The Dependence of the Characteristics of the Dispersion Curves on the Orientation Angle of the CARALL Structures

Hana Driss, Abderrahim El Mahi, Mourad Bentahar, Moez Beyaoui and Mohamed Haddar

January 3, 2023

The dependence of the characteristics of the dispersion curves on the orientation angle of the CARALL structures

Driss Hana1,2(&) , El Mahi Abderrahim1 , Bentahar Mourad1 ,

Beyaoui Moez2 , and Haddar Mohamed2

1 Laboratoire d'Acoustique de l'Université du Maine (LAUM) UMR CNRS 6613, Université du Maine, Av. O. Messiaen, 72085 Le Mans Cedex 9, France
Hana.Driss.Etu@univ-lemans.fr

2 Laboratoire de Mécanique, Modélisation et Production (LA2MP), Ecole Nationale d'Ingénieurs de Sfax (ENIS), Université de Sfax, Route de Soukra, 3038 Sfax, Tunisia

Abstract. This work aim purpose of this work is to present a parametric analysis of the effect of fiber orientation angle on elastic wave dispersion relations in the case of the CARALL composite. The material under consideration has 14 layers. Aluminum alloy and carbon/epoxy resin fibers are used to create the layers. The overall configuration of the CARALL composite is considered to be [Al, θ_5, Al]s, and the angle is the parameter whose values are taken in the range [0°, 90°]. The stiffness matrix approach is used to determine the dispersion curves. In addition, the computer program "Dispersion calculator" is utilized. The shift in phase velocity of the basic symmetric mode and the horizontal shear mode is mostly noticed due to differences in the fiber orientation angle. The other modes' sensitivity to the angle is negligible.

Keywords: Dispersion curve, CARALL, Lamb wave.

1. Introduction

Existing analytical and semi-analytical modeling approaches for analyzing guided wave propagation in anisotropic composites such as multi-layer wind turbine blade assemblies and tidal plant hydrofoils. Dispersion curves, which characterize the frequency dependence of guided wave velocities in a certain structure, must be computed to order to measure wave propagation velocity. The majority of dispersion curves are computed using analytical or semi-analytical finite element techniques. According to [Nayfeh, 1991], harmonic elastic waves propagate in n-layer anisotropic plates. Using the transfer matrix method, the solutions for each layer were identified and expressed as wave amplitudes, with stresses and displacements at the layer interfaces replacing the wave amplitudes [Nayfeh, 1991]. To take into consideration the attenuation and change in propagation direction brought on by the composite's anisotropic structure, Hosten and Castaings [1993] modified the transfer matrix formula. According to the transfer matrix technique, four waves with the same frequency and spatial characteristics should occur at each interface at the border of each layer of the N-layer composite laminate. An iterative root-seeking approach can yield an endless number of wave numbers at each frequency. To use matrix approaches for anisotropic layer simulation, the matrix must be expanded to six dimensions[Lowe, 1995]. The disadvantage of this strategy is that it gets unstable when the fd is huge (frequency multiplied by the plate thickness). [Knopoff, 1964] endorsed using the worldwide matrix method for excessive frequencies or thick plates. However, when the number of components increases, the matrix gets more complicated, and the approach becomes slower [Lowe, 1995; Knopoff, 1964]. For the computation of dispersion curves, the global matrix model has been incorporated in the commercial program "DISPERSE" [Pavlakovic et al., 1997]. Nayfeh and Chimenti [1988] advanced the equations of movement for a transverse isotropic plate linked to a fluid and presented a non-stop blending concept for uniaxial fibrous composites with transverse isotropy. For a fluid-coupled composite plate, general transmission curves had been calculated. In the example of circumferential wave propagation, Towfighi et al. used coupled differential equations to solve the problem of producing dispersion curves for curved anisotropic plates. Based on the Fourier series expansion of unknown values, they presented a method for an organized and comprehensive solution [Towfighi et al., 2002]. [Karpfinger et al., 2008] proposed a spectrum-primarily based totally technique that employs spectral differentiation matrices to discretize the underlying wave equations and solves the applicable equations as a generalized eigenvalue problem. The eigenvalues are the wave numbers of the various modes at a given frequency. This technique has the advantage of solving the generalised eigenvalue problem without the need of special functions. As a result, it is straightforward to employ in circumstances where traditional rootfinding methodologies are highly limited or difficult to apply due to attenuating, anisotropic, or poroelastic media.

2. Dispersion curves

There are three methods to plot the dispersion curves: the first is based on numerical simulations, the second on data collected by experiment, and the third on a specialized coding program like Matlab.

In this paper, a numerical simulation was performed for hybrid composite plates (CARALL) to calculate the dispersion curves.

[Sorohan et al., 2011] describes a method for getting all dispersion curves using simply numerical simulation and typical commercial finite element software. The approach is simply a series of modal assessments for a representative piece of the analyzed structure. Hora and Ervená [2012] provide Fourier transform methods for obtaining Lamb wave dispersion curves (FT). In both the frequency and spatial domains, propagating Lamb waves are sinusoidal. As a result, the temporal FT can be used to transition from the time domain to the frequency domain, followed by the spatial FT to the frequency-wavenumber domain, where individual mode amplitudes and wavenumbers can be monitored. [Schöpfer et al., 2013] provide a method for calculating dispersion curves using laser vibrometer measurements. The matrix pencil approach is used after Fourier translating the measurement data into the wavenumber domain.

Harb and Yuan [2015] offer a non-contact hybrid device consisting of an Air-Coupled Transducer (ACT) and a Laser Doppler Vibrometer for profiling the A0 Lamb wave dispersion of an isotropic aluminium plate (LDV). The ACT applies ultrasonic pressure on the surface of the plate. The pressure waves are re-fracted into the plate in part. The LDV is used to calculate the excited Lamb wave mode's out-of-plane velocity along the plate at various distances.

[Packo et al., 2014] shows how to compute dispersion curves and evaluate numerical models for directed waves. The proposed technique employs the wave equation and through-thickness-only discretization of anisotropic, multilayer plates to calculate the Lamb wave properties calculate the Lamb wave properties, the proposed technique employs the wave equation and through-thickness-only discretization of anisotropic, multilayer plates. [Honarvar et al., 2009] present an alternative strategy. [Honarvar et al., 2009] provide an alternate method for obtaining the frequency equation's solution from its three-dimensional representation in the form of dispersion curves. To begin, a three-dimensional depiction of the frequency equation's real roots is shown. Making a correct cut in the velocity frequency plane yields the dispersion curves, which are the numerical solutions to the frequency equation. Many researchers employ iterative strategies to solve frequency equations, such as linear Schwab and Knopoff [1970] and [Mal, 1988] or quadratic Schwab and Knopoff [1970] interpolation [Haskell, 1953] and [Press et al., 1961] or extrapolation [Lowe, 1995] algorithms that are extremely quick on a single root. When two roots are close together, such as at longitudinal mode dispersion curve intersections, the function changes sign twice, making such schemes unstable. Slower but safer iteration techniques such as Newton-Raphson, Bisection, and Mueller [Press et al., 1987] could be used to solve the frequency equations. These techniques, however, are difficult, slow, and time-consuming due to the huge number and diversity of operations, particularly in multilayered media [Honarvar et al., 2009].

2.1 Investigated CARALL structure

The dispersion curves for the CARALL composite of the following general configuration of corner layers, namely [Al, $+\theta_5$, Al]s, are calculated in this study. The composite under consideration has 14 layers, four of which are AluminumAl-loy1100 [Callister, 2002] and the remainder are Carbon/Epoxy prepreg [Rokhlin et al., 2011]. Table 1 shows the mechanical and physical parameters of the materials employed. CARALL's entire thickness is constant and equal to t=10 [mm]. The aluminum layers have a thickness of tAL=4 [mm], while the carbon/epoxy layers have a thickness of tCE=6 [mm].

Table 1. Mechanical properties of the used materials

Material	E1 [GPa]	E2 [GPa]	G12 [GPa]	υ12	υ23	ρ [g/cm³]
AluminumAlloy1010	69	69	25.9	0.33		2710
Carbon/Epoxy	150.95	12.80	8	0.46	0.45	1610

Figure 1 depicts a multicouche composite material with a local (stratified) coordinate system (x'1, x'2, x'3) and a global coordinate system (x1, x2, x3). Assume that the elastic wave propagates along the global coordinate system's axis x1. The angle defines the fiber's orientation as well as the local coordinate system (couche) for each layer (figure 1). The dispersion relationships are computed for the following coordinates: θ: =0º, 30º, 45º, 60º et 90º.

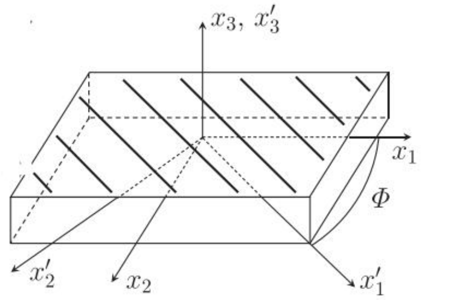

Figure 1. Layered plate for investigated CARALL with local (x'1, x'2, x'3) and global coordinate systems (x1, x2, x3), where x1 is the direction of elastic wave propagation.

2.2 Numerical Examples

We give our calculations on the Al-fibermatrix(carbon/epoxy) system using the Dispersion Calculator program in this part. (1) shows the stiffness matrix and the density of carbon-epoxy of 1610 kg/m3 (in GPa).

$$C_{\frac{cabon}{epoxy}} = \begin{pmatrix} 162 & 11.8 & 11.8 & 0 & 0 & 0 \\ & 17 & 8.2 & 0 & 0 & 0 \\ & & 17 & 0 & 0 & 0 \\ & Sym & & 4.4 & 0 & 0 \\ & & & & 8 & 0 \\ & & & & & 8 \end{pmatrix} \quad (1)$$

First, we show how to calculate the various mode families on [Al,0_5,Al]s and [Al,90_5,Al]s laminates with a thickness of 10 mm. Then, to demonstrate the method's capability on the orientation angle variation, we compute the dispersion curves corresponding to [Al,30_5, Al]s, [Al,45_5,Al]s, [Al,60_5,Al]s. Wave propagation is always along 0 for coherence considerations.

2.3 Dispersion diagrams

Figures 2 and 3 illustrate the dispersion curves produced for fiber orientation angles of 0° and 90°. The assumed frequency f and phase velocity Vp ranges are 0 [kHz] $\leqslant$ f $\leqslant$ 200 [kHz] and 0 [m/ms] $\leqslant$ Cp$\leqslant$ 20 [m/ms], respectively. The phase velocity of the antisymmetric fundamental mode A0 (bending wave) is substantially dependent on frequency in both circumstances, as illustrated for the comparatively low frequency f < 30[KHz]. As a result, this mode is quite dispersive. The basic horizontal shear mode SH0, on the other hand, is essentially constant. In both cases, the phase velocity of the SH0 mode Vp=2.733 [m/ms] up to frequency f=200[KHz]. For =0° and =90°, the phase velocity Vp of the fundamental symmetric wave mode S0 (pressure or longitudinal wave) is equal to (f=10[KHz]) Vp=7.51[m/ms] and Vp=4.35[km/s]. The initial greater shear horizontal mode SH1 appears at the same frequency f 80[KHz] in both laminates.

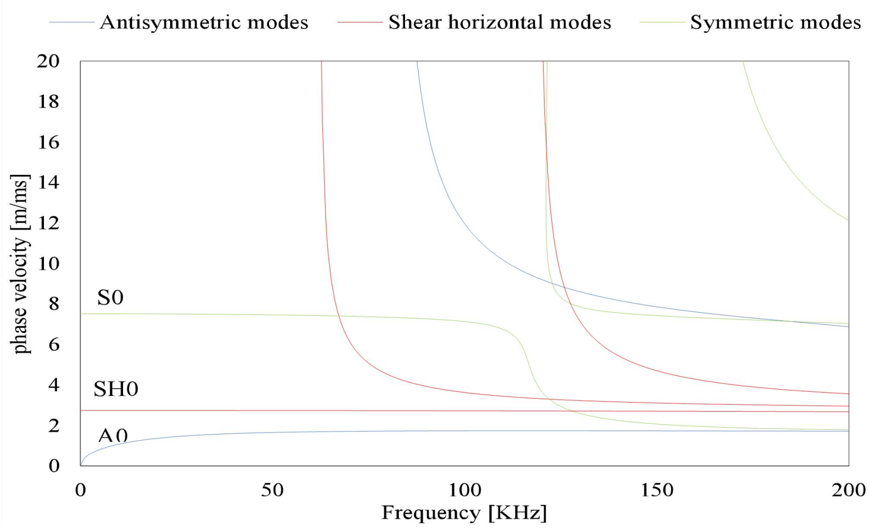

Figure 2: Dispersion curves for the investigated CARALL composite "fiber orientation angle θ=0°"

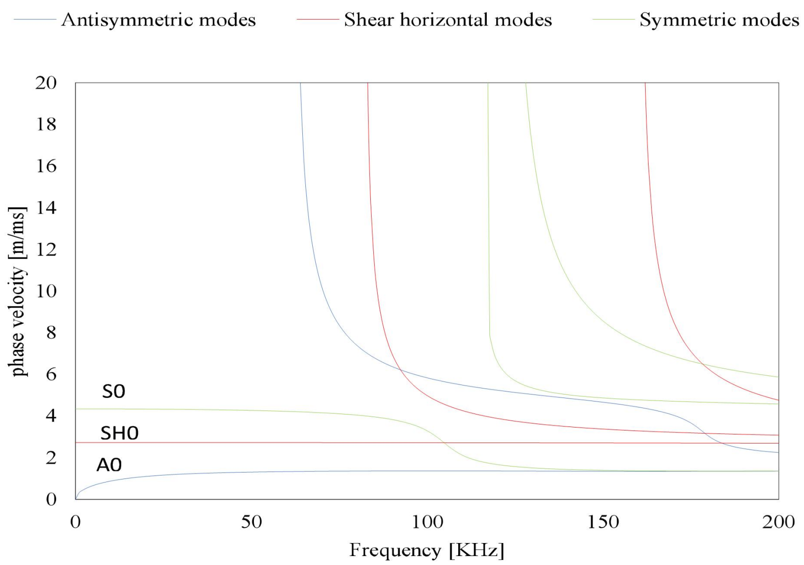

Figure 3. Dispersion curves for the investigated CARALL composite "fiber orientation angle θ= 90°"

Figure 4 displays the fundamental elastic wave modes A0, SH0, and S0 for 30°, 45°, and 60° fiber orientation angles. To improve clarity, the frequency range has been reduced to 0 [kHz] ≤f ≤80 [kHz]], with just the first upper modes visible. In practice, the remaining higher wave modes are insignificant. The introduction of higher modes significantly impedes adequate comprehension of the structure's dynamic response.

The phase velocity range vp does not change. As can be shown, the fiber orientation angle has no effect on the phase velocity of the fundamental antisymmetric mode A0. At =60°, however, the phase velocity Vp of the fundamental horizontal shear mode SH0 initially increases from Vp=3.05[km/s] to Vp=4.4[km/s]. As the fiber orientation angle rises, the phase velocity of the fundamental mode S0 decreases monotonically.

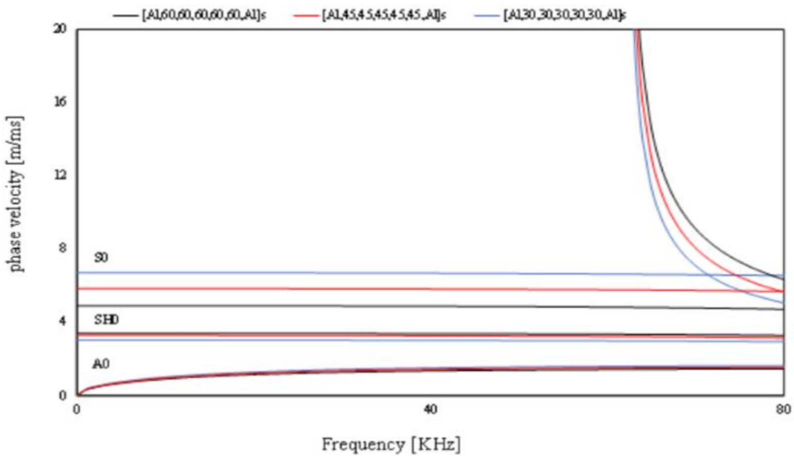

Figure 4. Dispersion curves (fundamental modes A0, SH0, S0) and fiber orientation angles θ=30°, 45°, and 60° were produced for the examined CARALL composite.

It should be noted that the frequency of coupling at which the initial horizontal shear mode SH0 occurs does not change appreciably across all parameter values studied, and the phase velocity is largely indifferent to the frequency f. As can be seen, changes in the orientation of the fiber have no influence on the speed of phase of the fundamental mode A0. The fundamental mode has the greatest influence on the phase velocity Vp; when the parameter value increases, the phase velocity increases monotonically. As previously stated, the effect of frequency fluctuation is very small up to a specific frequency; nevertheless, the larger the value of the fiber orientation angle, the greater the influence on frequency.

3. Conclusions

The current study looks at how the fiber orientation angle affects the dispersion relationships of the CARALL composite. The examined composite material has

14 layers, four of which are aluminum alloy and the remaining are prepreg Carbon/epoxy. The total thickness of the considered composite material is constant and equal to t=10 [mm]. The angle is the parameter with values ranging from [0°, 90°] and the typical configuration of the CARALL composite is [Al,+θ, Al]s. The gathered data may be summarized as follows. The existence of the upper horizontal shear mode SH1 limits the frequency's useful range. The basic antisymmetric mode A0 is almost insensitive to angle change in this region. The phase velocity of the basic modes SH0 and S0, on the other hand, is parameter-dependent. The unambiguous maximum is found at θ =60° in the first example, and the phase velocity falls monotonically with the rise and decrease of 60°. In the second situation (S0), the phase velocity is maximum at 0° and falls monotonically as the parameter value increases.

References

Hora, P., & Červená, O. (2012). Determination of Lamb wave dispersion curves by means of Fourier transform. *Applied and Computational Mechanics*, *6*(1).

Harb, M. S., & Yuan, F. G. (2015). A rapid, fully non-contact, hybrid system for generating Lamb wave dispersion curves. *Ultrasonics*, *61*, 62-70.

Honarvar, F., Enjilela, E., & Sinclair, A. N. (2009). An alternative method for plotting dispersion curves. *Ultrasonics*, *49*(1), 15-18.

Haskell, N. A. (1953). The dispersion of surface waves on multilayered media. *Bulletin of the seismological Society of America*, *43*(1), 17-34.

Hosten, B., & Castaings, M. (1993). Transfer matrix of multilayered absorbing and anisotropic media. Measurements and simulations of ultrasonic wave propagation through composite materials. *The Journal of the Acoustical Society of America*, *94*(3), 1488-1495.

Karpfinger, F., Gurevich, B., & Bakulin, A. (2008). Modeling of wave dispersion along cylindrical structures using the spectral method. *The Journal of the Acoustical Society of America*, *124*(2), 859-865.

Knopoff, L. (1964). A matrix method for elastic wave problems. *Bulletin of the Seismological Society of America*, *54*(1), 431-438.

Lowe, M. J. (1995). Matrix techniques for modeling ultrasonic waves in multilayered media. *IEEE transactions on ultrasonics, ferroelectrics, and frequency control*, *42*(4), 525-542.

Lowe, M. J. (1995). Matrix techniques for modeling ultrasonic waves in multilayered media. *IEEE transactions on ultrasonics, ferroelectrics, and frequency control*, *42*(4), 525-542.

Mal, A. K. (1988). Guided waves in layered solids with interface zones. *International journal of engineering science*, *26*(8), 873-881.

Nayfeh, A. H. (1991). The general problem of elastic wave propagation in multilayered anisotropic media. *The Journal of the Acoustical Society of America*, *89*(4), 1521-1531.

Nayfeh, A. H., & Chimenti, D. E. (1988). Propagation of guided waves in fluid-coupled plates of fiber-reinforced composite. *The Journal of the Acoustical Society of America*, *83*(5), 1736-1743.

Pavlakovic, B., Lowe, M., Alleyne, D., & Cawley, P. (1997). Disperse: A general purpose program for creating dispersion curves. In *Review of progress in quantitative nondestructive evaluation* (pp. 185-192). Springer, Boston, MA.

Packo, P., Uhl, T., & Staszewski, W. J. (2014). Generalized semi-analytical finite difference method for dispersion curves calculation and numerical dispersion analysis for Lamb waves. *The Journal of the Acoustical Society of America*, *136*(3), 993-1002.

Press, F., Harkrider, D., & Seafeldt, C. A. (1961). A fast, convenient program for computation of surface-wave dispersion curves in multilayered media. *Bulletin of the seismological society of America*, *51*(4), 495-502.

Press, W. H., Flannery, B. P., Teukolsky, S. A., Vetterling, W. T., & Chipperfield, J. R. (1987). Numerical recipes: the art of scientific computing: Cambridge University Press, Cambridge, 1986

Schöpfer, F., Binder, F., Wöstehoff, A., Schuster, T., Von Ende, S., Föll, S., & Lammering, R. (2013). Accurate determination of dispersion curves of guided waves in plates by applying the matrix pencil method to laser vibrometer measurement data. *CEAS Aeronautical Journal*, *4*(1), 61-68.

Schwab, F., & Knopoff, L. (1970). Surface-wave dispersion computations. *Bulletin of the Seismological Society of America*, *60*(2), 321-344.

Sorohan, Ş., Constantin, N., Găvan, M., & Anghel, V. (2011). Extraction of dispersion curves for waves propagating in free complex waveguides by standard finite element codes. *Ultrasonics*, *51*(4), 503-515.

S. I. Rokhlin, D. E. Chimenti, and P. B. Nagy, Physical Ultrasonics of Composites. Oxford University Press, Oxford, 2011.

Towfighi, S., Kundu, T., & Ehsani, M. (2002). Elastic wave propagation in circumferential direction in anisotropic cylindrical curved plates. *J. Appl. Mech.*, *69*(3), 283-291.

W. D. Callister, Material Science and Engineering. An Introduction. John Wiley & Sons, Inc., 2002.