Connection Between Prime Number Theory and Cosmological Models

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Abstract

Cosmology and prime number theory relate through the analysis of metric space in non-decelerative universe model with logarithmic integral function. It has been seen in history that the three components also exhibit relationships among each other, particularly through the zeroes of zeta functions, random matrix theory and energy levels in quantum chaotic systems. How large-scale structure of universe is related to some unknown prime number theories via mathematical tools. With this integration, new vistas are opened for refrigerators stored in mathematics and Cosmology.

1 Introduction

The physical world is recognized through observation. The observed things and phenomena are generalized and analyzed by mathematical methods. Mathematics is remarkably successful in analyzing natural phenomena. Along with dominant significance and application in natural sciences, mathematics has penetrated into humanities, psychology, game theory, economics, etc. It is natural to ask whether scientists create or discover mathematical laws. While we create the basic rules of mathematics, its semantics, syntax, and axioms, subsequent mathematical truths are discovered through specific methods. The actual world must be governed by certain laws and principles based on logic and causality, explaining why mathematics is so successful at describing physical laws. There is no alternative mode for describing physical laws as effectively as mathematical tools and procedures. If other technologically advanced civilizations exist in the Universe, they can recognize the identical laws using their own mathematical methods. Therefore, the answer to whether mathematics creates or discovers things lies somewhere in between. **Mathematics and the Universe** Mathematics appears more complex than the Universe itself. Using mathematics, we can create different types of universes that must be logical, causal, and consistent. Tegmark's approach suggests that everything in the Universe, including humans, is defined by fundamental mathematical structures[1]. Particles possess basic properties like spin, mass, and parity, all of which are purely mathematical. Spacetime dimensions and motion in a gravitational field are also described mathematically. Mathematics can offer predictions preceding observations, as exemplified by the discovery of Pluto and the evidence of the Higgs boson. **Mathematics in Theories** At present, string and loop theories have high ambitions in the application of mathematics to world description. Both theories are mathematically demanding and understood by few professionals. Loop quantum gravity (LQG) predicted that high-frequency photons would take longer to pass a certain distance due to interaction with spacetime structure, but this prediction was recently disproved. Despite the resources dedicated to string theories, they have not provided any unique predictions unknown from the standard model or cosmology. The theories propose a multiverse, suggesting we live in one of many universes. However, this multiverse concept holds little value for cosmology or the standard model. **Challenges to String Theory** In 2005,[2] physicist Washington Taylor and colleagues from MIT documented an infinite number of universes with a negative cosmological constant based on string models, a proof that has been verified and accepted. G. Ellis and others pointed out that we should, with higher probability, live in a universe with a negative cosmological constant, but we know from 1998 that the value is positive due to dark energy. This revelation questions the acceptability of string models for cosmology. **Riemann Hypothesis and Quantum Mechanics** One of the most fundamental issues in mathematics is the Riemann Hypothesis (RH)[3]. This issue is relevant not only to mathematics but also to quantum mechanics and quantum cryptography. Prime numbers are the basic constituents of mathematics. It has been found that the distribution of zeros of the Riemann zeta function is similar to the chaotic energy spectrum of a quantum system without time inversion symmetry. The distribution of adjacent zeros is nearly identical to a curve derived from the theory of random matrix. There is a correlation between prime numbers and periodic orbits in quantum chaotic systems. Quantum chaotic systems have hidden order and are responsible for the creation of complex structures, including life and consciousness. Thus, there is a deep relation between mathematics and quantum mechanics, with implications for the macroworld and cosmology.

2 The Expansive Nondestructive Universe (ENU) Model

Our model of the Universe (Expansive Nondestructive Universe, ENU)[4-6] is based on a simple assumption of constant velocity of the Universe expansion equal to the speed of light.

2.1 Scale Factor

The scale factor a is given by

$$a = ct_u \tag{1}$$

where a represents the scale factor (1.32×10^{26} m), c is the speed of light, and t_u denotes the cosmological time (1.38×10^{10} years).

2.2 Cosmological Constant

The cosmological constant Λ is specified as

$$\Lambda = 0 \tag{2}$$

2.3 Curvature Parameter

The curvature parameter k of the Universe is

$$k = 0 \tag{3}$$

2.4 Friedmann Equations with Conformal Time

The ENU dynamics can be expressed by Friedmann equations, which, subsequent to introducing dimensionless conformal time η, adopt the following form

$$\frac{d}{d\eta}\left(\frac{1}{a}\frac{da}{d\eta}\right) = -\frac{4\pi G}{3c^4}a^2(\varepsilon + 3p) \tag{4}$$

$$\left(\frac{1}{a}\frac{da}{d\eta}\right)^2 = \frac{8\pi G}{3c^4}a^2\varepsilon - k \tag{5}$$

where ε is the energy density, p is the pressure and a is the scale factor expressed as follows

$$a = \frac{da}{d\eta} \tag{6}$$

Equations (1) to (6) lead to

$$\varepsilon = \frac{3c^4}{8\pi G a^2} \tag{7}$$

$$p = -\frac{\varepsilon}{3} \tag{8}$$

The energy density can also be written as follows

$$a = \frac{2Gm_U}{c^2} \tag{9}$$

Equation (10) expresses the necessity of matter creation. The amount of the matter formed at 1 sec, δ will be

$$\delta = \frac{dm_U}{dt} = \frac{m_U}{t_U} = \frac{c^3}{2G} \tag{10}$$

2.5 Matter Creation in the Universe

The amount of matter created during 1 second corresponds to 10^5 Sun mass. It is the same amount of matter as that appearing from beyond the horizon in the inflationary model, *i.e.* just about 1 hydrogen atom in 1 km^3.

2.6 Energy Compensation in the Universe

It is logical that in the Universe the positive energy of matter is compensated by the negative gravitational energy and the total energy of the Universe remains thus zero, *i.e.* the conservation laws are not violated.

2.7 The Universe as an Absolute System

If the Universe is an absolute system, all its physical characteristics must be of zero value. In an opposite case, the characteristics should be observable from outside of the Universe and it would not be thus an absolute system.

3 Dimensionless Constants of Fundamental Physical Interactions

3.1 Strong Interaction Constant α_s

The constant of the strong interaction, denoted as α_s, is one of the dimensionless fundamental constants in particle physics. It characterizes the strength of the strong force, which is responsible for binding quarks together to form protons, neutrons, and other hadrons.

3.2 Definition and Significance

The strong interaction constant α_s is defined in the context of Quantum Chromodynamics (QCD), which is the theory describing the strong force. It can be expressed as:

$$\alpha_s = \frac{g_s^2}{4\pi} \tag{11}$$

where g_s is the coupling constant of the strong interaction. This dimensionless constant is crucial for understanding the behavior of quarks and gluons within hadrons and plays a significant role in high-energy physics.

3.3 Implications and Theoretical Considerations

Changes in α_s can have profound implications for our understanding of the early Universe. In theories attempting to resolve the initial singularity problem or to describe quantum gravity, it is essential that the value of α_s respects the fundamental conservation laws and remains consistent with experimental observations. Variations in α_s must be carefully considered to ensure they do not violate the principles of causality or the normalization of the Universe's wave function.

$$\alpha_s = 1 \tag{12}$$

Dimensionless constant of electromagnetic interaction α_{em} is

$$\alpha_{em} = \frac{e^2}{2\epsilon_0 \hbar c} = \frac{1}{137} \tag{13}$$

Dimensionless constant of weak interaction $\alpha_w$is

$$\alpha_w = \frac{8\pi^7 g_F^2 m_p^2 c}{h^3} \cong 10^{-6} \tag{14}$$

Dimensionless constant of gravitational interaction α_g is

$$\alpha_g = \frac{2\pi G m_p^2}{hc} \cong 10^{-38} \tag{15}$$

In relations (13) and (14), m_p is the proton mass.

3.4 Changing Fundamental Constants to Preserve Dimensionality

There are several ways to adjust fundamental constants to maintain the value of dimensionless constants. The simplest and most probable approach involves increasing the gravitational constant G and the Fermi constant g_F proportionally with cosmological time while simultaneously decreasing the mass of all elementary particles and the Boltzmann constant (which varies inversely with the square root of cosmological time)[7]. This approach could potentially explain a Universe without a definitive beginning or end. It would ensure that the ratios of physical interaction constants are conserved, while Planck length and time would gradually increase, and Planck mass would decrease. These adjustments in Planck quantities would effectively shift the moment of the Universe's creation relative to the observer's position on the time axis, which itself would be relative[7-9].

3.5 Historical Discussion on Changing Constants

The idea of altering fundamental constants has been discussed for a long time. References to the ongoing debate can be found in works such as [8] and [9]. These discussions revolve around how variations in constants could impact our understanding of the Universe's evolution and its initial conditions.

3.6 Recent Approaches, Dark Matter and Cosmic Fields

Recently, Stadnik and Flambaum proposed an original approach involving variations in fundamental constants induced by dark matter or other yet-to-be-discovered cosmic fields . This perspective offers a novel explanation for how fundamental constants might change over time and how such changes could influence the creation and evolution of matter in the Universe[10].

3.7 Influence on Illusive Matter Creation

In the next section, we will explore and explain how these unobservable changes in fundamental constants function and their influence on the creation of illusive matter in the Universe. This analysis will delve into the implications of constant variations on the formation and behavior of matter in a dynamically evolving cosmological framework.

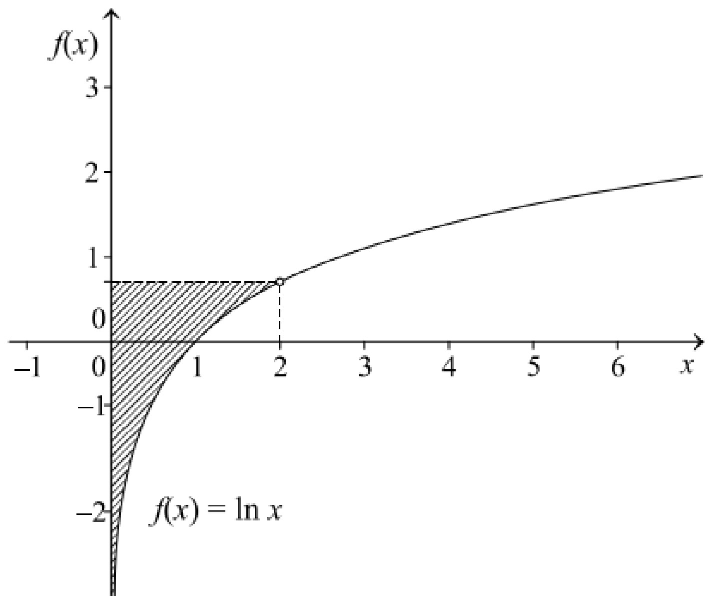

Fig. 1. Plot of logarithmic function

4 The Universe Expansion and Logarithmic Time

The concept of logarithmic time τ has been explored to describe the expansion of the Universe. The relationship between τ and the intrinsic cosmological time t is given by:

$$\tau = \ln t \tag{16}$$

Here, τ is the real logarithmic time, and t represents the intrinsic cosmological time of the Universe. In this framework, real time at present equals zero, was negative in the past, and will be positive in the future. The cosmological time at present is normalized to 1 and is proportional to the Universe scale factor a. For any observer, the present real time is always zero (see Fig. 1).

4.1 The Infinite Universe

When $t \to 0$, we have $\tau \to -\infty$. This implies that the Universe is infinite, without a beginning or an end. As cosmological time approaches zero, units like Planck time and Planck length are proportionally reduced due to changes in certain constants. This adjustment permits the Big Bang's existence, while the corresponding changes in constants do not affect the fundamental physical interactions and natural laws.

4.2 Logarithmic Time Advantages

The introduction of logarithmic time presents several advantages, especially in the cosmological context. Notably, it allows for a continuous and infinite description of the Universe's history. It also provides a framework where the concept of a beginning or end is not constrained, accommodating various cosmological models and theories.

4.3 Impact on Physical Constants and Laws

Despite the changes in constants as time progresses, the fundamental physical interactions and natural laws remain unchanged. This consistency ensures that the physical laws governing the Universe remain constant, even as our understanding and measurement of time evolve.

5 Interpreting Cosmic Acceleration Through Logarithmic Functions and Cosmological Time

The observed accelerated expansion of the Universe may be interpreted through the lens of logarithmic functions and the nature of cosmological time. According to this perspective, what we perceive as acceleration is actually a gradual increase in space and time units.

5.1 Cosmological Time and Expansion

The idea is that any observer would perceive real time as zero at any cosmological time. This leads to the assumption that the Universe's expansion velocity corresponds to the speed of light. Thus, the perceived acceleration can be understood as a function of the changing scale of space and time.

5.2 Analysis of the Area under the Curve

Consider the integral of the logarithmic function to analyze the areas under the curve. We have two regions: one representing the past and the other the future of the Universe.

5.3 Integral of Logarithmic Function

Define the integral of the logarithmic function as follows:

$$\int_0^t \ln t' \, dt'$$

This integral computes the area under the curve from $t = 0$ to t, representing the past universe.

5.4 Area Calculation

The total area under the curve up to time t can be split into the past and future parts. The equation for the total area is given by:

$$\int_0^t \ln t' \, dt' + t \ln t - \int_0^t \ln t \, dt$$

To ensure that the sum of negative and positive parts reaches zero, the following condition must hold:

$$\int_0^t \ln t' \, dt' + t \ln t - \int_0^t \ln t \, dt = 0$$

5.5 Solving the Equation

Solving this equation yields:

$$t = 2$$

This result aligns with previous findings, as shown in Figure 1. The cosmological time t is dimensionless and expressed on a logarithmic scale.

5.6 Causality and Time Magnitude

The increase in t from 1 to 2 represents 61 orders of magnitude. This is consistent with the Expected Normal Universe (ENU) for causality, where the time span is approximately 10^{71} years.

5.7 Interpretation of the Area under the Curve

The area calculation helps in understanding the universe's expansion in relation to real time. The curve can be seen as the worldline of the universe expanding in the direction of the τ-axis (real time). By comparing areas above and below the t-axis, we can analyze the expansion characteristics.

It holds for the metric of such universe

$$ds^2 = dt^2 - t^2\, d\tau^2 \tag{18}$$

and also

$$R^0_0 = R^1_1 = R^0_1 = 0 \tag{19}$$

It is flat Universe with the singularity in the point $t = 0$ (illusory beginning of the Universe). This is, in fact, de Sitter metric.

In Fig. 2 the plot of logarithmic integral function (*li*-function) is depicted.

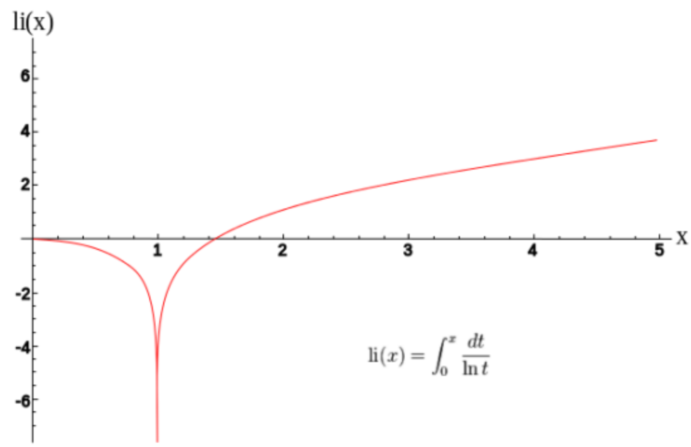

Fig.2 *Plot of li-function as a superposition of the causal past and future*

$$l(t') = \int_{-1}^{t} \frac{dt}{t} \tag{20}$$

5.8 Introduction to l-Function

The l-function is crucial for estimating the number of primes, and it is defined over the interval $0 \le t \le 1.45$. This function is symmetrical about the axis $t = 1$. It has singularities at the points $t = 0$ and $t = 1$.

5.9 Application to the Universe

We apply the l-function to cosmological models by considering time y as analogous to real logarithmic time. The y-axis represents time, extending from negative infinity up to the present, where $t = -1$ denotes the current time. For observers from the past, the time y-axis is negative, with the present moment at $y = 0$. The universe's spacetime expands at a constant velocity equal to the speed of light.

5.10 Present Time and Causal Horizon

In the context of this model, the present time is represented as the line $t = 1$. The entire history of the universe can be viewed as a series of present moments, with $t = 1$ demarcating the boundary between the causal and uncausal parts of the universe.

5.11 Causal and Uncausal Parts

The interval from $t = 0$ to $t = 1$ on the time axis represents the causal part of the universe. Beyond $t = 1$, in the interval from $t = 1$ to $t = 1.45$, is considered the uncausal part. This separation is significant in defining the causal horizon and logical horizon of the universe.

5.12 Cosmological Time Ranges

Our viewpoint of cosmological time ranges from 0 to 1 on the time axis, with $t = 1.45$ marking the boundary. The axis $t = 1$ represents the singular point that separates the causal horizon from the logical horizon.

5.13 Formulation of the l-Function

Based on the previous definitions, the l-function can be expressed to cover both the causal past and future of the universe. The integral's boundaries reflect this span, incorporating both causal and uncausal regions.

5.14 Expression of the l-Function

Thus, the expression for the l-function over the entire universe, considering the causal past and future, can be formulated as follows:

$$\text{Expression of } l\text{-function} = \int_0^1 \text{some integrand}\, dt + \int_1^{1.45} \text{some integrand}\, dt$$

where the specific integrands would depend on the detailed formulation of the l-function.

$$ds^2 = dt^2 - lnt^2 dt^2 \tag{21}$$

while

$$dt = \frac{dt}{lnt} \tag{22}$$

Nonzero components of the Ricci tensor are

$$R_0^0 = R_1^1 = \frac{1}{t^2 lnt} \tag{23}$$

For scalar curvature and Einstein field equations, it holds

$$R = \frac{1}{t^2 lnt} \tag{24}$$

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu} = 0 \tag{25}$$

It is obvious that the Universe is flat and all components of momentum-energy tensor are of zero value. It follows from (23) that for the past ($t < 1$) all components of Ricci tensor are negative, and for the future ($t > 1$) they are positive. This is why at given time there is only expansion, since expansion acceleration is observed.

6 Exploring Singularities and Metrics in the Universe

The metric (21) describes a scenario where singularities are present at $t = 0$ (the beginning) and $t = 1$ (the present), analogous to the Schwarzschild metric of black holes, which also features singularities at both the horizon and the center. This suggests an equivalency between black holes and the Universe.

6.1 Coordinate System and Singularities

In the Schwarzschild metric, singularities at the horizon can be eliminated by choosing a suitable coordinate system. Similarly, the negative curvature observed in the Universe's past and the positive curvature near black holes relate to the same underlying objects.

6.2 Curvature and Gravity

Despite the positive curvature for the future, the Ricci component R_{11} will always be zero, indicating a flat Universe. Gravity, considered local, contrasts with the global nature of our Universe's event horizon. Observers beyond this horizon would perceive our Universe as a local black hole, attributing nonzero values to R_{11} and registering gravity.

6.3 Future Observers and Flatness

For $t > 1$, the l-curve represents future observers for whom the Universe remains flat. Observers outside this curve would perceive gravitational effects, applying the Schwarzschild metric. These observers, however, will never be part of our Universe.

6.4 Singularity and Metric in the Present

The singularity at $t = 1$ reflects that questions about the beginning of expansion or the metric at the present moment are essentially meaningless. At this precise moment, there is no volume, energy, or physical reality other than the horizon surface accepting all information.

6.5 Antimatter and Multiverse Hypothesis

The hypothesis suggests that our Universe's singularity in metric (21) implies that the Universe could be viewed as a common point or a quantum of spacetime in an extensive Multiverse. This idea is in line with Feynman's concept that antiparticles are particles traveling backward in time, implying a phase change of our Universe to antimatter.

6.6 Asymmetry and Spacetime Expansion

The l-function contributes significantly to the spacetime structure, indicating that spacetime units expand differently for past and future causal horizons. This results in a zero area at $t = 1.45$, showing asymmetry between past and future horizons. The expansion factor of spacetime is approximately proportional to $t^{\frac{1}{2}}$, consistent with the theory predicting a decrease in the mass of elementary particles.

Metric (21) concludes that the Universe is infinite in both time and space, having accelerated expansion in the past. Constants and spacetime intervals must change accordingly. The Universe manifests as flat, and a single metric describes all causal past and future in a smooth variety.

7 Conclusions

The l-function has an important relationship with the metric that can be used to predict the number of primes and elementary quanta of timespace. We have paid attention to the l-function in order to find out deeper links between mathematics and cosmology.In mathematics, quantum mechanics and cosmology the function of l can be found. The metric (21) resembles de Sitter metric (18). For estimating the number of universes (primes), we can directly use the graph of l-function (see Fig. 2 for $t \geq 1$) as a function of dimension t(x). The plot should be considered for all scales, hence one can use it for estimation of the number of black holes – universes, visible in our Universe, which is Black Hole Multiverse. Our Universe has a scale factor of approximately 10^{26} m and black hole should possess gravitational diameter of not less than 1^4 m. The dimensionless ratio of these numbers is approximately $z = 10^{22}$ A number of universes P (black holes) for z is

$$P \approx \mathrm{Li}(z) \approx \frac{z}{\ln z} \approx 2 \times 10^{20}$$

Roughly, the visible universe has around 10^{23} stars. A recent detailed study [11] reports that of every thousand stars, one will become a black hole. That's a good agreement with prediction (26) and, at the same time, an advantage for our theory.

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