Modular Irregular Labeling on Complete Graphs and Complete Bipartite Graph

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MODULAR IRREGULAR LABELING ON COMPLETE GRAPH AND COMPLETE BIPARTITE GRAPH

Abstract

A graph that allows for a modular irregular labeling is a modular irregular labeling graph. A modular irregular labeling of a graph G of size n is a mapping of the graph's set of edges to 1, 2, . . . , k with the weights of all vertices distinct. The sum of a vertices's incident edge labels is its weight and the weight of all vertex, determined using the method of additive modulo n. The least biggest edge label that can be used for modular irregular labeling is the modular irregularity strength. This article shows a modular irregular labeling of complete graph $K_n, n \geq 3$ and some family of complete bipartite graph.

1. Introduction

Graph labeling is the process of mapping a collection of integers, known as labels, to graph elements, which are commonly represents either vertices or edges[2]. The labels are usually defines a positive integer. There have been numerous labeling systems established. Irregular labeling and modular irregular labeling are two examples[2]. Chartrand at. al. [3] were the first to adopt irregular labeling[13],[30] in 1988. There have been research on the irregular labeling of some graphs up to this point. Terminology that is not covered in this paper may be found here [4].

"A modular irregular labeling defined by $g : E \to \{1, 2, 3, ..., k\}$ with $k \in \mathbb{R}^+$, such that $w(g(x)) = \sum_{y \in N(x)} g(xy)$ is distinct vertices, where $N(x)$ is a incedent edges of vertex x. The irregularity strength $s(G)$ of a graph G is the minimum value of k for which G has irregular labeling with labels at most k. The irregularity strength $s(G)$ later proved that if the tree of a graph G is defined only for graphs containing at most one isolated vertex and no connected component of order 2. The lower bound of the irregularity strength of a graph G is $s(G) \geq \max\limits_{1 \leq i \leq \Delta(G)} \{n_i + i - 1/i\}$, where n_i vertices with degree i, as stated in theorem 2.2. For a regular graph G Przyboylo[5] has demonstrated a upper bound of

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an irregularity strength is $s(G) < \frac{16n+6d}{6}$. Aigner and Triesch[6] demonstrated that any tree with no vertices of degree two has an irregularity strength equal to the number of its leaves. Ferrara et al. [7] later proved that if the tree T has every two vertices of degree not equal to two at a distance of at least eight with number of leaves at least three, then $s(T) = n_1 + n_2/2$, where n_1 indicats the number of leaves and n_2 indicates the number of vertices of degree two. The survey of irregular labelling has beeen done by Baca et al." [8]. Many more outcomes have been discovered as a result of this survey report. For the most recent information, see Gallian's survey [2].

"A graph of modular irregular labeling is defined by $\phi : E(G) \rightarrow \{1, 2, ..., k\}$so that a bijective function defined by $wt(\phi(x)) = \sum_{y \in N(x)} \phi(xy)$ and distinct values. The set of the weights of the vertices is a family of labeling exists is called the modular irregularity strength $ms(G)$ of the graph G." The path, star, triangular graph, gear and cycle graph's modular irregularity strength was determined by Baca et al. [9]. The tadpole graph's modular irregularity and double cycle graph's modular irregularity were proven by Muthugurupackiam et al.[10]. Baca et al.[12], further demonstrated the fan graph's modular irregularity strength. Vidyanandini at. al. [13] analysed the edge irregularity strength of complete graphs and complete bipartite graphs. Sugeng at. al. [1] determined the modular irregular labeling on double friendship graph and star graph. In this paper, we present the modular irregular labeling of complete graph and complete bipartite graph.

2. Known Results

"There are some known results that we will use to prove the modular irregularity strength of the complete graph and complite bipartite graph that follows in this section."

Theorem 2.1. *(see[1]). "Let S_{kk}, k is atleast 1 be a regular double star graph then*

$$S_{kk} = \begin{cases} 2k, & k \text{ is odd} \\ \infty, & k \text{ is even"} \end{cases}$$

Theorem 2.2. *(see[3]). "Let G be a connected graph with an order more than 2, which has n_i vertices with degree i. Then, $s(G) \geq \max_{1 \leq i \leq \Delta(G)} \left\{\frac{n_i+i-1}{i}\right\}$. The relation between the iregularity strength and modular irregularit strength has been known and presented in the following theorem".*

Theorem 2.3. *(see[9]). "Let G be agraph without a component of order ≤ 2. Then, $s(G) \leq ms(G)$, Not all graphs can have modular irregular labeling. In the following theorem, Baca et al. give a requirement of a graph that cannot have a modular irregular labeling, denoted by $ms(G) = \infty$".*

Theorem 2.4. *(see[9]). "If G is a graph of orger $n, n \equiv 2(mod 4)$, then G has no modular irregular k-labeling, i.e., $ms(G) = \infty$".*

3. Main results

"In this section, we will prove modular irregular labeling for different vetex of complete graph and complete bipartite graph.

Theorem 3.1. *Complete graph K_n, $n \geq 3$ admit a modular irregular labeling.*

Proof. Let G be a simple undirected graph in which every pair distinct vertices are connected by a unique edge. Let $v_1, v_2, ..., v_n$ be the vertices and $e_1, e_2, ..., e_n$ be the edges. Let $|v(G)| = n$ and $|E(G)| \leq n$. The weights of the vertices are the sum of its incident edge labels and all vertex weights are calculated with sum modulo n. Define $f : V(G) \to 0, 1, 2, ..., n-1$ and $f : E(G) \to 1, 2, ..., n$."

Case(i) When $n = 3$, the complete graph K_3 illustrated.

The vertex labeling $V(G)$ has follows $v_i = i - 1, i = 1, 2, ..., n-1$ and the edge labels for

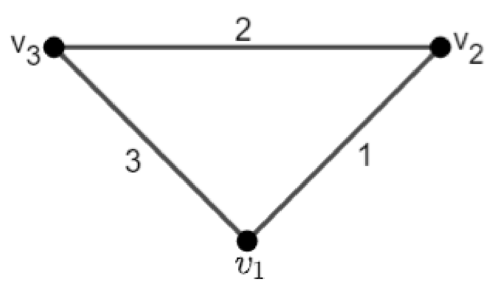

Figure 1. Complete Graph K_3

the complete graph K_3 defined by

$f(v_iv_j) = i, i = 1, j = 2$

$f(v_iv_j) = i + 2, i = 1, j = 3$

$f(v_iv_j) = i, i = 2, j = 3$

Case(ii)When $n = 4$, the complete graph K_4 illustrated.

The vertex labeling $V(G)$ has follows $v_i = i - 1, i = 1, 2, ..., n - 1$ and the edge labels for

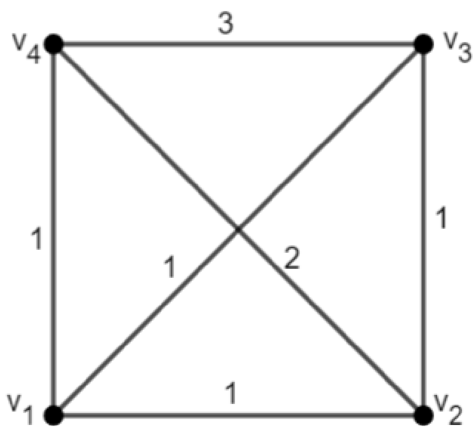

Figure 2. Complete Graph K_4

the complete graph K_4 defined by

$f(v_i v_j) = i, i = 1, j = 2$

$f(v_i v_j) = i, i = 1, j = 3$

$f(v_i v_j) = i, i = 1, j = 4$

$f(v_i v_j) = i, i = 2, j = 4$

$f(v_i v_j) = i - 1, i = 2, j = 3$

$f(v_i v_j) = i, i = 3, j = 4$

Case(iii) When $n = 5$, the complete graph K_5 illustrated.

The vertex labeling $V(G)$ has follows $v_i = i - 1, i = 1, 2, ..., n - 1$ and the edge labels for the complete graph K_5 defined by

$f(v_i v_j) = i, i = 1, j = 2,\ f(v_i v_j) = i, i = 1, j = 3$

$f(v_i v_j) = i + 1, i = 1, j = 4,\ f(v_i v_j) = i, i = 1, j = 5$

$f(v_i v_j) = i, i = 2, j = 3,\ f(v_i v_j) = i, i = 2, j = 4$

$f(v_i v_j) = i - 1, i = 2, j = 5,\ f(v_i v_j) = i, i = 3, j = 4$

$f(v_i v_j) = i - 2, i = 3, j = 5,\ f(v_i v_j) = i - 3, i = 4, j = 5$

Case(iv) When $n = 6$, the complete graph K_6 illustrated.

The vertex labeling $V(G)$ has follows $v_i = i - 1, i = 1, 2, ..., n - 1$ and the edge labels for the complete graph K_6 defined by

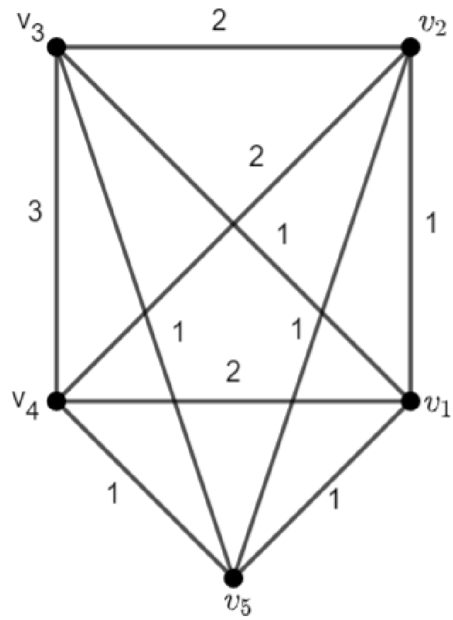

FIGURE 3. Complete Graph K_5

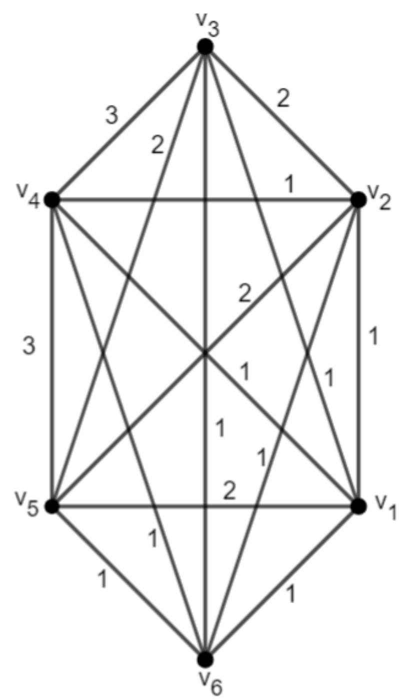

FIGURE 4. Complete Graph K_6

$f(v_iv_j) = i, i = 1, j = 2,\ f(v_iv_j) = i, i = 1, j = 3$

$f(v_iv_j) = i, i = 1, j = 4,\ f(v_iv_j) = i + 1, i = 1, j = 5$

$f(v_iv_j) = i, i = 1, j = 6,\ f(v_iv_j) = i, i = 2, j = 3$

$f(v_iv_j) = i - 1, i = 2, j = 4,\ f(v_iv_j) = i, i = 2, j = 5$

$f(v_iv_j) = i - 1, i = 2, j = 6,\ f(v_iv_j) = i, i = 3, j = 4$

$f(v_iv_j) = i - 1, i = 3, j = 5,\ f(v_iv_j) = i - 2, i = 3, j = 6$

$f(v_iv_j) = i - 1, i = 4, j = 5,\ f(v_iv_j) = i - 3, i = 4, j = 6$

$f(v_iv_j) = i - 4, i = 5, j = 6$

Case(v)When $n = 7$, the complete graph K_7 illustrated.

The vertex labeling $V(G)$ has follows $v_i = i - 1, i = 1, 2, ..., n - 1$ and the edge labels for

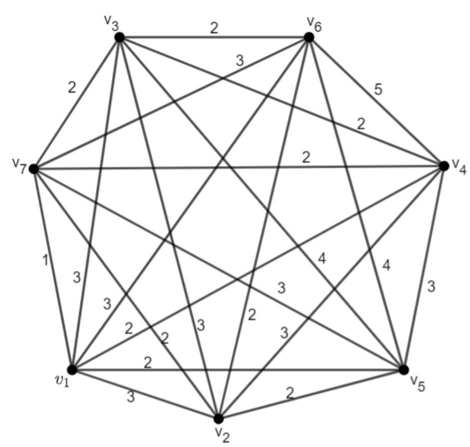

FIGURE 5. Complete Graph K_7

the complete graph K_7 defined by

$f(v_iv_j) = i + 2, i = 1, j = 2,\ f(v_iv_j) = i + 2, i = 1, j = 3$

$f(v_iv_j) = i+1, i = 1, j = 4,\ f(v_iv_j) = i+1, i = 1, j = 5$

$f(v_iv_j) = i+2, i = 1, j = 6,\ f(v_iv_j) = i, i = 1, j = 7$

$f(v_iv_j) = i+1, i = 2, j = 3,\ f(v_iv_j) = i+1, i = 2, j = 4$

$f(v_iv_j) = i, i = 2, j = 5,\ f(v_iv_j) = i, i = 2, j = 6$

$f(v_iv_j) = i, i = 2, j = 7,\ f(v_iv_j) = i-1, i = 3, j = 4$

$f(v_iv_j) = i+1, i = 3, j = 5,\ f(v_iv_j) = i-1, i = 3, j = 6$

$f(v_iv_j) = i-1, i = 3, j = 7,\ f(v_iv_j) = i-1, i = 4, j = 5$

$f(v_iv_j) = i+1, i = 4, j = 6,\ f(v_iv_j) = i-2, i = 4, j = 7$

$f(v_iv_j) = i-1, i = 5, j = 6,\ f(v_iv_j) = i-2, i = 5, j = 7$

$f(v_iv_j) = i-3, i = 6, j = 7$

Case(vi) When $n = 8$, the complete graph K_8 illustrated.

The vertex labeling $V(G)$ has follows $v_i = i-1, i = 1, 2, ..., n-1$ and the edge labels for

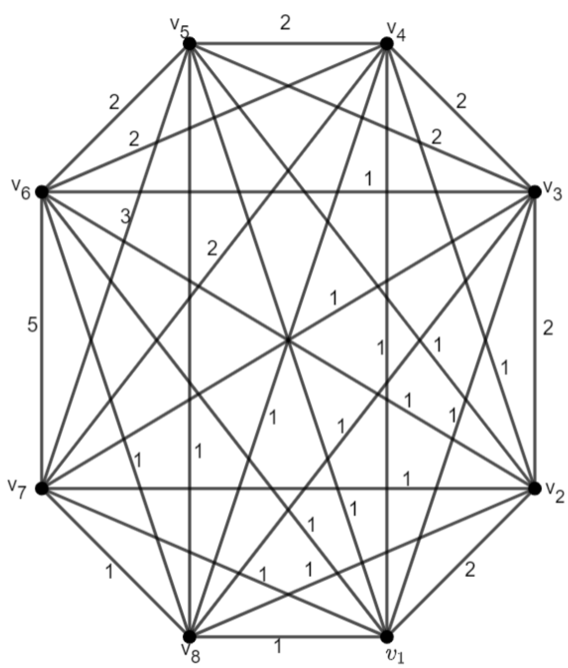

FIGURE 6. Complete Graph K_8

the complete graph K_8 defined by

$f(v_iv_j) = i+1, i = 1, j = 2,\ \ f(v_iv_j) = i, i = 1, j = 3$

$f(v_iv_j) = i, i = 1, j = 4,\ \ f(v_iv_j) = i, i = 1, j = 5$

$f(v_iv_j) = i, i = 1, j = 6,\ \ f(v_iv_j) = i, i = 1, j = 7$

$f(v_iv_j) = i, i = 1, j = 8,\;\; f(v_iv_j) = i, i = 2, j = 3$
$f(v_iv_j) = i-1, i = 2, j = 4,\;\; f(v_iv_j) = i-1, i = 2, j = 5$
$f(v_iv_j) = i-1, i = 2, j = 6,\;\; f(v_iv_j) = i-1, i = 2, j = 7$
$f(v_iv_j) = i-1, i = 2, j = 8,\;\; f(v_iv_j) = i-1, i = 3, j = 4$
$f(v_iv_j) = i-1, i = 3, j = 5,\;\; f(v_iv_j) = i-2, i = 3, j = 6$
$f(v_iv_j) = i-2, i = 3, j = 7,\;\; f(v_iv_j) = i-2, i = 3, j = 8$
$f(v_iv_j) = i-2, i = 4, j = 5,\;\; f(v_iv_j) = i-2, i = 4, j = 6$
$f(v_iv_j) = i-2, i = 4, j = 7,\;\; f(v_iv_j) = i-3, i = 4, j = 8$
$f(v_iv_j) = i-3, i = 5, j = 6,\;\; f(v_iv_j) = i-2, i = 5, j = 7$
$f(v_iv_j) = i-4, i = 5, j = 8,\;\; f(v_iv_j) = i-1, i = 6, j = 7$
$f(v_iv_j) = i-5, i = 6, j = 8,\;\; f(v_iv_j) = i-6, i = 7, j = 8$

therefore, by the definition vertex labels are obtained in complete graph $K_n, n = 3, 4, ..., 8$ □

Theorem 3.2. *Let K_n be a complete graph on n vertices then $ms(K_n) = \frac{ms(K_{n-1}) * ms(K_{n-2})}{3}$ for $n = 5, 6$*

Proof. The vertices are follows $V(G) = v_i = i - 1, i = 1, 2, ..., n-1$ and the edge labels for the complete graph defined by minimum of k by the definition of modular irregularity strength of G is obtained. Which completes the theorem. □

Theorem 3.3. *Let $K_{m,n}$ be a complete bipartite graph then $ms(K_{m,n}) = m + n - 2$*

Proof. Let us call the vertices of $K_{m,n}$ as $\{x_1, x_2, ..., x_m, y_1, y_2, ..., y_n\}$ where $x_i's$ and $y_i's$ are vertices. Let us define vertex labelling as follows $\Phi(x_i) = 3, 1 \leq i < 2$
$\Phi(x_i) = 4, 2 \leq i < 3$
$\Phi(x_i) = 6, 3 \leq i < 4$
$\Phi(x_i) = 1, 4 \leq i < 5$
$\Phi(y_j) = 2, 1 \leq j < 2$
$\Phi(y_j) = 5, 2 \leq j < 3$
$\Phi(y_j) = 7, 3 \leq j < 4$
$\Phi(y_j) = 0, 4 \leq j < 5$
the edge values are obtained $\{2, 3, 5, 6\}$

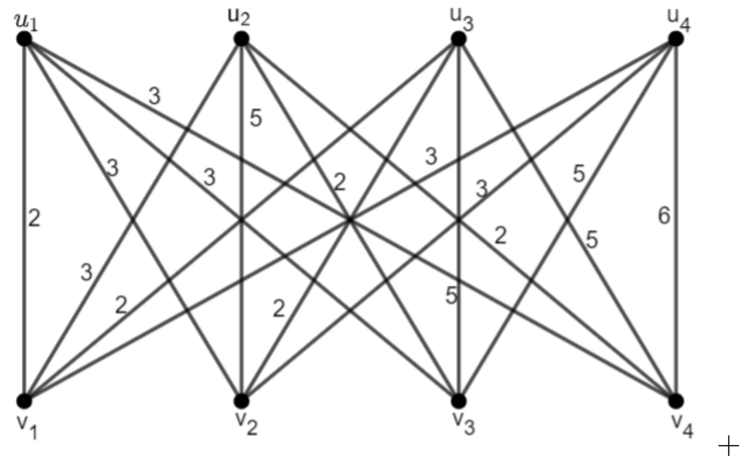

FIGURE 7. Bipartite Graph $K_{4,4}$

Let us define edge labels as follows

$f(x_iy_j) = i + 1, i = 1, j = 1,\quad f(x_iy_j) = i + 2, i = 1, j = 2$

$f(x_iy_j) = i + 2, i = 1, j = 3,\quad f(x_iy_j) = i + 2, i = 1, j = 4$

$f(x_iy_j) = i + 1, i = 2, j = 1,\quad f(x_iy_j) = i + 3, i = 2, j = 2$

$f(x_iy_j) = i, i = 2, j = 3,\quad f(x_iy_j) = i, i = 2, j = 4$

$f(x_iy_j) = i - 1, i = 3, j = 1,\quad f(x_iy_j) = i - 1, i = 3, j = 2$

$f(x_iy_j) = i + 2, i = 3, j = 3,\quad f(x_iy_j) = i + 2, i = 3, j = 4$

$f(x_iy_j) = i - 1, i = 4, j = 1,\quad f(x_iy_j) = i - 1, i = 4, j = 2$

$f(x_iy_j) = i + 1, i = 4, j = 3,\quad f(x_iy_j) = i + 2, i = 4, j = 4$

Hence from definition of modular irregular labeling, the vertex labeling of the complete bipartite graph $K_{m,n}$ obtained. □

4. Applications

In numerous fields of wireless networks, such as cognitive radio networks, Bcube for signals, big data, and cloud computing, bipartite graphs can mathematically simulate common scenarios as well as major problems[14]-[21]. Using modular irregular labeling in Bcube graph of networks we can connect more subscribers with minimum of connectivity[11]. In science, engineering, and technology, as well as medicine, the bipartite graph has a wide range of applications[32]. In cloud computing and cognitive radio networks, bipartite graphs and perfect matching algorithms may tackle a variety of challenges[22]-[29],[31]. Graphs are used to networks communication, data organisation and computational devices etc. Modeling cloud computing and cognitive radio network challenges will be the focus of future study. Bipartite graph applications in computer science, particularly in the above-mentioned fields, are rarely studied in the literature.

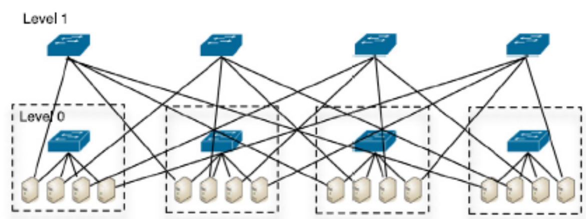

Figure 8. Bcube

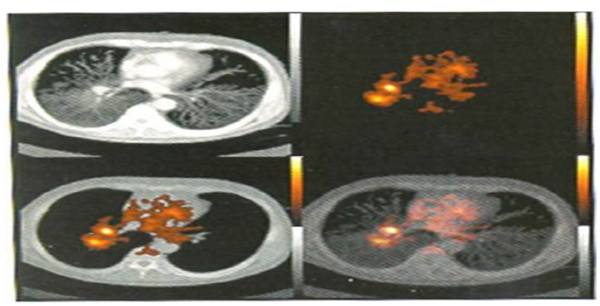

Figure 9. Radio Networks

Conclusions

Hence complete graph $K_n, n \geq 3$ and some family of complete bipartite graph admits modular irregular labelling.

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