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Abstract

Based on methodologies of variational calculus and differential entropy, we propose in this work a non-parametric model that provides a robust estimation of reliability to be used in a RCM scheme. A Weibull analysis is presented first in a case study within the usual RCM schemes. If the sample of data is severly reduced the weibull analysis lost preciscion, impacting in the RCM scheme. To solve this limitation, a maximum entropy aproach is propoused.

Differential entropy has been shown as a solid tool to model the response of a random variable when reduced sample size information is available. We take advantage of the formalism of the variational calculus to express a functional that obeys the Euler-Lagrange equations and supported by the Kolmogorov axioms, we extract a generalized non-parametric probability density. By subjecting this density to appropriate boundary conditions in terms of the first moments of the generalized probability density.

Keywords: Differential Entropy, Shanon, RCM, Variational Calculus, Euler-Lagrange, Weibull, Reliability.

1 Introduction

In the framework of reliability-centered maintenance (RCM) it is usual to have difficulties regarding the frequency of failure of vital components used in the industry [13,6]. Usually maintenance records, the information that manufacturers provide in the manufacturing of these components and the experience of maintenance personnel are the primary sources for creating a maintenance plan. In practice, most of these data are non-existent. At the same time, maintenance plans represent an investment in fixed assets, personnel training and supply of spare parts that exceed what medium-sized companies can afford in the short to medium term [14,6]. This segment is where the need for maintenance plans focused on reliability is detected for us, working primarily with very small samples or the experience of the staff. With this information and the concepts of entropy in information theory, it is intended to develop a generalized model that describes the probability of failure of a severely reduced sample of failure data in industrial components. One of the most used probability densities to study the frequency of failures in the RCM framework is the Weibull, the scale parameter is determined by the sample size, if the sample is small, the Weibull probability density is still able to give results on a small sample. On the other hand, if there is no faliure data, the data is censored [1,6], it is a fact that the components will fail. In this paper we propose to estimate, froma a very small data set of failure for replaceable systems, the Mean Time To Faliure (MTTF) and, based on the maximization [12,6] of Shannon's entropy, obtain a non-parametric probability density, subjecting the maximization process to the Kolmogorov axioms and carrying out estimates of the first moments of a generalized probability distribution. With these tools, our goal is to build a probability density that can estimate the probability of failure in industrial components from severely reduced samples, experience of the staff and warranty records.[2,6],[3,6], [4,6].

2 RCM methodology.

The usual schemes of reliability-centered maintenance require identifying in various stages the characteristics and processes to which various components are subjected in order to select the appropriate maintenance scheme that responds to the industrial process in which a component or a set of replaceable components are working. Roughly, the usual 6 stages of reliability-focused maintenance are: 1.-Industrial equipment selection. 2.-Failure mode identification. 3-.Define fail schemes. Identify consequences. 4-.Maintenance Strategy. 5-.Reliability. 6-. RCM. In this work, we focus primarily on the fifth stage. Reliability in the RCM scheme is to select the best model that represents the probability of failure and build with it, the entire reliability maintenance-centered scheme. Established in the fifth stage indicated, it is necessary to determine if the sample is large enough to justify a Weibull analysis [10,6].

3 Case Study.

The following data were obtained from the company Plastics and Metals of Coahuila (PYCSA), it is the replacement times in hours of valves that are used to control the pressure with which plastic is injected into various molds. They represent a system of replaceable parts and can be analyzed within reliability-centered maintenance schemes.

Tabla 1: Valves Time To Failure Sample

23890
24210
27142
25731
26584
28215
25321
21972
30121
25134

The experience of the technician indicates that the valves will fail around 28,000 hours of operation.

3.1 Weibull analysis

The data provided is a sample of failure times. In order to be able to offer a RCM scheme, we carried out an adjustment of Weibull parameters [8,6], through the maximum likelihood method. With the scale parameter on the average of the data and the shape parameter equal to one, these initial data provide through the Newton-Rhapson method the values: $\eta = 26818.4$, $\beta = 12.7290$. The cumulative probability function is then:

$$F(T) = 1 - e^{(-(T/26818.4)^{12.729})}$$

Therefore, reliability is given by: $R(T) = 1 - F(x \leqslant T)$

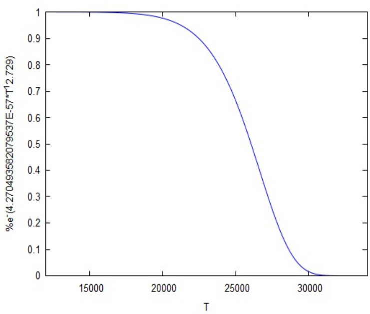

Figure 1: Weibull reliability.

3.2 Mean Time To Faliure (MTTF).

We then proceed to determine the MTTF (related to the expected value $E(t)$), since we have the reliability function, the MTTF turns out to be a value of critical importance to make decisions regarding the reliability-centered maintenance scheme.

$$E(t) = \int_0^{\infty} R(t) \mathrm{dt}$$

Likewise, the instantaneous fail frequency is given by:

$$h(t) = \frac{d}{\mathrm{dt}} \ln[R(t)]$$

The instantaneous frequency of failure is also known as a hazard function, it indicates the speed at which reliability decreases as the time in which the device in question is in operation.

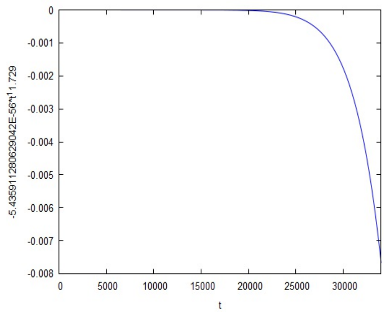

Figure 2: Hazard Function

The graph of the hazard function, such as reliability decreases sharply after the MTTF, inducing a risk process if inspection measures are not taken at least around 25,000 hours of operation on the valves.

3.3 Reliability.

With the data provided by the company Plasticos y Metales of Coahuila, we can establish the following reliability centered maintenance scheme.

Model	Parameters	MTTF
Weibull	$\eta = 26818.4$	25755.11
$R(T) = e^{-\left(\frac{x}{\eta}\right)^{\beta}}$	$\beta = 12.7290$	$h(t) > 25000$

In this way it is recommended to the company's administration that the valve replacements must be made before 25000 hours of operation without waiting to reach the MTTF, since the risk of failure steeply grows around 25700 hours. In addition, if we consider the value of the probability of failure in 28000 hours (suggested by experience of the technician) we can see that it represents a high risk quantile for the machinery:

$$F(28000) = 0.82293$$

4 Generalized non-parametric probability density.

In this section we elaborate a non-parametric approximation with the data to compare them with the results of the previous section. *It is necessary to understand that, what is being offered is an example of how to work with the maximum entropy model when the sample size is extremely small and we only have the experience of the technician. In no way the scheme that will be addressed below corresponds to a complete approach on how to perform the estimation of the moments in the equation (14), what we intend is to give an indicative idea of how to proceed once we have a well defined theoretical scheme* **to estimate the moments of the generalized non-parametric probability density**. As we do not have a sufficiently large data sample to ensure that the Weibull analysis is reliable, we developed the nonparametric scheme hereinafter. [10,6]

4.1 Theoretical considerations.

We begin with the entropy S. It quantifies the lack of information from a randomized experiment. If the experiment is governed by a probability distribution p_i then the function that measures the uncertainty is defined as:

$$S = -k \sum_{i=1}^{n} p_i \ln(p_i) \tag{1}$$

and we assume that $0 < p_i < 1$ which ensures the monotony property of entropy: $S \geqslant 0$ along the entire path of p_i, from a linear point of view, S represents a base that diagonalizes $\hat{p}$. Thus, when we have the maximum system information, S reaches its minimum value. In this idea, S will reach its maximum value when all p_i, have the same probabilities for each p_i. Following this hypothesis, we could assign probabilities through the relative frequency $p_i = 1/n$ that directly associates the entropy: $S = k\ln(n)$, so that S grows in direct proportion with n in the cited case. In general, we expect entropy to satisfy the following: [5,6]

Definition 1. *Principle of maximum entropy.*

«The statistical entropy of a random system reaches the maximum compatible with the restrictions imposed»

The basic idea behind this principle is that, there is no reason to privilege a particular state or event in a randomized experiment. In the continuous case, we can define a functional subject to conditions of continuity in its random variable, in this sense, Shanon's entropy [5,6] can be defined as:

$$S(P, P') = -\int_{\Omega} f(t) \ln[f(t)] \mathrm{d}t \tag{2}$$

P, are probabilities, and f represents probability densities. Subjecting (2) to restrictions:

$$\int_{\Omega} \mathrm{f(t)dt} = 1 \;\; ; \;\; \int_{\Omega} \mathrm{h_k(t)f(t)dt} = \mathrm{c_k} \quad \text{with k=0,1,2,...,n} \tag{3}$$

If: $F(f, f', t) = -f(t)\ln[f(t)] - \lambda_k \sum_k h_k(t) f(t)$, is a functional [6,6], finding its extreme points subject to the restrictions imposed by $\mathrm{h_k(t)f(t)}$ by the Lagrange multiplier method, then $F(f, f', t)$ also satisfies the Euler-Lagrange equation:

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial F}{\partial f'} - \frac{\partial F}{\partial f} = 0 \tag{4}$$

The direct consequence is clear, every problem propose in the Lagrange multiplier frame work, is equivalent to a set of differential equations that are obtained from the Euler-Lagrange equation. Moreover, Lagrange multipliers cannot ensure that the system of equations is overdetermined, consequently, Lagrange multipliers lead to a set of equations that include implicit derivatives, since it is not assured that the variables are all independent. In the case of a problem with a set of constraints and a single probability function, Lagrange multipliers and the Euler-Lagrange equation provide the same generalized model for probability density. However, in the case of modeling systems that respond with multiple probabilities [8,6], the Euler-Lagrange equations will always provide a decoupled set of second-order differential equations for each degree of freedom [6,6], which in this context will always be a probability density. Given the above, when submitted the functional: $F(f, f', t) = -f(t)\ln[f(t)] - \sum_k \lambda_k h_k(t) f(t)$ to the equation (4) we get:

$$\frac{\partial F}{\partial f} = -1 - \ln[f(t)] - \sum_k \lambda_k h_k(t) = 0$$

Also we can define: $\varphi(t) = 1 + \sum_{k=0}^{N} \lambda_k h_k(t)$ to obtain the generalized non-parametric probability density: [7,6]

$$f(t) = \mathrm{A}e^{-\varphi(t)} \tag{5}$$

The λ_k are the Lagrange multipliers, needed to establish the model, in $k=0$ we will have the normalization constant of the model: $\mathrm{A} = e^{\lambda_0}$. Now, to ensure the convergence of the probability model, we need to demand that:

$$N = 2m \quad \text{with } m \text{ a positive integer.} \tag{6}$$

Lagrange multipliers can be found using the following [9,6] procedure; taking the derivative of (5) and integrating over its domain we get:

$$\frac{\mathrm{d}}{\mathrm{d}t} f(t) = -f(t) \frac{\mathrm{d}}{\mathrm{d}t} \varphi(t) \tag{7}$$

$$\int_{-\infty}^{\infty} \frac{\mathrm{d}}{\mathrm{d}t} f(t) \mathrm{d}t = 0 \tag{8}$$

(8) vanishes by the boundary properties must be satisfy by a probability density, the integral of the left member of (8) is then expressed as:

$$\int_{-\infty}^{\infty} f(t) \sum_{k=1}^{N} \lambda_k \mathrm{kt}^{k-1} f(t) \mathrm{d}t = \sum_{k=1}^{N} \lambda_k k \int_{-\infty}^{\infty} t^{k-1} f(t) \mathrm{d}t = 0 \tag{9}$$

So we can see that the moments of the density in (5) are given by: $\mu_k = \int_{-\infty}^{\infty} t^k f(t) \mathrm{d}t$

And (9) give us one first equation for the λ_k:

$$\lambda_1 + 2\lambda_2 \mu_1 + 3\lambda_3 \mu_2 + \cdots + N\lambda_N \mu_{N-1} = 0 \tag{10}$$

The argument used that leads to (10) can be extended if we take the derivative from the probability model, multiplied by t^n and integrated over its entire domain, the new parameter n will count the restrictions that can be imposed on the functional that arises from the Euler-Lagrange equation. We proceed then:

$$t^n \frac{\mathrm{d}}{\mathrm{d}t} f(t) = -f(t) \sum_{k=1}^{N} \lambda_k \mathrm{kt}^{k+n-1} f(t) \tag{11}$$

$$\int_{-\infty}^{\infty} t^n \frac{\mathrm{d}}{\mathrm{d}t} f(t) \mathrm{d}t = t^n f(t) \Big/_{-\infty}^{\infty} - n \int_{-\infty}^{\infty} t^{n-1} f(t) \mathrm{d}t$$

Discarding the term evaluated for reasons already explained get:

$$n \int_{-\infty}^{\infty} t^{n-1} f(t) \mathrm{d}t = \sum_{k=1}^{N} \lambda_k k \int_{-\infty}^{\infty} t^{k+n-1} f(t) \mathrm{d}t \tag{12}$$

and again, using the definition of moments, we can conclude with:

$$n\mu_{n-1} = \lambda_1 \mu_n + 2\lambda_2 \mu_{n+1} + \cdots + N\lambda_N \mu_{n+N-1} \tag{13}$$

(10) and (13) form a system of equations that determine Lagrange multipliers based on the Moments of the distribution:

$$\begin{pmatrix} 1 & \mu_1 & . & \mu_{N-1} \\ \mu_1 & \mu_2 & . & \mu_N \\ \mu_2 & \mu_3 & . & \mu_{N+1} \\ . & . & . & . \\ . & . & . & . \\ . & . & . & . \\ \mu_{N+1} & \mu_{N+2} & \cdots & \mu_{2N} \end{pmatrix} \begin{pmatrix} \lambda_1 \\ 2\lambda_2 \\ 3\lambda_3 \\ . \\ . \\ . \\ N\lambda_N \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 2\mu_1 \\ . \\ . \\ . \\ (N+1)\mu_N \end{pmatrix} \tag{14}$$

In equation (14) we then need to feed the momentum operator with some reasonable estimation of them, and in that case, proceed to solve the system of equations for Lagrange multipliers.

5 Weibull vs the generalized non-parametric probability density.

The alternative we propose is a probability density that comes from maximizing entropy, that is, we propose an identical model to the one in equation (5), choosing a polynomial: $\varphi(t) = 1 + \sum_{k=1}^{N} \lambda_k t^k$ to set the model parameters in the model and equation (14) leads us to consider some system of equations, previously estimating the moments of the function $f(t)$, and of course, the question arises, how many terms must be included in $\varphi(t)$ to adequately represent the model? If we decide to include the mean, variance, asymmetry and kurtosis, [11,6], [10,6], [9,6] suggest consider the first four moments in absolute value, pointing that, even when the information in the moments operator is extremely difficult to obtain in the context of achieving base that diagonalizes the moment operator, the first two moments are sufficient to determine, in an acceptable way the probability density Lagrange multipliers. In that case, we would have a system of equations given by:

$$\begin{pmatrix} 1 & \mu_1 \\ \mu_1 & \mu_2 \end{pmatrix} \begin{pmatrix} \lambda_1 \\ 2\lambda_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{15}$$

5.1 Moments estimation.

To estimate the moments, we take the descriptive statistics of the data and obtain the first four moments. Likewise, for moments greater than the fourth, we assume a uniform distribution in order of completing the moments operator. The system of equations arises:

$$\cdot \begin{pmatrix} 1 & 25832 & 5379576 & 0.284135 \\ 25832 & 5379576 & 0.284135 & 0.316593 \\ 5379576 & 0.284135 & 0.316593 & \frac{1}{10} \\ 0.284135 & 0.316593 & \frac{1}{10} & \frac{1}{10} \end{pmatrix} \begin{pmatrix} \lambda_1 \\ 2\lambda_2 \\ 3\lambda_3 \\ 4\lambda_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 51664 \\ 16138728 \end{pmatrix}$$

Solving for multipliers, we obtain the probability density whose graph is:

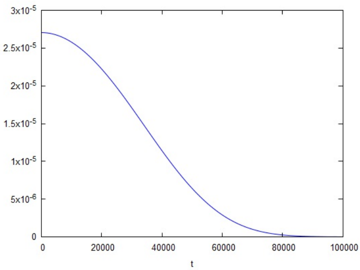

Figure 3: generalized probability density.

The MTTF for this distribution is then:

$$\int_0^\infty R(T)\mathrm{d}T = 22851.178637423861$$

Thus, the MTTF indicates that we are facing a lack of maintenance situation, putting the components of the entire injection system at risk. In this case, reliability suggests taking the following measures:

Model	MTTF	Inspections:
non-parametric	Wibull: 25755hr	3 years
$f(t) = \mathrm{A}e^{-\varphi(t)}$	Maximum Entropy: 22851hr	2/5 years

In this way, the company administration is recommended to prepare its inspection plans more frequently to avoid lack of maintenance.

6 Discussion and Conclusions.

The maximum entropy method provides us with a useful tool when establishing the reliability of a maintenance scheme, in the case presented, the lack of maintenance is detected by the generalized probability density and corrective measures are taken, in this case, increase the inspection frequency to avoid the abrupt risk of a valve failure. At the same time, we reduce replacement costs by ensuring an optimum inspection frequency within the scheduled shutdowns at the plant. Subsequent developments will focus on considering censored data and the technician's experience to estimate the moments in the equation (14). The state of the investigation indicates that we can develop reliability indications with the tool developed up to this point, the subsequent developments will strengthen the proposed framework and will give much more certainty to the maintenance plans that we develop for the industry. On the other hand, given that the formal language of probability is the mesure theory, we do not rule out entering this field to extract the necessary tools that allow us a significant improvement of the results and future investigations planned in this line of research.

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