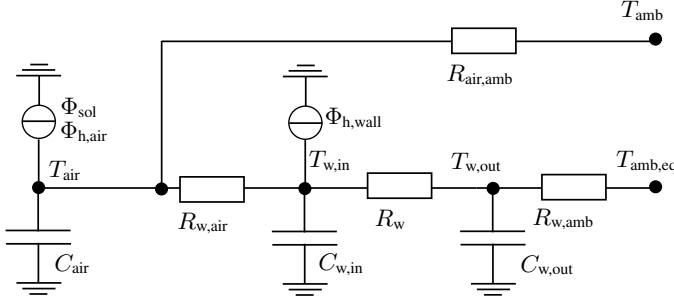


Fig. 3: RC thermal building model with 3R2C wall.

In contrast to the wall model in Fig. 2, the building model in Fig. 3 represents the thermal dynamics of the entire building. Therefore, the model contains not only wall elements, but also indoor air and connections to the ambient.

#### A. Thermal Building Model

The illustrated model in Fig. 3 consists of three inputs: the heating  $\Phi_h$ , the solar radiation  $\Phi_{sol}$ , and the ambient temperature  $T_{amb}$ . These inputs affect the building temperatures, which are dynamically described by states: one state for the indoor air temperature  $T_{air}$  and two for the walls, an inner wall element  $T_{w,in}$  and an outer  $T_{w,out}$ . The temperature dynamics are described by the differential Eq. (11) - (13), based on the Lumped Capacitance Model methodology in Eq. (10):

$$C_{air} \frac{dT_{air}}{dt} = \frac{T_{w,in} - T_{air}}{R_{w,air}} + \frac{T_{amb} - T_{air}}{R_{air,amb}} + \Phi_{sol} + \Phi_{h,air} \quad (11)$$

$$C_{w,in} \frac{dT_{w,in}}{dt} = \frac{T_{air} - T_{w,in}}{R_{w,air}} + \frac{T_{w,out} - T_{w,in}}{R_w} + \Phi_{h,wall} \quad (12)$$

$$C_{w,out} \frac{dT_{w,out}}{dt} = \frac{T_{w,in} - T_{w,out}}{R_w} + \frac{T_{amb,eq} - T_{w,out}}{R_{w,amb}}. \quad (13)$$

We describe the system inputs more precisely in Eq. (14) and (15). The heating in Eq. (14) is separated into a radiant and a convective part by the factor  $f_{heat,rad}$  (radiation heat flux contribution), where the convective part heats the air and the radiant part the inner wall:

$$\Phi_{h,air} = (1 - f_{heat,rad}) \Phi_h, \quad \Phi_{h,wall} = f_{heat,rad} \Phi_h. \quad (14)$$

The solar input in Eq. (15) is determined by the global radiation  $\phi_{global}$ . The sun directly affects the inside air temperature by  $\Phi_{sol}$ , which depends on the global radiation  $\phi_{global}$  and the solar heat gains factor  $f_{sol}$ . The heat transfer on the outside wall results from the equivalent ambient temperature  $T_{amb,eq}$ , which combines the ambient temperature  $T_{amb}$  and the global radiation  $\phi_{global}$ . This equivalent temperature  $T_{amb,eq}$  also depends on the parameters short-wave absorption coefficient  $h_f$ , and the exterior heat transfer coefficient  $h_A$  [30]:

$$\Phi_{sol} = f_{sol} \phi_{global}, \quad T_{amb,eq} = T_{amb} + \phi_{global} \frac{h_f}{h_A}. \quad (15)$$

#### B. Control Variables for Thermal Comfort

For thermal control of buildings, we define the heating  $\Phi_h$  as the control input  $u$  and the weather conditions, which is a combination of ambient temperature  $T_{amb}$  and global solar radiation  $\phi_{global}$ , as

measurable disturbances  $z$ . Instead of the indoor air temperature, we propose the control output  $y$  as the operative temperature from Eq. (5) by our simplified definition in Eq. (16). Therefore, we use the inner wall temperature  $T_{w,in}$  as radiant temperature  $T_r$  and assume negligible air flows:

$$y = T_{op} = 0.56T_{air} + 0.44T_{w,in}, \quad (16)$$

$$u = \Phi_h, \quad z = (T_{amb}, \phi_{global})^T.$$

For more information about the control application, we refer to our previous work [42].

#### C. Model Implementation and Validation

We implement the previously described model in Matlab and identify the thermal parameters with the *Matlab Identification Toolbox* [43]. We publish our model implementation as open-source code on Github<sup>2</sup>. Our implementation identifies a thermal building model from measurements and has two user-defined functions: (i) defining the model equation, and (ii) defining parameters to be identified. Given those two functions, our code provides easily applicable methods for identifying and validating the model with the use of input and output data.

For the identification, we use measurements with a sample rate of 2 min from May 06 – May 18, 2021 in a research building [44] under real-world conditions. We measure the indoor air temperature  $T_{air}$ , the heating (power of electrical heater)  $\Phi_h$ , the ambient air temperature  $T_{amb}$ , and the global radiation  $\phi_{global}$ . While we know that the research building is used as an office space, we have no information about the occupancy behavior, such as attendances or windows openings. We validate the model on the indoor air temperature with measurements from May 19 – May 31, 2021, without any re-calibration of the temperature. The results for the identification and validation are presented in Fig. 4 and Fig. 5.

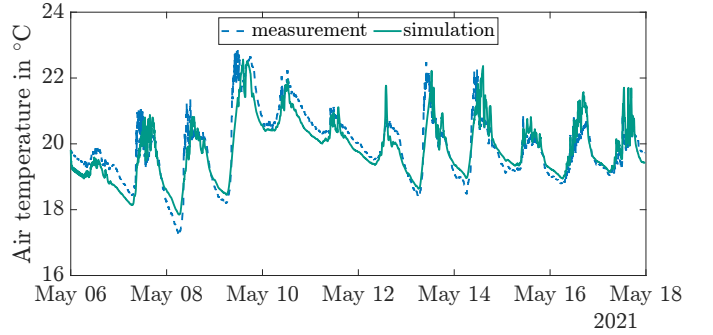


Fig. 4: Identification results

Over the entire validation period (May 19 – May 31, 2021), the model yielded a Mean Absolute Error (MAE) of 0.556 K and a Root Mean Square Error (RMSE) of 0.705 K. The error increases considerably on each of the weekends, May 22-24 and May 29-30, and decreases again on the weekdays, as the model does not account for occupancy behavior. While the building is used as an office space, the absence of occupants on the weekend can lead to lower internal heat gains and thus reduce the measured temperatures, compared to the simulation.

<sup>2</sup><https://github.com/Building-Measurement-to-Control-Toolbox/Matlab-Toolbox-Pub>

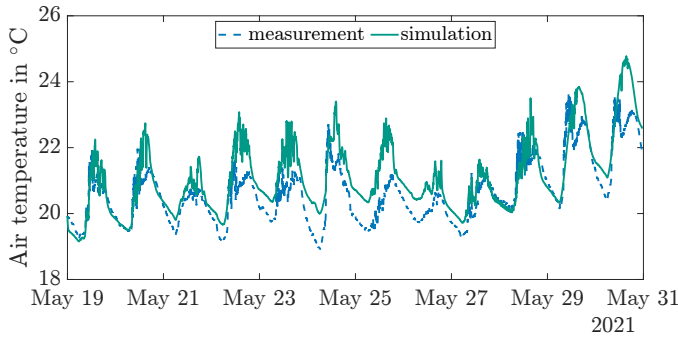


Fig. 5: Validation results

#### D. Discussion of Model

Our derived RC model is identified and validated with measurements from a research building under real-world conditions. Therefore, we use the Matlab Identification Toolbox and publish our open-source code. In general, the validation results demonstrate a minor deviation from the measurement. The most significant contribution to the mismatch to the measurement occurs on the weekend, resulting in an overall RMSE of 0.7 K over the validation period of 12 days. This indicates a different occupancy behavior in the office space during the weekend. The integration of occupancy behavior into thermal building models for control should be addressed in future work.

We extend our RC model by a formulation of thermal comfort based on the operative temperature  $T_{op}$  that combines two of the temperature states: the indoor air temperature  $T_{air}$  and the inside wall temperature  $T_{w,in}$ . As a result, we obtain a simple modeling approach to provide thermal parameters required for thermal comfort in a building. The model is identifiable and requires only measurements of temperatures, heating, and global solar radiation.

#### IV. CONCLUSION

The present paper answers 1) how to derive a minimalistic RC model for the control of thermal parameters required for thermal comfort, and 2) how to create a link between the theoretical derivation and the practical application.

Therefore, we derive an RC model from the laws of thermodynamics and heat transfer and also consider thermal comfort in the control equations. As control output, we propose the operative temperature  $T_{op}$  to represent the thermal comfort of humans in buildings. In that way, we consider not only convective heat transfer in the thermal environment, mostly depending on the indoor air temperature  $T_{air}$ , but also the radiating heat transfer with the surface wall temperature  $T_{w,in}$ .

We implement the resulting thermal building model as a grey box model in Matlab and publish this code open source. A grey box model uses a pre-defined model structure together with measurements to determine the dynamic thermal parameters (resistors, capacitors, and solar factor). To identify and validate the model, we measure the indoor air, outdoor air, and wall temperatures as well as the heating, and the global radiation. The major benefits of this grey box model are the high reliability and adaptability for MPC, when compared to black-box models (Cf. Sec. I).

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