Fuzzy Truth Maintenance System

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Fuzzy Truth Maintenance System for Non-monotonic Reasoning

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Abstract— **Knowledge is key factor in AI problem solving. Sometimes knowledge available to the AI problem is incomplete. John McCarthy proposed non-monotonic reasoning for incomplete problems in which inference is changed if added some knowledge. John There is no method to solve the non-monotonic reasoning. The fuzzy non-monotonic logic is able to solve problems of non-monotonic reasoning. In this paper, fuzzy non-monotonic logic is studied to solve problems of non-monotonic reasoning. Fuzzy truth maintenance system (FTMS) algorithm is studied for fuzzy non-monotonic reasoning. Some examples are discussed for fuzzy non-monotonic reasoning**.

Keywords— ***fuzzy Sets, non-monotonic reasoning, fuzzy non-monotonic reasoning, two fold fuzzy set, FTMS***

1. Introducdtion

Sometimes AI has to deal with incomplete knowledge. If knowledge base is incomplete then the inference is also incomplete.
In non-monotonic reasoning, some additional information is to be added the reasoning will be changed [4].
∀x(x is bird Λ x has wing) →x can fly
Penguin is bird Λ Penguin has wings →Penguin can fly
Ozzie is bird Λ Ozzie has wings → Ozzie can fly
"If x is not known then conclude y"
"If x is con not be proved in some amount of time, then conclude y"

x is bird Λ x has wings Λ x is not known to fly → x can fly
x is bird Λ x has wings Λ x is not known to fly → x can't fly
This is incomplete information.
it is default logic, in which one may be assumed it can fly or it can't fly
A method is needed to reasoning with non-monotonic logic to complete problem.

There are many theories to deal with incomplete information, but these theories are based on probable (likelihood]. Zadeh [12] fuzzy logic is based on belief rather than probable (likelihood). The fuzzy logic made imprecise information in to precise information.

The two fold fuzzy set Z= (A, B) for the proposition of the type "x is P", where A is likely support the knowledge and B is unlikely support the knowledge [9]
x is bird Λ x has wings Λ x is known to fly → x can fly
x is bird Λ x has wings Λ x is not known to fly → x can't fly
.
x is bird Λ x has wings Λ x is likely to fly → x can fly
x is bird Λ x has wings Λ x is unlikely to fly → x can't fly
Here is first case support the inference and in second case not support the information
The fuzzy non-monotonic reasoning will bring imprecise knowledge in to precise knowledge.

2. Fuzzy Logic

The possibility set may be defined for the proposition of the type "x is P" as

$\pi_P(x) \rightarrow [0,1]$
$\pi_P(x)=\max\{ \mu_P(x_i) \}, x \epsilon X$

$\mu_P(x)= \mu_P(x_1)/x_1+ \mu_P(x_2)/x_2+\ldots+ \mu_P(x_n)/x_n$
$\mu_{bird}(x)= \mu_{bird}(x_1)/x_1+ \mu_{bird}(x_2)/x_2+\ldots+ \mu_{bird}(x_n)/x_n$

$\mu_{bird}(x)$= 0.0/penguin +0.2/ozzie+ . 0.6/parrot+ 0.7/waterfowl + 0.9/eagle

Let P and Q be the fuzzy sets, and the operations on fuzzy sets are given below [10]

$PVQ=\max(\mu_P(x), \mu_Q(x)\}$ Disjunction
$P\Lambda Q=\min(\mu_P(x), \mu_Q(x)\}$ Conjunction
$P'=1- \mu_P(x)$ Negation
$PxQ=\min\{\mu_P(x), \mu_Q(x)\}$ Relation
$P \text{ o } Q==\min\{\mu_P(x), \mu_Q(x, x)\}$ Composition

The fuzzy propositions may contain quantifiers like “very”, “more or less” . These fuzzy quantifiers may be eliminated as
$\mu_{very}(x) = \mu_P(x)^2$ Concentration
$\mu_{\text{more or less}}(x) = \mu_P(x)^{0.5}$ Diffusion

The Zadeh [11] fuzzy condition inference s given by

if x is x is P_1 and P_2 …. X is Pn then Q =
min 1, $(1-\min(\mu_{P1}(x), \mu_{P2}(x), \ldots, \mu_{Pn}(x)) +\mu_Q(x)\}$

The Mamdani [5] fuzzy condition inference s given by
if x is x is P_1 and P_2 …. X is Pn then Q =
$\min\{1, (1-\min(\mu_{P1}(x), \mu_{P2}(x), \ldots, \mu_{Pn}(x)) +\mu_Q(x)\}$

The fuzzy rules are of the form “if <Precedent Part> then <Consequent Part>”
The Reddy [8] fuzzy condition inference given by “Consequent Part” is drawn from “Precedent Part”
if x is x is P_1 and P_2 …. X is Pn then Q =
$\min(\mu_{P1}(x), \mu_{P2}(x), \ldots, \mu_{Pn}(x))$

Quasi-fuzzy set
A quasi-fuzzy set is defined for the proposition “ x is P” as

$\mu_P(x) \rightarrow (0, 1)$

3. Fuzzy Predicate Logic

The fuzzy reasoning system[11] is complex processing system for incomplete information.. The fuzzy predicate logic (FPL) is transformed fuzzy facts and rules in to meta form (semantic form) [10]. These fuzzy facts and rules are modulated to represent the knowledge available to the incomplete problem.

The fuzzy modulations for Knowledge representation are type of modules for fuzzy propositions of the form “x is A”.
“x is A” is may be represented as
[A]R(x),
where A is fuzzy set, R is relation and x is individual in the Unversed of discourse X For instance
“x is bird” is modulated as
[bird]is(x)

The FPL is e combined with logical operators.
Let A and B be fuzzy sets.
x is ¬A
[¬A]R(x)
x is A or x is B
[A V B]R(x)
x is A and x is B
[A Λ B]R(x)
if x is A then x is B
[A → B]R(x)
Some of the Fuzzy Reasoning rules for FPL are given as
R1: [A]R(x)
[B](R(x) or R(y))
____
[AΛB]R(y)

R2: [A]R(x)
[B](R(x) or R(y)

[AVB]R(y)

R3: [A](R(x,y)
[B](R(y,z)

[AΛ B](R(x,z)
R4: [A](R(x) or R(y))
[B](R(y) or R(z))

[AV B](R(x) or R(z))
R5: : $[A_1]$R(x)
if [A]R(x) then [B]R(y)

[[Ao (A→B)]R(y)

x is bird
[bird]is(x)
if x is bird then x can fly
if [bird]is(x) then [fly]is(x)
or
[bird]→[fly]is(x)
if x is bird and x has wings then x can fly
if [bird]is(x) Λ [wings]is[x] then [fly]is(x)
[bird] Λ [wings]→[fly]is(x)
The fuzzy reasoning is given as
[long wings]]i(x)
if [wings]R(x) then [fly]is(x)

[long wings o (wings→fly)]is(x)

4. Fuzzy Non-Monotonic Logic

Zadeh [10] is defined the Z-Number {A,B] for the proposition of the type "x is P", where A support the P and B not support the P .

The fuzzy non-monotonic set may defined with two fold membership function using likely and unlikely .

Definition: Given some Universe of discourse X, the proposition " x is P" is defined as its two fold fuzzy membership function as

$\mu_P(x) = \{\mu_P^{likely}(x), \mu_P^{unlikely}(x)\}$
or
$P = \{\mu_P^{likely}(x), \mu_P^{unlikely}(x)\}$
Where P is Generalized fuzzy set and x Є X ,

$0 <= \mu_P^{likely}(x) <=1$ and, $0 <= \mu_P^{unlikely}(x) <=1$
$P = \{ \mu_P^{likely}(x_1)/x_1 + \ldots + \mu_P^{likely}(x_n)/x_n,$
$\mu_P^{unlikely}(x_1)/x_1 + \ldots + \mu_P^{likely}(x_n)/x_n$, x_i Є X, "+" is union

For example ' x will fly" , fly may be given as

$fly = \{\mu_{fly}^{likely}(x), \mu_{fly}^{unlikely}(x)\}$

= 0.0/penguin +0/2/ozzie+ .5/peacock+0.6/waterfowl + 0.8/parrot+ 0.9/eagle - .0/penguin + 0..1/ozzie+ 0.2/waterfowl+ 0.1/parrot+ 0.05/eagle}

In MYCIN [1], the certainty factor(CF) is defined as the deference between belief [MB] and disbelief [MD] of probabilities.

CF[h,e]=MB[h,e]-MD[h,e], where "e'is evidence and "h" is hypothesis.
CF[h,e],MB[h,e] and MD[h,e] are probabilities.

The fuzzy certainty factor (FCF) is defined by fuzziness instead of probability for the fuzzy preposition of the type " x is A"
CF[x, A]=MB[x, A]-MD[x,A],
The FCF is the difference between "likely" and "unlikely" and will eliminate conflict between "likely" and "unlikely" and, made as single membership function

$$\mu_A^{FCF}(x) = \mu_A^{likely}(x) - \mu_A^{unlikely}(x)$$

$$\mu_A^{FCF}(x) = \mu_A^{likely}(x) - \mu_A^{unlikely}(x) \quad >0$$
$$\mu_A^{FCF}(x) = \mu_A^{likely}(x) - \mu_A^{unlikely}(x) \quad =0$$
$$\mu_A^{FCF}(x) = \mu_A^{likely}(x) - \mu_A^{unlikely}(x) \quad <0$$

The above are interpreted as sufficient, redundant and insufficient information respectively.
The business intelligence has to take decision while taking decision. The fuzzy decision sets defined by

$$R= \mu_A^R(x) = \begin{cases} 1 & \mu_A^{FCF}(x) \geq \alpha, \\ 0 & \mu_A^{FCF}(x) < \alpha \end{cases} \quad (5)$$

where $\alpha \epsilon [0,1]$ and α-cut is decision factor.

$\mu_{fly}^{FCF}(x) = \{\mu_{fly}^{likely}(x) - \mu_{fly}^{unlikely}(x)\}$
= {0.0/penguin +0.2/ozzie+ . 0.6/parrot+ 0.7/waterfowl + 0.9/eagle - 0.0/penguin +0.1/ozzie+ . 0.1/parrot+ 0.15/waterfowl + 0.2/eagle }

= { 0.0/penguin +0.1/ozzie+ . 0.5/parrot+ 0.55/waterfowl + 0.7/eagle }
For instance " x can fly" for α>=0.5
Is given as
={0/penguin +0/ozzie+ 1/peacock+ 1/waterfowl+ 1/parrot+ 1/eagle}

Since formation of the fuzzy non-monotonic logic is simply to solve incomplete problem..

$\mu_P(x) = \{\mu_P^{likely}(x), \mu_P^{unlikely}(x) \}$

Suppose P and Q are fuzzy non-monotonic sets. The operations on fuzzy sets are given below for two fold fuzzy sets.

Negation
$P' = \{1- \mu_P^{likely}(x), 1- \mu_P^{unlikely}(x) \}/x$

Disjunction
$P \vee Q = \{ \max(\mu_P^{likely}(x), \mu_P^{likely}(y)), \max(\mu_Q^{unlikely}(x), \mu_Q^{unlikely}(y))\}(x,y)$

Conjunction
$P \wedge Q = \{ \min(\mu_P^{likely}(x), \mu_P^{likely}(y)), \min(\mu_Q^{unlikely}(x), \mu_Q^{unlikely}(y)) \}/(x,y)$

Implication
Zadeh fuzzy conditional inference
$P \rightarrow Q = \{\min(1, 1- \mu_P^{likely}(x) + \mu_Q^{likely}(y), \min(1, 1- \mu_P^{unlikely}(x) + \mu_Q^{unlikely}(y)\}(x,y)$

Mamdani fuzzy conditional inference
$P \rightarrow Q = \{\min(\mu_P^{likely}(x), \mu_Q^{likely}(y), \min(\mu_P^{unlikely}(x), \mu_Q^{unlikely}(y)\}(x,y)$

Reddy fuzzy conditional inference
$P \rightarrow Q = \{\min(\mu_P^{likely}(x), \mu_P^{unlikely}(y)\}(x,x)$

Composition
$P \text{ o } R = \{\min_x(\mu_P^{likely}(x), \mu_P^{likely}(x)), \min_x(\mu_R^{unlikely}(x), \mu_R^{unlikely}(x))\}/y$
The fuzzy propositions may contain quantifiers like "very", "more or less" . These fuzzy quantifiers may be eliminated as

Concentration
"x is very P
$\mu_{very\ P}(x) = \{\mu_P^{likely}(x)^2, \mu_P^{unlikely}(x)\mu_P(x)^2\}$

Diffusion
"x is more or less P"
$\mu_{more\ or\ less\ P}(x) = (\mu_P^{likely}(x)^{1/2}, \mu_P^{unlikely}(x)\mu_P(x)^{0.5}$

For instance, consider logical operations on P and Q

$P = \{0.8/x_1 + 0.9/x_2 + 0.7/x_3 + 0.6/x_4 + 0.5/x_5,$
$0.4/x_1 + 0.3/x_2 + 0.4/x_3 + 0.7/x_4 + 0.6/x_5\}$

$Q = \{0.9/x_1 + 0.7/x_2 + 0.8/x_3 + 0..5/x_4 + 0.6/x_5,$
$0.4/x_1 + 0.5/x_2 + 0.6/x_3 + 0.5/x_4 + 0.7/x_5\}$

$P \vee Q = \{0.9/x_1 + 0.9/x_2 + 0.8/x_3 + 0.6/x_4 + 0.6/x_5,$
$0.4/x_1 + 0.5/x_2 + 0.6/x_3 + 0.7/x_4 + 0.7/x_5\}$

$P \wedge Q = \{0.8/x_1 + 0.7/x_2 + 0.7/x_3 + 0.5/x_4 + 0.5/x_5,$
$0.4/x_1 + 0.3/x_2 + 0.4/x_3 + 0.5/x_4 + 0.6/x_5\}$

$P' = \text{not } P = \{0.2/x_1 + 0.1/x_2 + 0.3/x_3 + 0.4/x_4 + 0.5/x_5,$
$0.6/x_1 + 0.7/x_2 + 0.6/x_3 + 0.3/x_4 + 0.4/x_5\}$

$P \rightarrow Q = \{1/x_1 + 0.8/x_2 + /x_3 + 0.9/x_4 + 1/x_5,$
$1/x_1 + 1/x_2 + 1/x_3 + 0.8/x_4 + 1/x_5\}$
$P \text{ o } Q = \{0.8/x_1 + 0.7/x_2 + 0.7/x_3 + 0.5/x_4 + 0.5/x_5,$
$0.4/x_1 + 0.3/x_2 + 0.4/x_3 + 0.5/x_4 + 0.6/x_5\}$

$\mu_{very\ P}(x) = \{\mu_P^{likely}(x)^2, \mu_P^{unlikely}(x)\mu_P(x)^2\}$
$= \{0.64/x_1 + 0.81/x_2 + 0.49/x_3 + 0.36/x_4 + 0.25/x_5,$
$0.16/x_1 + 0.09/x_2 + 0.16/x_3 + 0.49/x_4 + 0.36/x_5$

$\mu_{more\ or\ less\ P}(x) = (\mu_L^{ikely}(x)^{1/2}, \mu_U^{nlikely}(x)\mu_P(x)^{1/2}\}$
$= \{0.89/x_1 + 0.95/x_2 + 0.84/x_3 + 0.77/x_4 + 0.70/x_5,$
$0.63/x_1 + 0.55/x_2 + 0.63/x_3 + 0.81/x_4 + 0.77/x_5\}$

quasi-fuzzy non-monotonic set is defined as

$\mu_P(x) = \{\mu_P^{likely}(x), \mu_P^{unlikely}(x)\}$
$\mu_P(x) \rightarrow (0, 1)$

Consider the fuzzy non-monotonic inference

"x is bird $\wedge$ x has wings $\rightarrow$ x can fly"
if [bird]is(x) $\wedge$ [wings]is[x] then [fly]is(x)

$\mu_{bird}(x) = \{\mu_{bird}{}^{likely}(x), \mu_{bird}{}^{likely}(x)\}$

$\mu_{bird}(x)$= {0.0/penguin +0.2/ozzie+ . 0.6/parrot+ 0.7/waterfowl + 0.9/eagle - 0.0/penguin +0.1/ozzie+ . 0.1/parrot+ 0.15/waterfowl + 0.2/eagle }

$\mu_{wings}(x) = \{\mu_{wings}{}^{likely}(x), \mu_{wings}{}^{likely}(x)\}$

$\mu_{wings}(x)$= {1.0/penguin +1.0/ozzie+ .1.0/parrot+ 1.0/waterfowl + 1.0/eagle – 0.0/penguin +0.0./ozzie+ . 0.0/parrot+ 0.0/waterfowl + 0.0/eagle }

where $\mu_{wings}(x)$ is quasi fuzzy set.

. "x is bird Λ x has wings Λ x will fly → x can fly"

x can fly may be given as using Reddy fuzzy conditional inference "consequent part "may be derived from "precedent part".

[fly]is(x)=[bird]is(x) Λ [wings]is[x]

= [bird] Λ [wings] is[x]

= [min{bird, wings} is[x]

=0.0/penguin +0.2/ozzie+ . 0.6/parrot+ 0.7/waterfowl + 0.9/eagle - 0.0/penguin +0.0/ozzie+ . 0.0/parrot+ 0.0/waterfowl + 0.0/eagle } is(x)

$\mu_{bird}(x)$ ={0.0/penguin +0.2/ozzie+ . 0.6/parrot+ 0.7/waterfowl + 0.9/eagle - 0.0/penguin +0.0/ozzie+ . 0.0/parrot+ 0.0/waterfowl + 0.0/eagle } is(x)

$\mu_{bird}{}^{FCF}(x)$= 0.0/penguin +0.2/ozzie+ . 0.6/parrot+ 0.7/waterfowl + 0.9/eagle

the decision of " x can fly" for α>=0.5 is given by
= 1/parrot+ 1/waterfowl + 1/eagle

The parrot, waterfowl and eagle can fly.

Here fuzzy logic made imprecise information to precise information's. Some birds can fly and some birds can't fly.

For instance, The parrot, waterfowl and eagle can fly and, penguin and ozzie are can't fly
"

Zadeh[13] defined fuzzy granularity for the proposition of type " x is A is λ" where λ is granular variable likely, unlikely, very likely not very likely, more or less likely, etc.

For instance, the inference for "x is bird is not very likely " is given as
1- $\mu_{bird}(x)$ 2

Fuzzy granular non-monotonic position "x is P is not very likely " is given by
$\{ 1-\mu_P{}^{likely}(x)^2 , 1-\mu_P{}^{unlikely}(x)^2\}$

The g fuzzy granular non-monotonic position "x is P is not very unlikely " is given by

$\{ \mu_P{}^{likely}(x) , \mu_P{}^{unlikely}(x)\}$

Granular variables may be apply on respective functions

The fuzzy granular vales may be applied on respective fuzzy membership functions.
"x is P is very likely " is given as

$\{ \mu_P^{\text{very likely}} (x) , \mu_P^{\text{unlikely}} (x)\}$
"x is P is more or less unlikely " is given as

$\{ \mu_P^{\text{likely}}(x) , \mu_P^{\text{more or less unlikely}}(x)\}$
For instance, "Ozzie is bird is very likely " is given as

$\{ \mu_{bird}^{\text{likely}} (\text{Ozzie}) , \mu_{bid}^{\text{unlikely}} (\text{Ozzie})\}$

"Ozzie is bird is very likely " is given as
$P\{ \mu_{bird}^{\text{likely}} (\text{Ozzie})^2 , \mu_{bird}^{\text{unlikely}} (\text{Ozzie})\}$
"Ozzie is bird is more or less unlikely " is given as
$P\{ \mu_{bird}^{\text{likely}} (\text{Ozzie}) , \mu_{bird}^{\text{unlikely}} (\text{Ozzie})^{0.5}\}$

$\mu_{bird}(x) = \{\mu_{bird}^{\text{likely}} (x), \mu_{bird}^{\text{likely}} (x)\}$

. "x is bird Λ x has wings Λ x will fly → x can fly"

5. Fuzzy Truth Maintanance System

Doyel studied truth maintenance system TMS] for non-monotonic reasoning
The fuzzy truth maintenance system (FTMS) for fuzzy non-monotonic reasoning using fuzzy conditional inference as

FTMS is having There is list of justification and conditions.

if x is bird and x has wings then x can fly

List L(IN-node, OUT-node), FCF-node
IN-node evidence is likely
OUT-node evidence is unlikely
FCF is (likely-unlikely)
Consider the proposition " if x is bird then x is fly)
x is bird
IN 0.9
OUT 0.1
FCF 0.8
Conclusion : fly (if FCF>=0.5 fly otherwise can't fly)
x is bird
IN 0.3
OUT 0.1
FCF 0.2
Conclusion : can't fly (if FCF>=0.5 fly otherwise can't fly)

6. Conclusion

In non-monotonic logic, multiple inferences are giving, if knowledge is goes on adding. In fuzzy non-monotonic, the inference is given for the incomplete problem. FTMS is algorithm to solve non-monotonic logic.

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