The Dual Problem Application and Economic Interpretation

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***Abstract*— The idea of this publication is based on the fact that through the dual problem of linear programming to find the optimal solution that reflects the full information for the solution of original problem. In particular, the dual problem provides important information in economic terms in connection with specific conditions imposed at the time of formulation of problem. The investigation is made by an example of sensitivity analysis-economic interpretation as a potential tool for optimizing the production process, supposing that we have a primary problem of mixed products and the goal is to determine the levels of production that provide maximum profit.**

***Keywords*— the problem of transport, optimisation, dual problem, sensitivity analysis, optimal solution, economic interpretation, etc.**

I. Introduction

Daily activity of business enterprises contains different situations which might be either favorable or unfavorable that should be managed more rationally. The business environment where economic entities have these activities is constantly changing, therefore its managers are facing many problems that should be solved, find alternatives that can be singled out as the best, alternatives that should be excluded and ultimately should take a decision for the future of the company.

Rapid economic developments are associated with the increased complexity of management problems and increased difficulties for solving them. One of the ways that can facilitate problem solving is its presentation by mathematical model, thus the quantitative methods might be used to find a solution. Implementation of these models and techniques occupies a very important place because it helps the decision-making process to be "the best possible" and make them important management tools. Generally, the use of these quantitative methods and algorithms for solving such problems require processing of a large amount of information and performing a large number of calculations in order to achieve the optimal solution.

In this paper we will investigate the sensitivity analysis of the optimal solution against the changes in the coefficients of the goal function and the values of free terms in the right side of the equation. In addition it will be necessary to determine the sensitivity analysis of optimal solution from the changes on products, constrains, system parameters and conditions.

Supposing that we have a primary problem of mixed products and the goal is to determine the levels of production that provides maximum profit for the firm. The firm is using 3 resources from which they extract three different products, as shown in the Table 1:

Production factors (resources)	Types of products				Reserves (unit)
	I	II	III	IV	
Raw material (kg)	a_{11}	a_{12}	a_{13}	a_{14}	b_1
Work (h)	a_{21}	a_{22}	a_{23}	a_{24}	b_2
Administration	a_{31}	a_{32}	a_{33}	a_{34}	b_3
	c_1	c_2	c_3	c_4	

Table. 1 Inputs of production system

The formulation of primary problem is given as follows:

$$f = c_1x_1 + c_2x_2 + c_3x_3 + c_4x_4 \tag{1}$$

with conditions:

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4 \le b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + a_{24}x_4 \le b_2 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + a_{34}x_4 \le b_3 \\ \quad x_1 \ge 0,\ x_2 \ge 0,\ x_3 \ge 0,\ x_4 \ge 0 \end{cases} \tag{2}$$

where:

a_{ij} - The amount of resources i for unit production j,

c_{ij} - Profit from the sale of unit production of j,

b_i- reserve of resource i,

f – Goal function,

x_i – the amount of products that must be produced.

The dual problem of the primary problem is:

2. Goal function to be minimised:

$$g = b_1y_1 + b_2y_2 + b_3y_3 \tag{3}$$

with conditions

$$\begin{cases} a_{11}y_1 + a_{21}y_2 + a_{31}y_3 \ge c_1 \\ a_{12}y_1 + a_{22}y_2 + a_{32}y_3 \ge c_2 \\ a_{13}y_1 + a_{23}y_2 + a_{33}y_3 \ge c_3 \\ a_{14}y_1 + a_{24}y_2 + a_{34}y_3 \ge c_4 \\ \quad y_1 \ge 0, y_2 \ge 0, y_3 \ge 0 \end{cases} \tag{4}$$

where:

y_i – is the marginal value or unit shadow prices for the resources 1, 2 and 3,

g – the total value of resources.

The first formulation is constructed mathematical model in order to find the production plan of the production quantities x_i , which will result with maximum profit for the company. It is clear that this profit is conditional and depends on factors of production reserves, raw materials or used resources that are each time in limited quantities. Therefore, the raw material b_1 which is used for production of x_1 products P_1 is $a_{11}x_1$, for production of x_2 products P_2 is $a_{12}x_2$, for production of x_3 products P_3 is $a_{13}x_3$, for production of x_4 products P_4 is $a_{14}x_4$, and the total quantity of raw materials consumption for the production plan is $a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4$ and this should not exceed the raw material reserves - b_1 unit.

Thus: $a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4 \leq b_1$.

Second formulation is based on the fact that the profit resulting from the sale of manufactured products cannot be made without the use of resources (raw materials, workforce). Starting from this fact it can be concluded that the production factors have given value for the firm, thus it's of a great interest to determine this value.

In the analogy with the primary problem is also formulated the so-called dual problem, whose objective is the distribution of resources in a way that the total amount of resources is minimized. In order to achieve this that to each foregoing production factor is associated an artificial unit price y_i, (the optimal value of these variables in the economy will be called as shadow price or hidden price). y_i values, are calculated in such a way that the sum of these values of resources used to be equal to the total profit resulting from the product sale. So, from what was said above we can write the goal function (the shadow price or hidden price needs to be minimised for the resources 1 and 2):

$$g = b_1y_1 + b_2y_2 + b_3y_3 \ \ (min) \tag{5}$$

The value assigned to the resources used for production of a unit product 1, which must be at least as profit contribution of a unit of product 1.

$$a_{11}y_1 + a_{21}y_2 + a_{31}y_3 \geq c_1 \tag{6}$$

By analyzing the second the in-equation (6), we can come to two observations:

a) If the value that the company gets from the usage of resources in order to produce a unit product is equal to his contribution to profit, as a result of the sale of his product, then this product belongs to the optimal production plan from the first formulation,
b) If the value of resources used to produce a unit product exceeds the unit profit that this product brings, then this means that the resources can be used in the most profitable way for production of other products.

Example

Company produces three types of products using 3 production factors: raw materials, workforce and energy, as shown in Table 2.

Production factors (resources)	Types of products				Reserves (unit)
	I	**II**	**III**	**IV**	
Raw material (kg)	1	1	1	1	100 t
Work (h)	10	4	5	4	600 h
Energy (kW)	2	2	6	6	300 kW
Unit profit (€)	**10**	**6**	**6**	**8**	

Table 2. Inputs of production for company

The problem from the Table 2 it can be formulated as the problem of linear programming:

$$f = 10x_1 + 6x_2 + 6x_3 + 8x_4 \quad (max) \tag{7}$$

With conditions:

$$\begin{cases} x_1 + x_2 + x_3 + x_4 \leq 100 \\ 10x_1 + 4x_2 + 5x_3 + 4x_4 \leq 600 \\ 2x_1 + 2x_2 + 6x_3 + 6x_4 \leq 300 \\ x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0,\ x_4 \geq 0 \end{cases} \tag{8}$$

And as next step to solve this problem is its transformation in standard form:

$$f' = -f = -10x_1 - 6x_2 - 6x_3 - 8x_4 \quad (min) \tag{9}$$

with conditions:

$$\begin{cases} x_1 + x_2 + x_3 + x_4 + x_5 = 100 \\ 10x_1 + 4x_2 + 5x_3 + 4x_4 + x_6 = 600 \\ 2x_1 + 2x_2 + 6x_3 + 6x_4 + x_7 = 300 \\ x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0,\ x_4 \geq 0, x_5 \geq 0, x_6 \geq 0, x_7 \geq 0 \end{cases} \tag{10}$$

As standard features requires that the goal function is minimized so we can write f '=- f, but again the goal function (f' min) is equivalent to that (f max) which is required by a given task, because for the same solutions (x_1, x_2, x_3, x_4) for which the function f would have max values, the function f ' will have minimum value. Also, the problem in standard form is consisted only from the equations therefore we add new unknown non-negative parameters x_5, x_6 and x_7 in order to equal the 2 sides in-equation and the same will become equation.

Before we start with simplex method the problem must be in canonical form as given in the following:

$$f' = 0 - (10x_1 + 6x_2 + 6x_3 + 8x_4) \quad (min) \tag{11}$$

with conditions:

$$\begin{cases} x_5 = 100 - (x_1 + x_2 + x_3 + x_4) \\ x_6 = 600 - (10x_1 + 4x_2 + 5x_3 + 4x_4) \\ x_7 = 300 - (2x_1 + 2x_2 + 6x_3 + 6x_4) \\ x_i \geq 0,\ \forall i = \overline{1,7} \end{cases} \tag{12}$$

So, we can build a simplex table with coefficients before the variables. Initially should be noted that x_5, x_6 and x_7 are basic unknown parameters (PB) and 100, 600 and 300 are free terms (TL) (see Table 3).

Table 3 – Simplex table 1

PB	TL	x_1	x_2	x_3	x_4	x_5	x_6	x_7
x_5	100	1	1	1	1	1	0	0
x_6	600	**10**	4	5	4	0	1	0
x_7	300	2	2	6	6	0	0	1
f'	0	10	6	6	8	0	0	0

Since 10 is key, instead of x_6 on the basic unknown parameter is introduced x_1 in the Table 3. At the same the row in which is the key (10) is divided by 10 and is written in the table but instead of x_6 now we have x_1, thus the procedure is repeated and we gain the following tables (Table 4, 5 and 6).

Table 4 – Simplex table 2

PB	TL	x_1	x_2	x_3	x_4	x_5	x_6	x_7
x_5	40	0	0.6	0.5	0.6	1	-0.1	0
x_1	60	**1**	0.4	0.5	0.4	0	0.1	0
x_7	180	0	1.2	5	**5.2**	0	0.2	1
f'	-600	0	6	6	8	0	0	0

Table 5 – Simplex table 3

PB	**TL**	**x_1**	**x_2**	**x_3**	**x_4**	**x_5**	**x_6**	**x_7**
x_5	19.23	0	**0.46**	0.07	0	1	-0.077	-0.115
x_1	46.15	1	0.31	0.115	0	0	-0.115	-0.077
x_4	34.62	0	0.23	0.96	1	0	-0.038	0.19
f'	0	0	1.08	-2.84	0	0	-0.84	-0.77

Table 6 – Simplex table 4

PB	**TL**	**x_1**	**x_2**	**x_3**	**x_4**	**x_5**	**x_6**	**x_7**
x_5	19.23	0	**0.46**	0.07	0	1	-0.077	-0.115
x_1	46.15	1	0.31	0.115	0	0	-0.115	-0.077
x_4	34.62	0	0.23	0.96	1	0	-0.038	0.19
f'	0	0	1.08	-2.84	0	0	-0.84	-0.77

Table 7 – Optimal solution for primary problem

PB	**TL**	**x_1**	**x_2**	**x_3**	**x_4**	**x_5**	**x_6**	**x_7**
x_2	41.67	0	1	-0.167	0	2.16	-0.16	-0.25
x_1	33.33	1	0	0.167	0	-0.67	0.167	0
x_4	25	0	0	1	1	-0.5	0	0.25
f'	-783.33	0	0	-2.67	0	-2.33	-0.667	-0.5

Since in the fourth row there is no any positive number then the problem is solved and because the value of -783.33 is the min of *f'* then 783.33 is the required max from the beginning at *f*: $x_1 = 33.33$, $x_2 = 41.67$, $x_3 = 0$, $x_4 = 25$.

The dual problem

$$g = 100y_1 + 600y_2 + 300y_3 \quad (min) \tag{13}$$

with conditions

$$\begin{cases} y_1 + 10y_2 + 2y_3 \geq 10 \\ y_1 + 4y_2 + 2y_3 \geq 6 \\ y_1 + 5y_2 + 6y_3 \geq 6 \\ y_1 + 4y_2 + 6y_3 \geq 8 \end{cases} \tag{14}$$

The system of conditions is transformed in to a system of equations

$$\begin{cases} y_1 + 10y_2 + 2y_3 - y_4 = 10 \\ y_1 + 4y_2 + 2y_3 - y_5 = 6 \\ y_1 + 5y_2 + 6y_3 - y_6 = 6 \\ y_1 + 4y_2 + 6y_3 - y_7 = 8 \end{cases} \tag{15}$$

Then the above system is transformed in to canonical form:

$$g = 0 - (-100y_1 - 600y_2 - 300y_3) \quad (min) \tag{16}$$

$$\begin{cases} y_4 = -10 - (-y_1 - 10y_2 - 2y_3) \\ y_5 = -6 - (-y_1 - 4y_2 - 2y_3) \\ y_6 = -6 - (-y_1 - 5y_2 - 6y_3) \\ y_7 = -8 - (-y_1 - 4y_2 - 6y_3) \end{cases} \tag{17}$$

The dual simplex Table 8, is created by the following actions:

1. column of free terms has negative elements,
2. select the min (-10, -6, -6, -8) =- 10 (y_4 is out from the base),
3. from the field of negative elements are selected reports: $min\left\{\frac{-100}{-1}, \frac{-600}{-10}, \frac{-200}{-2}\right\} = 60$,
4. Key element is -10 and through this value the procedure is continued based on the Gauss method.

Table 8. Dual simplex tables

PB	**TL**	**y_1**	**y_2**	**y_3**	**y_4**	**y_5**	**y_6**	**y_7**
y_4	-10	-1	-10	-2	1	0	0	0
y_5	-6	-1	-4	-2	0	1	0	0
y_6	-6	-1	-5	-6	0	0	1	0
y_7	-8	-1	-4	-6	0	0	0	1
g	0	-100	-600	-300	0	0	0	0

Table 8.1. Dual simplex tables

PB	**TL**	**y_1**	**y_2**	**y_3**	**y_4**	**y_5**	**y_6**	**y_7**
y_2	10	1	10	2	-1	0	0	0
y_5	-2	-0.6	0	-1.2	-0.4	1	0	0
y_6	-1	-0.5	0	-5	-0.5	0	1	0
y_7	-4	-0.6	0	-5.2	-0.4	0	0	1
g	600	-40	0	-180	-60	0	0	0

Table 9. Dual simplex tables

PB	**TL**	**y_1**	**y_2**	**y_3**	**y_4**	**y_5**	**y_6**	**y_7**
y_2	8.47	0.77	10	0	-1.15	0	0	0.38
y_5	-1.08	0.46	0	0	-0.304	1	0	-0.23
y_6	2.84	0.07	0	0	-0.116	0	1	-0.96
y_3	4	0.6	0	5.2	0.4	0	0	-1
g	738.46	-19.23	0	0	-46.15	0	0	-34.61

Table 9.1. Dual simplex tables

PB	TL	y_1	y_2	y_3	y_4	y_5	y_6	y_7
y_2	6.67	0	10	0	-1.65	1.67	0	0
y_1	1.08	0.46	0	0	0.304	-1	0	0.23
y_6	2.67	0	0	0	-0.12	0.15	1	-1.62
y_3	2.6	0	0	5.2	0	1.30	0	-1.3
g	783.33	0	0	0	-33.33	-41.67	0	-25

Since the free terms are all positive and the coefficients of goal function is non-negative it can be concluded that the optimal solution is achieved. In the following table are made some basic transformations in order to gain the values of basic unknown parameters (y_1, y_2, y_3, y_6).

Finally, can be written that shadow prices or marginal values of production factors are as follows: y_1 = 2:33, y_2 = 0667, y_3 =0.5, y_4 = 0, y_5 = 0, y_6 = 2.67 and *min g* = 783.33.

So, as these resources are used to generate profit 783.33€ = *max f*, also the derived values attributed to them should be 783.33€ which is confirmed in the following:

$$g = 100y_1 + 600y_2 + 300y_3 = 10x_1 + 6x_2 + 6x_3 + 8x_4 = f$$

$$100 \cdot 2.33 + 600 \cdot 0.667 + 300 \cdot 0.5 = \\ = 10 \cdot 33.33 + 6 \cdot 41.67 + 6 \cdot 0 + 8 \cdot 25$$

783.33=783.33

Table 10. The dual simples table

PB	TL	y_1	y_2	y_3	y_4	y_5	y_6	y_7
y_2	0.667	0	1	0	-0.165	-0.167	0	0
y_1	2.33	1	0	0	0.667	-2.17	0	0.5
y_6	2.67	0	0	0	-0.12	0.15	1	-1.62
y_3	0.5	0	0	1	0	0.25	0	-0.25
g	783.33	0	0	0	-33.33	-41.67	0	-25
					$-x_1$	$-x_2$	$-x_3$	$-x_4$

Detached from the above discussion we can say that the attributed value of sources:

- For the production of one unit of second product is given by $y_1 + 4y_2 + 2y_3 = 2.33 + 4 \cdot 0.667 + 2 \cdot 0.5 = 6$ which means that it is equal to the unit profit from the second product, so the second product is in the optimal solution of the primary problem or is in the production plan.
- For the production of one unit of third product is given by $y_1 + 5y_2 + 6y_3 = 2.33 + 5 \cdot 0.667 + 6 \cdot 0.5 = 8.67 \geq 6$ which means that the value of resources which are used for producing of one unit of third product will diminish the profit of the company with 8.67-6 = 2.67€ so this product will not be involved in the production plan of primary problem.
- Another indicators is the value of additional variable y_6 = 2.67 that endorses that the value of resources for the production of one production unit exceeds the first product with 2.67 € the contribution to a profit which this product is giving. This fact indicates that resources can be used in more profitable way for the production of 3 other products. For more on the following table is given the link between variables from which it can be seen that x_3 = 0 and y_6 is optimal dualit basis

II. SENSITIVITY ANALYSIS

In the example which is given above of a mixed product is supposed that the unit profit is also provided for each product, reserves of the resources available for use and rates of the system conditions that express the expenditures of each production factors (source) for each production unit are known. In practice these parameters are not ever known with certainty since the cost of raw materials, labor cost may experience changes which will affect the unit profit of product.

This sensitivity of the optimal solution to the change of the unit profit is of great importance for the manager and is measured by tolerance where the unit profit can very without causing changes in the optimal solution. For illustration lets uses the example above where the second product brings profit of 6 €/unit and that this product is included in the optimal solution. With this sensitivity analysis is proved that the current solution remains optimal as long as unit profit for the second product is between 4.92 and 8. So this is an information for the manager that a decrease in the unit profit of product with more than 1.08€ and an increase for more than 2 € will lead to another optimal solution.

In the following is explained technique of the profit elasticity.

Changing the coefficients into the goal function

Sensitivity of the optimal solution to a changes of the coefficients cj of the goal function is analyzed by adding to this coefficient a variable δ and thus become the coefficient $\delta + c_j$.

Table 11. Dual simplex table

PB	TL	x_1	x_2	x_3	x_4	x_5	x_6	x_7
x_2	41.67	0	1	-0.167	0	2.16	-0.167	-0.25
x_1	33.33	1	0	0.167	0	-0.67	0.167	0
x_4	25	0	0	1	1	-0.5	0	0.25
f'	-783.33	0	0	-2.67	0	-2.33	-0.667	-0.5

As the second product is in BP thus we add δ, and new coefficient becomes $\bar{c}_2 = c_2 + \delta = 6 + \delta$ and the goal function will take the following form:

$$\bar{f} = 10x_1 + (6 + \delta)x_2 + 6x_3 + 8x_4 = f + \delta x_2 \quad (max) \qquad (18)$$

Since x_2 is on base (PB) then from the last simplex table (optimal) are obtained:

$$x_2 = 41.67 - (0.167x_3 + 2.16x_5 - 0.167x_6 - 0.25x_7) \quad (19)$$

$$f = 783.33 - (2.67x_3 + 2.33x_5 + 0.667x_6 + 0.5x_7) \quad (20)$$

Substituting the equations (19) and (20) in to the equation (18) is derived the following equation:

$$\bar{f} = (783.33 + 41.67\delta) - [(2.67 + 0.167\delta)x_3 + (2.33 + 2.16\delta)x_5 + (0.667 - 0.167\delta)x_6 + (0.5 - 0.25\delta)x_7]$$

And the new table will look as follows (table 12):

Table 12. Dual simplex 'Optimal table'

PB	TL	x_1	x_2	x_3	x_4	x_5	x_6	x_7
x_2	41.67	0	1	-0.167	0	2.16	-0.16	-0.25
x_1	33.33	1	0	0.167	0	-0.67	0.167	0
x_4	25	0	0	1	1	-0.5	0	0.25
$\bar{f}'$	$-783.33-41.67\delta$	0	0	$-2.67-0.167\delta$	0	$-2.33-2.16\delta$	$-0.667+0.167\delta$	$-0.5+0.25\delta$

Since the current solution remains optimal the values δ must meet the following system of non-equations:

$$\begin{cases} -2.67 - 0.167\delta \le 0 \\ -2.33 - 2.16\delta \le 0 \\ -0.667 + 0.167\delta \le 0 \\ -0.5 + 0.25\delta \le 0 \end{cases} \qquad (21)$$

By solving of system (21) we find that $-1.08 \le \delta \le 2$, which means that the optimal solution (production plan) does not change while the coefficient $\bar{c}_2 = c_2 + \delta$ of the goal function is $6 - 1.08 \le c_2 \le 6 + 2$, so $c_2 \in [4.92, 8]$.
Since c_2 changes, the value of *maxf* also changes according to *max f=783.33+41.67*δ for$\delta \in [-1.08, 2]$.

This analysis tells us that an increase in the coefficient of the goal function means that this product has become more profitable and the resources are shifted from production of products to the production of most profitable product.

In the same way are chosen the values profit boundaries per unit production of other products x_1, x_4 and x_3.

$10 - 4 \le c_1 \le 10 + 3.5$, thus $c_1 \in [6, 13.5]$,

$8 - 2 \le c_4 \le 8 + 4.667$, thus $c_2 \in [6, 12.667]$,

$-\propto \le c_3 \le 6 + 2.67$, thus $c_3 \in [-\propto, 8.67]$.

Changing of the free terms of system conditions (stock of factors of production)

The sensitivity analiysis for the optimal solution of primary problem is performed by the table 12 by changing the reserves. For example, for raw material b_1 we add λ_1 and the new vector of free terms is obtained as follows:

$$B_1 = \begin{pmatrix} 100 + \lambda_1 \\ 600 \\ 300 \end{pmatrix} \qquad (22).$$

Values of basic variables given by $X_B = D^{-1}B_1 \ge 0$. The boundaries of tolerance for b1 is determined by the requirement that X_B positive.

D^{-1} matrix can be obtained by optimal table 12 and consist of elements of the pillars of the initial basic variables (x_5, x_6, x_7), while B_1 is a vector matrix of gree terms in initial simplex table (b_1,b_2,b_3).

$$X_B = D^{-1}B_1 = \begin{pmatrix} 2.16 & -0.16 & -0.25 \\ -0.67 & 0.167 & 0 \\ -0.5 & 0 & 0.25 \end{pmatrix} \begin{pmatrix} 100 + \lambda_1 \\ 600 \\ 300 \end{pmatrix} \ge \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \; (23)$$

Thus, from the expression (23) the following values have been obtained:

$$\begin{cases} 216 + 2.16\lambda - 96 - 75 \ge 0 \\ -67 - 0.67\lambda + 100 \ge 0 \\ -50 - 0.5\lambda + 75 \ge 0 \end{cases} \sim \begin{cases} 2.16\lambda + 45 \ge 0 \\ -0.67\lambda + 33 \ge 0 \\ -0.5\lambda + 25 \ge 0 \end{cases} \sim \begin{cases} \lambda \ge 20.8 \\ \lambda \le 49.2 \\ \lambda \le 50 \end{cases}$$

This means that the basic variables x_1, x_2, x_3 and x_4, remain in the optimal basis until $-20.8 \le \lambda \le 49.2$, while $100-20.8 \le b_1 = 100+49.2$, finally $79.2 \le b_1 \le 149.2$.

Which menas that while raw material reserves are in this interval, the basic variables x_1, x_2, and x_4, will remain in optimal solution but with different values which are dependent on the value of λ that are:

$x_1 = -0.67\lambda+33$, $x_2 = 2.16\lambda+45$, $x_4 = -0.5\lambda+25$

max f =10x₁+6x₂+8x₄=7883.33+2.26λ

III. CONCLUSIONS

- The dual problem may reflect significant economic information about production factors used in the production process.
- The application of the dual problem for production problems aims to find the minimum value of resources as a whole, respectively the distribution of resources according to a production plan so that the total value of resources to be much smaller: $g = b_1y_1 + b_2y_2 + b_3y_3 \; min$
- Solving the dual problem in economic value is interpreted as marginal or artificial price of a unit of production factors. This means that through this solution is highlighted the attributed value (merit value) of resources as production factors.
- Through dual problem is reflected the contribution of a unit of resources in profit generation, specially economic argumentation for producing or not producing that kind of product.
- Through this method is shown the tolerance interval of profit from the production of products involved in the production plan and also the needed price of sale for the product which is not included to the production plan (if c_3 >8.67€ then convenient to produce product 3);
- Through this analysis, we find profitable products for production in case of change of the raw material reserves.

REFERENCES

[1]. Donald A. Pierre: Optimisation theory with applications, http://books.google.com 227-248.

[2]. Ruseti, B., Mitre, M.: Applied mathematics for the Faculty of Economics, (2008), 69-113.
[3]. Simonnard, M.: Linear programing.
[4]. J. Pula: " Transportation methods of linear programming," University of Pristina, 2005,
[5]. Shpëtim Shehu "Operational Research", Tirana 2004,
[6]. L.W. Winston: "Operations Research. Applications and Algorithms, Duxbury Press, Belmont, 1996,
[7]. R. Likaj: "Operational Research", lectures authorized, Faculty of Mechanical Engineering, Prishtina 2007.
[8]. Wireless LAN Medium Access Control (MAC) and Physical Layer (PHY) Specification, UDCS Std. 802.11, 1997.