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Abstract— The target area of the research is the assessment of the mechanical excitation frequencies of three-phase asynchronous motors, primarily caused by bearing and shaft adjustment, and the understanding of the wear process. Measurements and calculations are basically based on a wear map, not only using spectrum analysis methods. Spectrum analyzes do not solve the steps for developing error detection methods in all cases. The processing of measurement results was preceded by several measurement cycles, the results of which were used to create the wear map.

Keywords— shaft adjustment, bearing frequency, outer ring frequency, inner ring frequency, wear map

I. INTRODUCTION

The target area of the research is to assess the mechanical excitation frequencies of three-phase asynchronous motors, primarily due to bearing and shaft adjustment, and to learn about the wear process. The field of science is one of the widely studied branches, however, it has a niche area that does not provide clear guidance (in the case of current signal shape analysis) in the cycles of bearing wear. The research foundations are partly derived from vibration diagnostics.

The design of the wear characteristic fields is based on four elements, the outer and inner ring of the bearing, rolling elements, primarily running rolling elements and the bearing basket, as well as the analysis of errors resulting from shaft alignment errors. In the case of axis adjustment, analysis of excitation errors resulting from perpendicularity and parallelism, angle errors in the amplitude and frequency range. The first sketch of the wear process and setting up step rules, creation of simplification paths. In the process of attrition, the development of the structure of points and forecasts returns. This is based on the examination of the motor current signal based on phase current and Park-vector component analyses. The first part of the thesis deals with the presentation of the background of the research, the second with questions based on the wear process.

Electric machines are part of an integrated system (frequency converter, sensors, software, etc.) and work with it indispensably. In this flowchart, it is possible to set up an operational map of the machine. There are many rotating machines in the industry, cooperation and control are necessary, continuous condition monitoring. Here, the goal is not only to detect error signals, but to manage and develop a source of information that goes beyond this, moving towards the field of self-learning electric drives. The joint work of machines and systems, the analysis and correction of each other's operating characteristics, and the drawing of conclusions. The course work primarily deals with signals of mechanical origin and creates analysis possibilities from them. The field of expertise is a wellestablished and widely cultivated sector in domestic and international terms. I am looking for the place and raison d'être of my own work in this extremely rich field of literature. The international literature provided extremely rich knowledge for the creation of the thesis, which I will examine with my own measurement results.

II. DEVELOPED BEARING DEFECTOMETRY

Motor current signature analysis is an extremely widely used method, with an already developed equipment park and measurement procedures, and I examine its possibilities in this chapter. Primarily, the diagnostic application of Park vectors (space vectors) is given priority. The international literature extensively discusses spatial vector examinations, and the study of this method has given new possibilities in diagnostics.

The chapters contain several spectrums so that the widest possible range of signal processing options can be explored. In signal analysis studies, amplitude frequencies play a significant role in the representation of time-varying signals. The issue of information recording is, on the one hand, the motor's power supply (mains or frequency converter), and, in summary, the waveforms and frequency components of the voltage and current connected to the motor and their appropriate sampling.

Setting up the bearing fault map (in part mechanical life cycle map) is the first step that lays the foundation for setting up

the map to be drawn that can be used to infer wear. It is not possible to create an algorithm without setting up fault metrics.

If wear is assumed at specific geometric points of the bearing. If, for example, several errors occur on the inner ring which is not precisely known in reality - then the error frequencies must be multiplied by an integer multiple. In addition to the bearing frequencies, the excitation frequencies resulting from the axis adjustment must also be associated, so the number of examined signals increases significantly. The experimental results showed that not only the bearing frequencies should be calculated and checked, but also the shaft adjustment frequencies. It is not advisable to deviate from this, because a shaft adjustment error will affect the bearing and vice versa.

The basic concept of the measurement is that the tested engine is also a sensor. All errors are detected through the air gap. The resulting radial movement cannot be of the order of millimeters. The examined asynchronous motor has an air gap of the order of a tenth of a millimeter, the rotor rubs against the stator, the detected mechanical faults appear based on axial magnetic pull-compression displacement, which the rotor transmits with the iron, the stator core. The mediating tool of the test is the magnetic field, including the rotating field, so its characteristics must also be taken into account. The space vector theory does not necessarily solve the fault finding problems (phase symmetry, the mechanical component will be present, because the three-phase vectors do not separate the versions of the error signals in the shape fidelity of a track geometry). The field shape description and path change method included in space vector theory often make error analysis difficult and the effect of errors on each other cannot be clearly followed, so I came to the conclusion that I will create a wear matrix that shows the reasons. and the results of the wear process.

III. BEARING LIFE CYCLE MODEL, WEAR ALGORITHM

The aim of the first phase of the research is to create a process analysis map that goes beyond the usual analyzes of function transformations. The research is not complete without a process that sheds light on the characteristic fields generated by a single row deep groove ball bearing.

- In addition to the four bearing fault frequencies, bring the axis adjustment frequency to a common path in the process of wear.
- Is it possible to parallelize the frequency ranges
- At which element does the error signal start first and in what frequency range
- How do gault frequencies interact?
- By increasing the harmonic number, how much noise is added to the measurement processing

These questions can only be answered with a life cycle model.

A model was made according to the x table. The four bearing error frequencies and one shaft adjustment frequency are calculated together with the harmonics. The calculation does not differ from this for now. There are components appearing at lower frequencies, e.g. basket wear and signals with a significantly higher frequency range, such as: inner ring.

The measurements and calculation results of the first semester can be seen in Figure 1. Life cycle model, wear process diagram. The figure refers to a deep groove ball bearing or, in general, a single row deep groove ball bearing. The calculation takes into account the frequency converter power supply and calculates it as a function of the power supply frequency. The frequency components are calculated by Fourier transformation. The horizontal axis is time, the vertical axis is frequency.



Fig. 1. Life cycle model of a single-row deep groove ball bearing, assuming continuous wear, based on motor current signature analysis.

Field parts are as follows:

- Green field: Bearing outer ring sector
- Purple field: Bearing inner ring sector
- Brown field: Bearing cage sector
- Gray field: Bearing rolling elements sector
- Black field: shaft misalignment setting fault part sector

A. Step rule

The diagram is small, so the frequency value is not visible, but it is a process diagram that can be followed based on colors, which is also the purpose of this. Lifetime goes from left to right by touching the red fields along the blue field route. Its size is such that you can see all the fields together. The route goes through the blue field.

B. Step process

The start of the error signal moves from left to right in the blue field (yellow is the intervention range). The red fields are specific error frequencies calculated by Fourier transformation. The initial wear starts from the red field on the left (inner ring error frequency, purple field). The goal is to reach the red fields on the right, but then the drive system is completely destroyed.

You can't skip a red field, you can't go diagonally, provided it doesn't cross a blue field and doesn't create an ambiguous situation. The number of steps can be reduced, but then there may be a situation where the process reaches the top of the next red field, then it becomes ambiguous and gives an anomaly, because it either skips a step or analyzes a red field twice, neither of which is allowed.

Blue squares cannot cross each other in any area. You have to touch the red fields and continue through them. Red fields can be next to each other (they are), the reason for this is that the error frequencies are close to each other. So the interaction is easier to see than in spectra. You have to enter the red field from the long or short side, you cannot avoid it, then you would miss an error frequency, and the interpretation of the wear process would be meaningless.

IV. TIMELINESS OF MACHINE SHUT DOWN

The basic question of the process: Where should the machine be stopped?

When entering the brown field, the bearing basket starts to get damaged, at which point it is no longer possible to prevent an unplanned shutdown in time, because it is possible that the failure phenomenon will be accelerated in a higher load condition.

Yellow field, critical shutdown, provided that there are no sudden load changes during the operating time. The frequency and values are not visible in the figure, but its important role is to show the behavior of mechanical defects in a map-like manner, as a relief map, which is divided into heights



Fig. 2. Bearing wear process diagram with final stop field, critical stage.



Fig. 3. Three-phase asynchronous motor performance test and measurement of shaft coupling heating.



Fig. 4. Three-phase asynchronous motor performance test and measurement of shaft coupling heating.



Fig. 5. Three-phase asynchronous motor performance test and measurement of shaft coupling heating.



Fig. 6. Three-phase asynchronous motor performance test and measurement of shaft coupling heating.



Fig. 7. Three-phase asynchronous motor performance test and measurement of shaft coupling heating.



Fig. 8. Three-phase asynchronous motor performance test and measurement of shaft coupling heating.

The research uses proven mathematical transformations as input analysis to the algorithm, fast-time and short-time Fourier transform (fig. 9-12).



Fig. 9. The following transformations show the motor current waveforms test path fast-time Fourier transformations.



Fig. 10. The following transformations show the motor current waveforms test path fast-time Fourier transformations. Finding and identifying fault frequencies.



Fig. 11. The following transformations show the motor current waveforms test path fast-time Fourier transformations. Finding and identifying fault frequencies.





Fig.12. The following transformations show the motor current waveforms test path short-time Fourier transformation. Finding and identifying fault frequencies.

The number of steps is compared in a table in the next chapter.

V. STEPS AND THE LIFE CYCLE FOR THE BEARING AND SHAFT ROT AT ION FREQUENCY (KNOCK FREQUENCY)

The table shows the change in mechanical fault frequencies of the asynchronous motor as a function of the life cycle (duration).

Sector	Element	All Steps	Steps in Yellow Field	Difference step Reduction Option
Green	Outer ring	113	36	77
Purple	Inner ring	194	59	135
Brown	Cage	216	60	155
Gray	Rolling elements	220	63	157
Black	Shaft knock	81	53	28
Summary		833	307	526

Tab. 1. Step numbers and their comparison is based on Fig. 2.

The purpose of the step counter is to analyze the stopping lane and the interaction of the fault frequencies that appear there.

The figure shows the widened stop field (yellow route). The reason for the widening is that the analyst waited for the inner ring error frequency, which previously only occurred in the higher frequency range, to re-enter the wear path (purple grid field) in the first stage of the cycle. However, increasing wear on the bearing can be assumed at this point. It is advisable to stop as planned in the yellow field.

Process analysis is accompanied by a lot of noise, so the stop field must also be reviewed, with measurements and faster calculations. In practice, the key question is always: How much safety margin is left? In the yellow field, the figure is already gradually decreasing, at which point the bearing has exhausted its middle-aged life cycle.

What could be its purpose? Reducing the yellow field bar, but it does not necessarily solve the problem, because this map represents continuous operation.

VI. MOT OR CURRENT TRACK GEOMETRY CALCULATION OPTION

Field geometry questions: Archimedes [22] approximated the circumference of the circle with the circumference of the polygons written on the curve, in the case of n=96, the method gives a value for π as the circumference of the unit diameter. For the coordinates given by the space vector diagram of the asynchronous motor current with the points appearing in the given time interval Iy and Iy, Ix on the y-axis and Iy on the xaxis. Definition: The length of the curve y=f(x), a $\leq x \leq b$: If f is continuous and differentiable on the closed interval [a,b], then the curve y=f(x) x=a and x= the length of the section between b [22]:

$$L = \int_{a}^{b} \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, dx = \int_{a}^{b} \sqrt{1 + [f'(x)]^2} \, dx.$$

The calculation had the following basic motive. Defining the shape fidelity of the space vector trajectory geometry with a specific numerical value, the further calculations are compared to this, with the appropriate calculations and multiplications. Note that this calculation can be used for this trajectory, it will differ in practice, which means that the integrand will change due to the change in the geometric shape.

Examination of the shape fidelity of the asynchronous motor stator current field vector path geometry with an arc length calculation algorithm, characterized by the fact that the calculation interprets the shape of the path arc and selects the corresponding integrand, then stores the value of the arc length and compares it with the next measurement result, and the search algorithm interprets the frequency of the tested signal and its amplitude. Each signal component forms a different track geometry, mapping this is the basic task of the calculation. In addition, the search algorithm also has specified functions of the failure frequency of bearings as shown in the first statement, that is, another task of the algorithm is to generate the space vector model to assist the calculation and/or to perform lifetime estimation.

The elements of the statement are individually known solutions, the search for novelty can be explored in the joint effect.

VII. PREPARATION OF INTELLIGENT DRIVES

Learning mechanical errors using Kohonen's self-organizing map. Setting up a necessary topology, the characteristics of the asynchronous machine are reflected by the grid points of neurons arranged in a plane and the locally discrete topology of their responses. One of the properties of a map is the "feature map". A vital issue is the layout and how many steps the signals can be processed in. Taking into account the modulating effect created by the switching frequency of the frequency converter semiconductors in the excitation of the components of the frequency components caused by the bearing and shaft adjustment, and to help set up interpretation sets for higher frequency ranges. Task-oriented priority management of incoming signals.

Bearing frequency search realized with the help of a spectrum formed from current-field vector components of a three-phase asynchronous motor (power supply via mains or frequency converter), characterized by the fact that after and/or during the production of the current spectrum, a search function (algorithm) is run that specifically searches for the reference to the bearing signals, the basic carrier of which takes into account the amplitude change before and after frequency and filtering. The search algorithm is able to find signals and multiply signals. It can also calculate the frequency components of the signals based on the description of the path geometry of the current space vector with probability of occurrence. The algorithm learns the machine error and produces reference spectra and restores a space vector pattern from them.

CONCLUSION

The reason for the simplification may be the fact that the interaction of information and noise must be decided. If a smaller signal works in a completely different frequency range, don't run an algorithm that analyzes a part that has more noise than useful information. Here we can talk about a controlled learning process.

Further development opportunities

- Exchange of bearing frequency fields, reduction of field crossing points.
- Currently, it arranges steps based on frequency components, changing this.
- Examination of characteristic fields based on amplitude distribution.
- •Analysis of nearby frequency connections.
- •Is fault frequency skipping possible (red field)?
- What simplification process can be produced?
- •Examination of the number of withdrawals

The axis impact frequency field cannot be omitted. The technological process of the electric drive largely determines this question and in many cases they focus on the axis adjustment. In theory, it can be examined in this way as an early prediction, but in the case of the largest harmonic, it is necessary to examine when it enters the characteristic field.

A. Preparation for the development of a self-learning electric drive system

The measurement results are inputs to fuzzy systems. The presentation of the sets refers to the mechanical results of the rotary machine. The purpose of this chapter is to prepare the discussion in the fuzzy system. The basic idea starts from the fact that I assign the changes of the four bearing frequencies and associated amplitude values to ranges. The reason for this is that many errors affect each other and the signals must be compared not only to each other.

During the measurements and analyses, the simple numerical data comparison did not necessarily lead to results, because multiple modulations also occur. As a consequence of this, signal amplification also affects the reading result, especially in the higher frequency ranges.

Purpose, to create the sets that will be presented with respect to the bearing. The selection of ranges follows classical set theory, i.e. whether the given value is an element of a set or not.

Under given conditions, the amplitude changes of the four bearing frequencies (if only the fundamental harmonics are taken into account, which is quite simple), can be designated as a uniform range, which can be called the fundamental set (universe).

The analysis must also be extended to harmonics generated by bearing frequencies. This should be characterized by their license plate number, which increases the value of this domain quite a bit. As a result, they have to be divided into separate ranges using the registration numbers belonging to the harmonics, this selects the sets and helps to create the membership functions.

During the measurements, a number of questions arose as to which ranges a signal can be assumed to be faulty. The purpose of the calculations is to approximate non-contradiction.

After forming the membership functions, the rules between the linguistic terms of the fuzzy sets and the input and output values must be established. The formation of the rules of the fuzzy system depends significantly on the nature of the regulated equipment.

Adaptive fuzzy systems: Fuzzy controls can be static fuzzy based on state variation. The static fuzzy controller is robust and suitable for controlling complex or poorly defined dynamic systems, but it does not behave adaptively in the event of unknown parameter and structure changes of the controlled process and its environment.

Adaptive fuzzy control is similar to conventional adaptive control in its basic structure, principles, and mathematical tools of analysis and design.

There are two fundamental differences between them. The fuzzy controller has the same universal nonlinear structure even in the case of different regulated equipment and processes, while the classic adaptive controller changes from task to task. In contrast to the conventional version, the dynamics of the controlled system and human knowledge about the control method can also be incorporated into the fuzzy controller.

Diagnostic testing of electric rotating machines is a dynamically developing field. After the measurements, it was necessary to develop an analysis strategy that reaches the standard of international scientific works, which is why the previously mentioned mathematical sets were included: Fast Fourier Transform (FFT), Discrete Fourier Transform (DFT), short-time Fourier transform (Short-time Fourier transform, STFT), continuous Wavelet transform (Continuous Wavelet Transform, CWT), Constant Q Gabor transform (Constant-Q Gabor Transform, CQT)), Hilbert Huang transform (Hilbert-Huang Transform, HHT), Wigner-Ville Distribution (WVD), Fuzzy systems, neural networks, statistics and machine learning (Statistics and Machine Learning. The purpose is not only data analysis comparison, conclusion, deduction, but also the development of self-learning systems, learning the direction towards machine intelligence. For this, it is necessary to measure the continuous operation of the motor, analyze current waveforms, and train in error metrics. Searching for a frequency indicating an axis adjustment error and setting up analysis sets. Application of operations research in the functions created about the fault.

REFERENCES

- Jean-Claude Trigeassou, Electrical Machines Diagnosis, First published, 2011, Great Britian and the United States by ISTE Ltd and John & Sons, Inc. ISBN 9781 848212633
- [2] B. Noureddine, P. Remus, R. Raphael and S. Salim, "Rolling Bearing Failure De-tection in Induction Motors using Stator Current, Vibration and Stray Flux Analysis Techniques," IECON 2020 The 46th Annual Conference of the IEEE Industrial Electronics Society, Singapore, 2020, pp. 1088-1095, doi: 10.1109/IECON43393.2020.9254401.
- [3] S. R. Kapoor, N. Khandelwal and P. Pareek, "Bearing fault analysis by signal energy calculation based signal processing technique in Squirrel Cage Induction Mo-tor," 2014 International Conference on Signal Propagation and Computer Techno-logy (ICSPCT 2014), Ajmer, India, 2014, pp. 33-38, doi: 10.1109/ICSPCT.2014.6884922.
- [4] J. Xinjie, H. Malik and S. K. Panda, "An Optimized Intelligent Technique for Bearing Fault Diagnosis using Motor Current Signal Analysis," 2022 International Power Electronics Conference (IPEC-Himeji 2022- ECCE Asia), Himeji, Japan, 2022, pp. 730-735, doi: 10.23919/IPEC-Himeji2022-ECCE53331.2022.9807128.
- [5] M. S. Moiz et al., "Health Monitoring of Three-Phase Induction Motor Using Cur-rent and Vibration Signature Analysis," 2019 International Conference on Robotics and Automation in Industry (ICRAI), Rawalpindi, Pakistan, 2019, pp. 1-4, doi: 10.1109/ICRAI47710.2019.8967356.
- [6] J. Jung et al., "Monitoring of journal bearing faults based on motor current signa-ture analysis for induction motors," 2015 IEEE Energy Conversion Congress and Exposition (ECCE), Montreal, QC, Canada, 2015, pp. 300-307, doi: 10.1109/ECCE.2015.7309702.
- [7] W. Zhou, T. G. Habetler, R. G. Harley and B. Lu, "Incipient Bearing Fault De-tection via Stator Current Noise Cancellation using Wiener Filter," 2007 IEEE Inter-national Symposium on Diagnostics for Electric Machines, Power Electronics and Drives, Cracow, Poland, 2007, pp. 11-16, doi: 10.1109/DEMPED.2007.4393064.
- [8] R. Pusca, R. Romary, N. Bessous and S. Sbaa, "Comparative Study between Two Diagnostic Techniques Dedicated to the Mechanical Fault Detection in Induction Motors," 2020 International Conference on Electrical Engineering (ICEE), Istanbul, Turkey, 2020, pp. 1-8, doi: 10.1109/ICEE49691.2020.9249884.
- [9] J. Jung et al., "Monitoring Journal-Bearing Faults: Making Use of Motor Current Signature Analysis for Induction Motors," in IEEE Industry Applications Magazine, vol. 23, no. 4, pp. 12-21, July-Aug. 2017, doi: 10.1109/MIAS.2016.2600725.
- [10] P. Pareek, N. Khandelwal and S. R. Kapoor, "A new approach for bearing fault analysis in Squirrel Cage Induction Motor," 2016 IEEE 1st International Conference on Power Electronics, Intelligent Control and

Energy Systems (ICPEICES), Delhi, India, 2016, pp. 1-6, doi: 10.1109/ICPEICES.2016.7853090.

- [11] G. Avalos, S. Aguayo, J. Rangel-Magdaleno and M. R. A. Patemina, "Bearing fault detection in induction motors using digital Taylor-Fourier transform," 2022 In-ternational Conference on Electrical Machines (ICEM), Valencia, Spain, 2022, pp. 1830-1835, doi: 10.1109/ICEM51905.2022.9910779.
- [12] N. Bessous, S. E. Zouzou and A. Chemsa, "A new analytical model dedicated to diagnose the rolling bearing damage in induction motors simulation and experi-mental investigation -," 2016 4th International Conference on Control Engineering & Information Technology (CEIT), Hammamet, Tunisia, 2016, pp. 1-9, doi: 10.1109/CEIT.2016.7929085.
- [13] E. Elbouchikhi, V. Choqueuse, F. Auger and M. E. H. Benbouzid, "Motor Cur-rent Signal Analysis Based on a Matched Subspace Detector," in IEEE Transactions on Instrumentation and Measurement, vol. 66, no. 12, pp. 3260-3270, Dec. 2017, doi: 10.1109/TIM.2017.2749858.
- [14] N. Khandelwal, P. Pareek and S. R. Kapoor, "Start-up transient current analysis for Squirrel Cage Induction Motor," 2016 IEEE 1st International Conference on Po-wer Electronics, Intelligent Control and Energy Systems (ICPEICES), Delhi, India, 2016, pp. 1-6, doi: 10.1109/ICPEICES.2016.7853261.
- [15] A. Soualhi, G. Clerc and H. Razik, "Detection and Diagnosis of Faults in In-duction Motor Using an Improved Artificial Ant Clustering Technique," in IEEE Transactions on Industrial Electronics, vol. 60, no. 9, pp. 4053-4062, Sept. 2013, doi: 10.1109/TIE.2012.2230598.
- [16] Yong Li and Tengxi Wang, "Signal segmentation for isolating the influence of PQ variation and machine manufacturing imperfections on bearing fault detection," 2013 International Electric Machines & Drives

Conference, Chicago, IL, USA, 2013, pp. 734-741, doi: 10.1109/IEMDC.2013.6556175.

- [17] A. Husna, K. Indriawati and B. L. Widjiantoro, "Discriminant Feature Extraction of Motor Current Signal Analysis and Vibration For Centrifugal Pump Fault De-tection," 2021 International Conference on Instrumentation, Control, and Automa-tion (ICA), Bandung, Indonesia, 2021, pp. 207-212, doi: 10.1109/ICA52848.2021.9625679.
- [18] E. T. Esfahani, S. Wang and V. Sundararajan, "Multisensor Wireless System for Eccentricity and Bearing Fault Detection in Induction Motors," in IEEE/ASME Transactions on Mechatronics, vol. 19, no. 3, pp. 818-826, June 2014, doi: 10.1109/TMECH.2013.2260865.
- [19] B. Raison, G. Rostaing, O. Butscher and C. S. Maroni, "Investigations of algo-rithms for bearing fault detection in induction drives," IEEE 2002 28th Annual Con-ference of the Industrial Electronics Society. IECON 02, Seville, Spain, 2002, pp. 1696-1701 vol.2, doi: 10.1109/IECON.2002.1185536.
- [20] Y. Tian, D. Guo, K. Zhang, L. Jia, H. Qiao and H. Tang, "A Review of Fault Diagnosis for Traction Induction Motor," 2018 37th Chinese Control Conference (CCC), Wuhan, China, 2018, pp. 5763-5768, doi: 10.23919/ChiCC.2018.8484044.
- [21] [21] S. Zhao et al., "The Inter-turns Short Circuit Fault Detection based on External Leakage Flux Sensing and VMD-HHT Analytical Method for DFIG," 2021 Interna-tional Conference on Sensing, Measurement & Data Analytics in the era of Artificial Intelligence (ICSMD), Nanjing, China, 2021, pp. 1-5, doi: 10.1109/ICSMD53520.2021.9670783.
- [22] [191] George B. Thomas, Maurice D. Weir, Joel Hass, Frank R. Giordano, Thomas-féle Kalkulus 2. Második kiadás. II. kötet, Typotex Budapest 2010