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Type-II emphasizing Type-II(B) in Ramond–Ramond Sector has been analysed and computed from the Atiyah–Hirzebruch spectral sequence taking E_i sheets for the concerned values of $i \equiv 4 = \infty$ and for $E_n^{p,q}$ for $n = 1,2,3$; several varieties of K–Theories where a transitive approach has been shown from the KK–Theory to K–Theory to String Theory concerning Fredholm modules of Atiyah–Singer Index Theorem and the Baum–Connes conjecture with respect to the Hilbert–A, Hilbert–B module and c*-algebras also in the reduced form taking Morita equivalence and the Kasparov composition product where extended relations has been provided between the equivalence of noncommutative geometry and noncommutative topology channelized through Poincaré Duality, Thom Isomorphism and Todd class.

I. INTRODUCTION

Any map[1] from a domain to a codomain with the mapping parameter $\theta : \zeta \longrightarrow \zeta'$ can provide a continuous set of functions when ζ and ζ' is endowed with a metric which when attempt for any representation of a Topological structure considering two sets $\{\zeta\}\ and\ \{\zeta'\}$ there norms even a bijection[1] between them $\zeta \ \longleftrightarrow\ \zeta'$ which for a defined function f over a value of $f(x)$ there involves a structure of a vector space with concerned operations through a continuous linear transformation, that space for that function carries a Topology best known as Hilbert space. The specified module that carries the $c*-$ algebra[2] for that space is defined as $c*-$ Hilbert modules[3] through the inner product.

For any group ^ with a subgroup l the representations Γ_l^{Λ} makes it easier to construct new representations through the subgroup or the smaller group l over certain parameters that when categorize through the constructive modules of Hilbert's $c*$ then this extent the $c* -$module to $c* -$algebras through the non$-$ commutative formulations[4,5].

Furthermore, any derived pathway to construct the non$-$ commutative geometry provides a framework for the moulder category to represent an equivalence over $(left - right) - symmetric$ rings[6,7,8,9] as established afterwards with rings R and R'; then for the $ring - representations$, studying the category of those modules; there exists Morita equivalence[10] for the isomorphic commutative form or in general norms in the case of $non - commutative$ rings[11].

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For the constructions of $KK - Theory$; Morita equivalence is an important tool to $c *-$ algebras where for the inequality on the two modules $A\ and\ B$; for the moulder form E on A and B for the moulder form E on A and $E\cdot$ on B (*as appeared later in the paper*) a homotopy invariant bifunctor can make a Morita equivalence for the $KK - Theory$ through $KK(A, B)\ and\ KK(B, C)$ for A, B, C as $c *-$ algebras; there's for the modular form E having elements ε, ϵ the inequality represents the form $< \varepsilon, \epsilon >< \epsilon, \varepsilon > \leq || < \varepsilon, \varepsilon > \ || < \epsilon, \epsilon >$ where for the $A - module$; the above relation holds and taking the $B - module$ representing the $c *-$ algebraic pair $KK(A, B)\ and\ KK(A, C)$ where one finds the combined form over the composition product representing $KK(B, C)$ such that $B\ and\ C$ supposedly (strong) Morita equivalent to be represented in a non$-$ trivial way as to be proved throughout the paper[12,13].

Over the compact Hausdorff spaces[14] and considering the Fredholm modules of Atiyah–Singer Index Theorem[15] for a relatable definition of A, B, C in $c *-$ algebras the Kasparov's product $KK(B, C)$ for $KK(A, C)\ and\ KK(A, B)$ will be established over a closed interval taking an elliptic differential operator $\varrho^0_{M_s} or\ \varrho^0_{M_n}$ for $s - smoothness\ or\ n - \dim$ and through extensive analysis of that operator which indeed suffice the Fredholm module making a relatable framework for $K -$ *Homology* and $K - Theory$ [16,3,5,6] in interval $[0,1]$; The Thom isomorphism is established for the Chern Character Ch over a mapping parameter ι through a $rank - n$ vector bundle $v_{1(n)}$ with v_2 having the first related to a unit sphere bundle. This in turn induces the categorical correspondence between a relational establishment over noncommutative geometry and noncommutative topology taking the function f over a bounded structure through linear transformations that bounds the concerned subsets $I\ and\ J$ for a mapping parameter ρ_η in the same Hilbert space H.

This will deduce for a much more concrete formalism of the $K - Theory\ to\ K -$ *Homology* with an extension of $c *-$ algebras to $reduced\ c^* - a\lg e\ bras$ for parent group (^) that defined the ℓ^2 norm of Hilbert space taking into consideration the $KK - Theory$ with Gromov's $a - T - menable$ property for all the necessary formulations concerned before except Morita equivalence that when established through $5 - parameters$ through an assembly mapping parameter ד over discrete torsions gives the ultimate relation of $KK - Theory$ in $Baum - Connes\ conjecture$ taking into account both the $Novikov\ conjecture$ and $Kadison - Kaplansky\ conjecture$ for injectivity and surjectivity respectively connecting to noncommutative topology[17,18,19].

Extensions have been made in the operator and Topological aspects in the cohomology class where several classifiers are shown with distinct property to suffice the Sp_c – Structure and the Atiyah – Hirzebruch spectral sequence for the Type II (II-A and II-B) as concerned on the complex Topology space T* where the Atiyah – Singer Index Theorem taking the Fredholm modules as necessary for K-Theory with Bott – Peiodicity is taken and a cannelization is made to Grothendieck – Riemann – Roch; for the transition of KK-Theory to Strings; Hodge dual, Gauge symmetry, charge density for the required Lagnagian in RR-fields through D-Brane Potential, De Rham Cohomology, and GSO – Projections are shown. P-form electrodynamics ans P-Skeleton are considered for the purpose. NS 3-form and its relation to RR-flux in both D-Brane charge density and supergravity is established. The spectral sequence of Atiyah-Hirzebruch is taken and operator over $E^{p,q}_n$ for n taking the values 2,3,∞ over a consideration of several orders of K-Theory as such Topological, Algebraic, and Twisted. etale cohomology and its representation is shown for Algebraic K-Theory and the Kahler (without any specific consideration of compact and Ricci flatness) has been shown in general terms for K-Theory in a Twisted formalism in E_i for $i = 4 = \infty$.

II. ESTABLISHING MORITA EQUIVALENCE WITH KK-THEORY FOR HILBERT C*-MODULE

For a Hibert space H with a c*-module H_c one can define a c*-algebra for the metric g on a Riemann manifold M (*having the form* M_g) with a vector bundle V there exists a compact neighbourhood being locally variant on a small patch; over an isomorphism of the Hilbert space of that vector bundle V in a continuous way for a commutative c*-algebra through the vanishing infinity.

For the modular form of c*-algebra the Hilbert module for the non–commutative form is the generalized norm taking the algebra over a topological field $\mathcal{T}$ in unital formulation for the unit parameter $\imath$ as such for every ϵ in the algebra there exists $\epsilon = \imath\epsilon = \epsilon\imath$.

Representing over the induced form for any finite group ^ with $\ell \subset$ ^ for the vector bundle $\mathcal{V}$ on the Hilbert space $\mathcal{H}$, any construction can be defined over the k –elements of the group ^ over L defined a parameter $\mathcal{P}$ as[19],

$$\mathcal{P} = \sum_{k=1}^{n} L_k$$

This gives for each k, the induced representation through group ^ in the same $L_k^+ \in L_k$ for $\ell \subset$ ^ through the vector representation $\mathcal{V}$ of subgroup ℓ being $\ell \subset$ ^ in Hilbert space $\mathcal{H}$ parametrized through[3,6],

$$\mathcal{X}_{(\pi,\mathcal{V})}$$

Thus, one gets,

$$\text{for every} \bigoplus_{k=1}^{n} L_k^+\mathcal{V} \ \ \text{there is,}$$

$$\sum_{k=1}^{n} L_{(1,\ldots\ldots,n)k}\pi(L_k^+)\mathcal{E}_k$$

Representing $\mathcal{E}_k \in \mathcal{V}$, three non-trivial actions can be noted for the constructions[20,21],

1. $\mathcal{E}_k \in \mathcal{V}$
2. $L_k^+ \in L_k \forall \ell \subset$ ^
3. $\ell \subset$ ^

This takes a pre-Hilbert Hausdorff space to construct c*-algebra satisfying the operations of an inner product through the Hilbert A–module being non–negative and self-adjoint. Taking the inner product of the complex manifold representing $\mathcal{M}^*$ through,

$$\mathcal{M}^* \times \mathcal{M}^* \longrightarrow A$$

Thus, for any sequence of set that is countable over the Topological space $\mathcal{T}$ with a proper representation for the previously encountered manifolds $\mathcal{M}^{\mathcal{T}}$ taking k^{th} countable order of infinity,

$$\{\mathcal{M}_k^{\mathcal{T}}\}_{k=1}^{\infty}$$

When merged with the unital form taken before $\epsilon = \imath\epsilon = \epsilon\imath$ such that for every unit parameter $\imath$ there exists ϵ in the algebra; where for any c*-algebra there holds the Banach–algebra for a compact $\mathcal{F}$, that if provided there exists three forms taking $B_0(\mathcal{F})$[3,19,20,22],

1. $Typical\ form$ – For the complex space $\mathcal{M}^*$; the locally compact Hausdorff space for vanishing infinity norm gives $B_0(\mathcal{F})$ for continuous functions on $\mathcal{M}^*$.
2. $Unital - \begin{cases} \text{if is commutative} \\ \text{identity element of having norm 1} \\ \mathcal{F} \text{ in } B_0(\mathcal{F}) \text{ is compact} \end{cases}$
3. For Point [2] to have a congruent transformation, there is Banach algebra $B_0(\mathcal{F})$ in A–form where the congruent transformation is unital for a closet set $[A]$.

For the compact Hausdorff (here parameterizing $\mathcal{F}_0^+$) with vector bundles $\mathcal{V}$ for the labeling of $\mathcal{F}_0^+$ - 0 for positive to extend over Bott Periodicity with + as adjoint through 8–periodic homotopy groups from π_0 to π_7 such that[12],

$\pi_{0,1,2,3,4,5,6,7}$ gives 3 − category tables in unitary U, orthogonal $\mathcal{O}$, symplectic Sp,

$$\begin{array}{cccc} \underline{U} & \underline{\mathcal{O}} & & \underline{Sp} \\ \pi_k \longrightarrow \pi_{k+2} & \pi_{k+8} & \overset{=}{\longrightarrow} & \pi_{k+4} \quad \forall k = 0,1 \ldots \\ & \pi_{k+4} & \overset{=}{\longleftarrow} & \pi_{k+8} \end{array}$$

Thus, for Hausdorff $\mathcal{F}$; the underlying K-Theory $K(\mathcal{F})$ there is[12,23];

I. Topological K-Theory $\Longrightarrow$ *on* $\mathcal{M}^{\mathcal{T}}$ *for* $K(\mathcal{F})$
II. Reduced K-Theory $\Longrightarrow K_{red}(\mathcal{F})$ *for* $S^n \exists n > 0$ relates the Bott for *positive* 0 *for* $\mathcal{F}_0^+$ and *adjoint* + in Hausdorff $\mathcal{F}$ for $K_{red}(\mathcal{F}_0^+)$ in non–commutive form.

Where Point [I] relates the Banach–algebras for the locally compact Hausdorff over a abelian module on any sequence of set countable over Topological space $\mathcal{T}$ (as previously mentioned) on c*-algebras for bivariant forms suffice the proper framework for the Hilbert c*-module on rings R and R' for modular homeomorphisms on R such that the biproduct exists in finitary over a defined functor δ preserving equivalence and additive properties[9,16,23],

$$\delta : mod - R \longrightarrow mod - R'$$

$$\delta' : mod - R' \longrightarrow mod - R$$

$$\begin{cases} \delta : mod - R \longrightarrow & mod - R' \\ \delta' : mod - R' \longrightarrow & mod - R \\ & \begin{cases} suffice\ Morita\ Equivalence\ (strong) \\ for * -operations\ on\ c * -algebras \end{cases} |_{R^{Morita} \approx R'^{Morita}} \end{cases}$$

For the naturally induced isomorphism for functors δ and δ' for a finite module ring R for the bi–module (R, R') suffice the natural isomorphism iff for $X_{(R,R')}$ *and* $Y_{(R',R)}$ there is[2,3,9,16,23],

$$\begin{array}{lll} (R, R') - bimodule & \Longrightarrow & X_{(R,R')} \otimes_{R'} Y_{(R',R)} \cong R \\ (R', R) - bimodule & \Longrightarrow & Y_{(R',R)} \otimes_{R} X_{(R,R')} \cong R' \end{array}$$

Moreover, if we consider A, B *and* C as c*-algebras then if there is a Hilbert B -module that is fully countably generated in the form of E , then for that c*-subalgebras of B there exists a strong Morita equivalence between A and B provided for the B module there is $\varphi\ (E) \cong A$ and for A module there is $\varphi\ (E\ \cdot\ ) \cong B$ where for the c*-algebraic pair (A, B), over a homotopy invariant bifunctor the constructions can be taken for A , B and C in such a way that for the defined abelian group $KK(A, B)$ and combining it with $KK(B, C)$ a strong Morita equivalence can be established in the form[2,16,24],

$KK(A, B) \cong KK(A, C)$ ∃ Combining the elements of $KK(A, B)$ AND $KK(B, C)$, there exists the intersection product and the non-trivial assumptions that B and C are strongly Morita equivalent.

III. RELATING NONCOMMUTATIVE GEOMETRY WITH NONCOMMUTATIVE TOPOLOGY

Now, for the linkage of $K - Theory\ to\ K -$ *Homology* and c*-algebras for the locally compact Hausdorff spaces, there can be a relatable definition of the c*-algebra through noncommutative topology where there exists a detailed constructions to be discussed below[13,17,18,25].

The mostly related theorem that suffice this duality with an equivalence between noncommutative geometry and noncommutative topology; just like the formulations of the Kasparov's composition product over A, B, C in c*-algebras giving the result $KK(A,C)\ for\ KK(A,B) \times KK(B,C)$ with the associated Morita Equivalence (as stated and established earlier in the paper) for the closed interval $[0,1]$ any abelian group taking a trivial parameterization $\gamma(A)$ or can represent the Atiyah – Singer Index Theorem for the vector bundle V having the elements $v_1\ and\ v_2$ which over the smooth manifold M_s with $'s'$ representing the smoothness property and the elliptic differential operator for the mapping over smooth sections on M_s as,

$$\varrho^0_{M_s} : v_1 \longrightarrow v_2$$

Where for this elliptic differential operator $\varrho^0_{M_s}$ implying the Fredholm modules on the Hilbert space H for c*-algebras there is the Chern character $Ch(\varrho^0_{M_s})$ giving Thom isomorphism with the mapping of vector bundles of $rank - n$ through,

$$\iota : v_{1(n)} \longrightarrow v_2$$

Taking the unit sphere bundle $S(v_1)\ and\ v_2$ representing the Chern character $Ch(\varrho^0_{M_n})$ for $n -$ *dimensional* compact manifold over the relation through a complex Tangent bundle T' as,

$$\left[H^k(T\,';\mathbb{Q}) \longrightarrow H^{n+k}\left(v_2(T\,')\right) / S\left(v_2(T\,')\right);\mathbb{Q}\right]^{-1} = Ch\left(\partial\left(v_{1(\iota^*)}, v_{2(\iota^*)}, \xi\left(\varrho^0_{M_n}\right)\right)\right)$$

$$\exists v_2(T\,') \Longrightarrow\ vector\ bundles\ v_{1(\iota^*)}\ and\ v_{2(\iota^*)}$$

Where $T\,'$ represents the complex tangent bundle of Todd class $Td(T\,')$

Where $\xi\left(\varrho^0_{M_n}\right) \Longrightarrow\ isomorphisms\ on\ S(T\,')\ in\ Td(T\,')$

Which establishes the $KK - Theory$ through Fredholm module $\varrho^0_{M_n} or\ \varrho^0_{M_s}$ for $n - \dim\ \ or\ s - smoothness$ where both are considered for the purpose of the constructions of Atiyah–Singer Theorem.

For the relation between noncommutative geometry and noncommutative topology it is now easy to show the c*-algebras for the dual category of the Hausdorff spaces over *— isomorphism through the operator theory for a bounded structure over a function f operating through linear transformations through two vector spaces $I\ and\ J$ that are bounded through the image of the function $f(\eta)$ for the η taking control over the mapping parameter ρ as ρ_η for $\rho_\eta : I \longrightarrow J$ where ρ_η makes the transformations that bounds the subsets of I to subsets of J on the same Hilbert space H.

Towards the establishment of noncommutative topology as described above in the paper the relation between noncommutative topology with noncommutative geometry over a non–trivial prescriptions of $c\ *-$ Hilbert modules and Hausdorff space that gets channelized further to establish the $KK - Theory$ and Morita equivalence; a considerable fact is that for the proper extensions of $c\ *-$ algebras there is a defined category of the operator formalisms in the algebraic notions of $K - Theory$ where it can be shown that for the parent group (*taken before*) ^ with the $c\ *-$ algebra, any reduced category for the completion of c^*_{red} (^) formalism through a locally compact Topological group (*denoting with a trivial notation just for the formulations*) as ^' for a translation invariant norm through bounded functions; this c^*_{red} (^) has an isomorphism for $c\ *$ (^) where any defined $c\ *-$ algebra can be expressed taking the c^*_{red} (^) as a quotient of $c\ *$ (^) for the Hilbert space H having the defined norm H_{l^2} there exists $5 - parameters$ that connects the $K - Theory\ to\ K -$ *Homology* for making $KK - Theory$ which provides a relation to Gromov hyperbolic groups[26] along with the groups that defined ^' for a translation invariant norm through bounded functions in $SL_3(\mathbb{Z})$along with other $rank - 1$ Lie Groups and other discrete Lie Groups $SO(n,1)$ and $SU(n,1)$ with Gromov's $a - T - menable$ property for the assembly mapping parameter ℸ (*which will be extremely useful later in the paper*) for isomorphism having the representation of[16,18,25],

$$Baum - Connes\ conjecture^{injectivity\ \Rightarrow\ Novikov\ conjecture}_{surjectivity \Rightarrow Kadison-Kaplansky\ conjecture}$$

For ρ_{free} (^) $\exists\ \rho$ represents discrete torsion for group (^)

Where the $5 - parameters$ are the $5 - classifiers$ viz.,

1. $A - module$
2. $B - module$
3. $\rho^*_{free} - Kadison - Kaplansky$
4. c *− algebra
5. $KK - Theory$

$for\ action\ S \implies S^{\rho^*_{free}}_{over\ A} through\ c *{-}automorphisms$

Where $a - T - menable$ group for Hilbert space H on the (*previously taken*) $\ell \subset$ ^ giving three non–trivial connections to conclude the paper,

For $\sum_n^{\ell}$ summing over $n - elements$

For $H * \in H$ where $\sum_n^{\ell}\ \ H * \longrightarrow \infty$

For c^*_{red} (^) $\longrightarrow$ assembly mapping parameter ד over a norm $|N|^2$provides the relation,

$$(\sum n|N|^2)_{ד}{}^{\frac{1}{2}}_{c^*_{red}}(\hat{}) \coloneqq sus\ (\ ||N\ *(\ \ell \subset\ \hat{}\ )\ ||_2:\ ||\ell \subset c\ (\hat{})\ ||_2:\ = 1\ )$$

IV. GELFAND TRANSFORM AND POINCARE DUALITY FOR C*–ALGEBRA IN k^{th} HOMOLOGY GROUP IN Sp_c

Considering an involution ι_0 for the Topological group ^ with the defined Harr measure μ in a locally compact Hausdorff space F there exists a commutative spectrum S^{σ} where for the unital element i being the element of S^{σ} for the Gelfand space G representing,

$$i \in S^{\sigma}\ in\ G$$

There exists a commutative form for an algebraic isomorphism α^* in two categories of algebras,

1. c *− algebra for an enveloping c * norm in $\alpha^* -$ isomorphism.
2. Banach algebra for the continuous function f_c

Considering Point [2], one gets the transform of G representing as G_c for $c -$ continuous form through $2 - norms$ for the group action of group ^ defined ℓ_+(^) and ℓ_{++}(^) where for the spectrum S^{σ} in Point [1], gives the modified form of a Fourier Transform as Gelfand Transform for G_c.

$\exists\ \exists\ for\ \ell_+(\mathcal{R})\ in\ G_c$ and $f_c \in \ell_+(\mathcal{R})$ any c *− algebra for the Hausdorff space F over a two− way mapping π: F $\longleftrightarrow$ F′ where F′ is also a Hausdorff space there exists;

Gelfand− Naimark Transform $\Longrightarrow$ c*(F) and c*(F′) in noncommutative c *− algebras the spectrum S^{σ} can be defined over π' for Hausdorff F in $G_c - norm$ in $\alpha^* - isomorphism$ as,

$$\pi'\colon \mathrm{F} \rightarrow c_+\left(\alpha^*_{c^T}\right)\forall^{T\ in\ an\ identfiable\ Topo\log ical\ space\ in\ c^T}_{c_+ in\ a\ spacial\ case\ of\ c^*-norm}$$

Where $c_+(\mathrm{F}) \rightarrow c_+\left(\alpha^*_{c^T}\right)$

The two norms in group action for group ^ namely, ℓ_+(^) and ℓ_{++}(^) for a Borel measure β for ℓ_+(^) $\cong \mu \exists \mu$ represents the Harr measure (*as considered earlier*) through the involution ι_0 one gets a generalized notion as,

Noncommutative geometry established over $f_c \ \forall \ell^2_{++}$(^)$-norm\ -norm \exists f_c \in \ell^2$(^) there are,

1. Subspace $c^*_{sub}$for $c^* - a\lg e\, bra$
2. The generalized norm ℓ_{++} (^)
3. Banach algebra B_0 in Banach space$B_0(X)$ with a Borel measure β in continuous f^*_c for subgroup $\ell \subset$ ^ in $c^* - subspace$ in $c^* - a\lg e\, bra$ suffice the form,

$\sum_{\mathcal{M} \in c^* - sub} d\mu\ Tr\big(f^*_c(\mu)\big)\mu_\epsilon$(^)

Where μ_ϵ acts on the Haar measure μ for group ^ over the action μ_ϵ(^).

Now, for the Gelfand— Naimark Transform; a generalized application of the Fourier Transform (*rather Gelfand Transform*) with its application in noncommutative geometry for the isomorphism over a 'assembly mapping parameter ᒣ' that we considered earlier, there can be the application for both c^*_{red} and $c^* - a\lg e\, bra$ for an $index^0_1$ in *— over $c^*_{red} - a\lg e\, bras$ the common notion that arises is of,

1. Taking a discrete torsion-free parameter $\psi -$ compact for an equivariant $k -$ homology for the norm of 'right-side accessible form is always difficult than left-side accessible form' — the Baum— Connes conjecture can be extended for a proper action (*without considering any classifying space S^{cl}_c* for ψS^{cl}_c); the 'assembly mapping parameter' ᒣ takes the mapping denoted by $\psi -$ subscript for action $\dot{S}$ given,

$$\dot{S} = ᒣ_{\psi - com} \varphi^{\psi - com(R)}_{0or1}(E\psi^{\dot{S}}_{-com}) \longrightarrow \varphi_{0or1}\big(c^*_{red}(\varphi - com)\big)$$

2. Taking the same discrete and torsion-free parameter (which has been considered $\varphi\ or\ \rho$ throughout the paper for notational significance (without any reduced form); which now acts on the classifying space φS^{cl}_c for the complex integers denoting Λ ; the $c^* - a\lg e\, bra$ gets the Gelfand-Naimark Transform in the commutative way that becomes accessible for the Poincare duality to consider upon. However, for the case considered here, any automorphisms acting on c^* for $A - module$ gives the Baum-Connes conjecture in the form of ᒣ with $A\ and\ \varphi$ as,

$$ᒣ_{A,\varphi} : R\varphi\varphi^{\psi(R)}\big(E\varphi^{\dot{S}}_A\big) \longrightarrow \varphi_{0or1}\big(A \ltimes_\gamma \psi\big)$$

For the trivial parameterization of as considered where any parameter— less $A\ for\ \delta$ as considered above else the parameter $A\ for\ the\ A - module$ without δ being considered otherwise.

Thereby, Poincare duality can be defined through $KK - Theory$ for complex integer Λ on classifying space ΛS^{cl}_c in $c^* - a\lg e\, bra$over discrete parameter ψ ; taking the Thom Isomorphism for Topological $K - Theory$ in the homology theory for a generalized norm defining[22,24],

$Spin^c -$ structure Sp_c on Riemann manifold $\mathcal{M}$ with metric representation $\mathcal{M}_g$ for the parameter (*mentioned above*) as Λ structuring,

$$Sp(n)_{\times\Lambda} \times U(1) \longrightarrow SO(n) \times U(1) - 1\ for\ Spin^q_c$$

$\exists q$ Representing morphisms over Λ_2 for the sequence S ,

$$S = 1 - \Lambda_2$$

Representing the $Chern\ class$ for $U(1)^{B}_{Chern\ class} \in H^2(\mathcal{M}_g \Lambda)$

Thus taking $\mathcal{M}_n$ as the $n - \dim$ manifold; Poincare duality can be expressed in,

$$\mathcal{M}_n(compact,\ closed\ and\ oriented)_{isomorphic\ to}\ \mathcal{M}_{n,n^{\circ}}$$

For $n^{\circ} - integers$; whereas expressed earlier Sp_c^q being the spin– structure on morphisms for the action on a manifold that is orientable in Topological $K - Theory$.

For isomorphisms on any integers of n° in $\mathcal{M}_{n,n^{\circ}}$ any $mod - 2$ (without any orientation assumption) – Poincare conjecture holds for,

$H^{n^{\circ}}(\mathcal{M}_n, \Lambda)$ in $(n - k)$ –homology group of n in $[\mathcal{M}_M]$ class taking the Thom Isomorphism in $M^{\mathrm{hom\,o\,log\,y}}_{cross-product}$ for $H_{\wedge}$ as,

$H_{\wedge} M \otimes H_{\wedge} M$

Suffice the form $H_{n^{\circ}-k}(M) \cong H^{n^{\circ}} for\ integers\ n^{\circ}$; thereby establishing the Poincare duality.

V. Sp_c WITH TYPE–II STRINGS IN ATIYAH-HIRZEBRUCH FOR RAMOND–RAMOND SECTOR

The $K - Theory$ for the operator and Topological aspects in the cohomology class; there exists distinct classifiers for the $D - Branes$ or Dirichlet Branes in the Ramond–Ramond (RR)– Sector of Type II-B Strings sufficing the $3 - \dim$ integral class property. There is the cohomology class for the transformation–twist giving the $mod - 2$ torsion quantum corrections considering the Freed–Witten discrepancies as and when considered in the peculiar $K - Theory$ in the reconsiled aspects over Atiyah–Hirzebruch spectral sequence.

The non–trivial aspect to discuss in high energy physics for the Topological K–Theory taking the Type–II (II-A and II-B) superstrings is to consider the RR–fields in $P - form$ electrodynamics considering the $10 - \dim$ $Supergravity$ for the potential $\mho^{\circ}$ over $\Omega_{P+1} - field$ defined through the Hodge duals $*_d$ in the form Ω^{*d}_{9-P} there exists $4 - classifiers$ that will ultimately result the approach of $K - Theory$ in the complex Topological space T^* on manifold M over a representation M^*_Trelates not only the Atiyah–Singer Index Theorem (for the Fredholm modules, Bott–Periodicity as taken earlier) but also gives the Grothendieck–Riemann–Roch Theorem on bounded complex Λ^* on sheaves $S_{,,}$ over a relation $S_{,,}^{\Lambda^*}$taking the morphism $\sigma_m : X \longrightarrow Y$ for $\sigma_m : A(X) \longrightarrow A(Y)$ over the Tangent sheaf T_{Λ^*} of Λ^* on $\sigma_m!$ to suffice $ch(\sigma_m! \Lambda^*)$ gives,

$$\Lambda^*_{\sigma_m}\left(ch(S_{,,})Td\left(T_{\sigma_m}\right)\right)$$

All suffice through the $4 - classifiers$ as mentioned above[27,28],

1. Hodge dual $*_d$
2. Gauge symmetry $\overset{\circ}{g}_{P-form}$
3. Equations of motion $\partial * g^{\circ} = *\mathbf{J}$ for $\mathbf{J}_{\text{P–vector}}$
4. Charge density C_ρ through the Lagrangian for ζ_{C_ρ} in $RR - fields$ for ϖ_{10-P} through the D–Brane potential $(10 - P)$ gives the equations of motion $S^{\times}$ for $(10 - P)$having a replacement order of $P\ to\ (7 - P)$ for the previously taken charge density C_ρ giving two non–trivial relations[29,30],

A. $De\,Rham\,Co\,\hom o\log y$ with $H - twist$ for the exterior derivative ∂ with charge density C_ρ for the parameter χ gives,

$$\partial\chi_{9-P} + H \times \Omega_{9-P}$$
$$= \partial\chi_{P+1}$$
$$= \partial^2 \varpi_{7-P}$$
$$= C_{9-P}$$

B. The action for Type II (II–B being both T and S–dual to itself) for non–invariant GSO – projections in subdomains where for the existence of 32–supercharges in Type II–B ($\mathcal{R}^{8,1} \times S^1$) the action $S_{,,}$ of P–form electrodynamics on a manifold M through gauge symmetry can be represented by $\mathring{g}_{P-form}$ gives,

$$S_{,,} = \int_M \left[\frac{1}{2} \mathring{g} \chi * \mathring{g} + (-1)^P B\chi * J\right]$$

Which gives the nilpotent potential in manifold M over a spacetime coordinates (σ, τ) as,

$$\partial\Omega_{P+1} + \chi_{9-P} + \Omega_{\ (\sigma,\tau)}$$
$$= \partial\Omega_{P+1\ (\sigma,\tau)}$$
$$= \partial^2\, \varpi_{P\ (\sigma,\tau)}$$
$$= 0_{\ (\sigma,\tau)}$$

All of these suffice for Sp^c in the extension of Poincare duality in a generalized norm of orientability of homology theory taking the Thom Isomorphism in complex form of Topological $K - Theory$ relating Atiyah–Singer Index Theorem and Fredholm modules, Bott–Periodicity, Atiyah–Hirzebruch, Grothendieck–Riemann–Roch with KK–Theory[31-34].

Additionally, to discuss furthermore about the Type II Superstrings formalism as associated with supergravity for a homology class there is a relation between the Dirac quantization conditions and RR–fields where in the Lie group structure[27,37,38],

$$U(1) \times SU(2) \times SU(3) \subseteq SU(5) \subseteq SO(10) \subseteq E(8)$$

The Photon being represented by $U(1)$ the related methodology of the charge quantization and the magnetic monopoles where their independent nature relates the breaking of gauge group from $D(1)$heavy branes when the distance is infinite for a path υ suffice the relation[37,38],

$$\prod_{\upsilon} \left(1 + ieA_j \frac{dx^j}{d(\upsilon)} d(\upsilon)\right) = \exp\left(ie \int A \cdot d(\upsilon)\right)$$

$$\exists\, e \oint_{\partial\cap} A \cdot d(\upsilon) = \int_{\cap} Bd(\upsilon)$$

Considering a cycle σ_{cy} in the homogeneous Lie group, the movement can ultimaltely results in lifting the Lie group that originates over identity structres through,

$$2 - times(\sigma_{cy})\ and\ 3 - times(\sigma_{cy})$$

Where the $2 - times(\sigma_{cy})$ where a covering parameter $\Im$ for $SO(2)$ can maintain the Type II superstring actions over the $Twisted\ K - Theory\ (over\ Topo\log i\ cal\ norms)$.

One category of Type II superstrings (Type II-B) which has been extended to $12 - \dim$ where in the t'Hooft limit, for Yang–Mills $N = 4$, F-Theory being encountered under $SL(2, \mathbb{Z})$, the D–Brane analogy being extended where there exists some non–trivial aspects being existent over RR–Fields and its relation to the Twisted K–Theory making up these points[35,36,27,39],

1. $GSO -$ Projections for an eliminated Tachyon and preserved Supersymmetry.
2. Distinct classifiers for Type II into II^{II-A}_{II-B}.
3. $SL(2, \mathbb{Z})$ for a CFT for a worldsheet periodicity as concerned for Fermion–projections giving 3 sub–relations,
    a. Invariance over $SL(2, \mathbb{Z})$.
    b. Modular diffeomorphisms as expressed on Torus for Point [3] to get rid of gravitational anomalies.
        i. This in turn establishes the integral for $Kalb - Ramond\ (K - R)\ field$ with the relation to the **B** field for ƛ as,

$$- \int_{KR} ƛ^i ƛ^j \boldsymbol{B}_{ij}$$

Thus, for the correspondence to $KR\ NS - NS\ B - field$; a far more concrete relation can be attained for $H - flux^{NS}_{D-Brane}$ where the $P - form\ for\ P - skeleton$ represents a complicated structure later but for the cohomology integral coefficients for a D-Brane absent RR–flux the relation can be stated over[35-37,27,38,39],

$$NS\ 3 - form^{\otimes RR-flux \cong ch\arg e\ density\ of\ D-Brane}_{+RR-flux \cong equations\ of\ motion\ (\sup ergravity)}$$

Extending Type II for Type II–B the representation when made for a manifold M for the group operators Og in the quotient space $q\ with\ q^{\partial-rescalling}$ Type II–B represents the Orientifold over the operator relation where $\partial\ in\ \partial - rescalling$ being taken trivially for the involution parameter, the non–empty operator represents the orientifold for the operator Og^2_p such that for the operator $P\sim$ there is Type II–B for,

$\partial(P\sim)$

Where through the splitting another structure represents $II - A$ for the $(1 - 1) - form$.

The $P - skeleton$ as stated above in turn gives the Topological K–Theory over **B** for the fibre $f.$in the cohomological space $M.$ over a Serre fibration parameter $S_{f.}$: $M. \to B.$ in the $(p, q) - norm$representing the cohomolgy pair $(M._{(p)}, M._{(p,q)})$ for $k^{th} - co\hom o\log y\ \ group$ through,

$$\bigotimes_{p,q} H^k(M._{(p)})$$

$$\bigotimes_{p,q} H^k(M._{(P)}, M._{(P),} M._{(P-1)})$$

For the Atiyah–Hirzebruch taking the space $M.$ and the spectral sequence associated with it for the fibres $f.$ there exists the $E_n - sheet$ taking $(p,q) - norms$ for $E_n^{p,q}$ for n taking the values 2,3, ∞ ; the spectral sequence can be in respect of the differentials E_d where there is,

1. Atiyah–Hirzebruch spectral sequence
2. Twisted K–Theory
3. Topological K–Theory
4. Algebraic K–Theory
5. Complex δ
6. E_n for different values of n providing;
    a. Serre spectral sequence for E_1
    b. Topological K–Theory for E_2^p
    c. Twisted K–Theory for E_3 over the differential $E_{3_{(d)}}$ such that for the $E_n^{p,q}$; n takes an equality for E_2 *and* E_3.
    d. For [Point *b*] in complex parameter $\delta. = 2k + 1$denoting complex projective $\mathbb{C}\mathrm{P}^{\delta}$ there exists two foundations,
        i. Collapsing for $even\ 2k$
        ii. Non collapsing for $odd\ 2k + 1$
            1. Where Topological K–Theory as associated with Atiyah–Hirzebruch for $2k + 1$ over space $M.$; a nice relation can be expressed in $E_2^{p,\delta.}(M.)$.
7. For the Kahler where any compact Kahler having Ricci flatness is a Calabi–Yau[40,41] for all the threefold being non–trivial in superstring theory, any Kahler (*without any consideration of being compact*) can give the twisted formalism of K–Theory for E_i such that $i \equiv 4 = \infty$.
8. Algebraic K–Theory having a relation to the *etale* cohomology[42,43] for the scheme M_T^e where M_T is a Topological space; any representation can be done in the local isomorphism such that for the category taking M_T^e for $etale\ representation\ et(M_T^e)$suffice isomorphism for the Topological space[44] $T(or\ M_T)$ which provides the Atiyah–Hirzebruch spectral sequence[27] for E_2 in $(p,q) - norms$ thereby establishing the Quillen– Lichtenbaum conjecture for $E_2^{p,q}$ with the *etale* cohomology M_T^e.

VI. DISCUSSIONS

Representing a finite group of two elements for a specified vector bundle acting on the Hilbert space for A – module being non–negative and self–adjoint constructions are made over a complex manifold such that taking that topological space through a pre–Hibert Hausdorff order there exists the c*-algebra and Banach–algebra where the 3–points emphasized here a 8–periodic homotopy groups can be established through Bott Periodicity for Unitary, Orthogonal and symplectic category over a compact Hausdorff suffice K-Theory in the Topological and reduced form over two functors in (left–right) ring representation makes the well-defined Morita Equivalence taking the naturally induced isomorphism for those two functors in bi–modular forms.

The linkage of $K - Theory\ to\ K -$ *Homology* and $c* -$ algebras have been established taking the same Morita equivalence for Kasparov's composition product where through the elliptic differential operator representing the Fredholm modules it has been established by Atiyah–Singer Index Theorem and Thom Isomorphism for the associated Chern character to establish the structures taking over the operator theory for the linear transformations with $n - \dim$ vectors through a specified mapping parameter over $n - \dim$ spaces for the complex tangent bundle channnelizing the way to represent the dual category of the Hausdoff spaces in *− isomorphism that bounds the two subsets taken in this paper giving the relation between noncommutative geometry and noncommutative topology where the noncommutative topology is sufficed over a connectivity channelling from $K - Theory\ to\ K -$ *Homology* considering operator $K - Theory$ taking $Baum - Connes\, conjecture$ through the $5 -$ classifiers for a concrete relation to $KK - Theory$.

Considering the two categories of algebra c* and the Banach where for the defined group action for the associated 2–norms as defined for Banach; there's a two spectrums satisfying c* taking the same group actions where a modified form of Fourier transform, i.e., a Gelfand Transform can be established and given related to the same Hausdorff space for a Gelfand–Neimark transform over a^*-isommorphism considered. The properties of noncommutative geometry can be perceived through a generalized notion occupying Borel measures, involutions and Harr measures given the second norms taking group action (^) as $\ell_{++}(\wedge)$. This also provides the subspace of c*-algebra as c^*_{sub} over the previously mentioned transforms and associated parameters with the necessary mapping operator in $index^0_1$ there is a c^*_{red}-algebras in the $k - homology$ for a right side assemble parameter taking the same Baum–cones conjecture for a classifying space $S^{c\ell}_c$. This torsion–free parameter in the A–module with Baum–Connes conjecture and the Hilbert A–module over the complex parameter δ gives the Poincare duality through the KK–Theory over another complex integer Λ where Thom isomorphisms have been considered for a Topological K-Theory and the Sp_c-structure over the associated Chern class in the n–dim manifold.

Different forms of K–Theory as Twisted, Topological, Algebraic is considered taking $E^{p,q}_n$ and E^p_n- norms for defined value of $n = 2,3,4 = \infty$ where the last value is expressed in terms of Kahler manifold (*which if is compact with vanishing Ricci curvature can give the Calabi–Yau and iff this CY is of threefold then a non–trivial expression of string theory is defined for various values of supersymmetry*). The K–Theory in the cohomology class with the topological aspects gives the Ramond–Ramond sector sufficing 3–dim integral class property for Type–II(B) strings. For the potentials of supergravity and the classifiers that are concerned through various representation–forms gives the Atiyah–Singer Index Theorem with Fredholm modules and the Grothendieck–Riemann–Roch Theorem through the equations presented in the paper.

De Rahm cohomology with H–Twist is taken with a charge density and GSO–Projections for concerned Type–II action (on II-A and II-B) in P–form electrodynamics. Along with RR–fields the NS–NS B–field in the same P–form over P–skeleton where the NS 3–form is shown for the RR–flux on D–Brane RR–flux on supergravity. For the extension of Type–II(A) being considered through splitting and (1–1)-forms where the extended notion of Serre fibration is shown and also the Atiyah–Hirzebruch spectral sequence is established for $E_n - sheets$ with $n = 2,3,4,\infty$ giving distinct categories of K–Theories.

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