# Contact Modelling in Co-simulation of Mechanical Systems Using Model Order Reduction

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**Abstract**
There are different ways to account for contact interactions in a mechanical system; each approach has its own advantages and disadvantages. In this work, we aim to compare two different contact modelling possibilities in a co-simulation setup. Co-simulation involves the simultaneous simulation of multiple interconnected physical systems using different simulation tools. The process of a co-simulation involves defining the *interface* between different subsystems and exchanging interface variables between them at specific communication points called *macro time steps*. To determine the variables at the interface between the communication points, we consider two model-based modellings which involve creating a *reduced interface model* (RIM) of the mechanical system to mimic the behaviour of the full model at the interface. The first method, the *smooth RIM*, assumes that the contact state will remain unchanged during the macro time step. The second method, the *non-smooth RIM*, involves identifying potential contact pairs and accounting for them through the solution of a linear complementarity problem during the time step, allowing for the possibility of contact attachments or detachments between communication points. Using a robotic arm as an example, it was found that the smooth RIM can produce inaccurate results in certain cases. One solution to this issue is to regularize some of the constraints by adding constitutive relations, but that can change the physics the model represents, and determining the proper stiffness and damping coefficients can be challenging. In contrast, the non-smooth RIM was found to produce results that were more accurate and in line with the reference solution without changing the system model.

## 1 Model-based co-simulation of mechanical systems

Consider a multibody system subjected to both unilateral interactions and bilateral constraints. For now, we will ignore the effects of friction. This multibody system has n generalized velocities $\mathbf{v}$ and a set of $\mathrm{n}_{\mathbf{q}}$ generalized coordinates $\mathbf{q}$, which are connected through the transformation $\dot{\mathbf{q}} = \mathbf{N}\mathbf{v}$, where $\mathbf{N}$ is a $\mathrm{n}_{\mathbf{q}} \times \mathrm{n}$ transformation matrix. The dynamics equations for this system can be expressed as:

$$\begin{cases} \mathbf{M}\dot{\mathbf{v}} + \mathbf{c} = \mathbf{f}_\mathrm{o} + \mathbf{A}_\mathrm{i}^\mathrm{T}\boldsymbol{\lambda}_\mathrm{i} + \mathbf{A}_\alpha^\mathrm{T}\boldsymbol{\lambda}_\alpha + \mathbf{A}_\mathrm{t}^\mathrm{T}\boldsymbol{\lambda}_\mathrm{t} + \mathbf{A}_\mathrm{p}^\mathrm{T}\boldsymbol{\lambda}_\mathrm{p} \\ \mathbf{0} \leq \dot{\mathbf{w}}_\mathrm{p} \perp \boldsymbol{\lambda}_\mathrm{p} \geq \mathbf{0} \end{cases} \tag{1}$$

where $\mathbf{M}(\mathbf{q})$ is the $\mathrm{n} \times \mathrm{n}$ the mass matrix; $\mathbf{f}_\mathrm{o}$ and $\mathbf{c}$ are the $\mathrm{n} \times 1$ array of generalized applied forces and the Coriolis and centrifugal forces. Here, we are also interested in four subspaces. The three subspaces of active, tight and potential constraints that represented with the subscripts a, t and p, and also the interface subspace is shown with the subscript i. The corresponding force, velocity and Jacobian of these subspaces are also denoted by $\boldsymbol{\lambda}$, $\mathbf{w}$ and $\mathbf{A}$, respectively. Active/tight constraints are the ones that will remain closed/open until the next communication point. However, a new group of *potential* contacts are defined here that are prone to change state, e.g., active contacts that might open during the macro step.
In a co-simulation, we look for the sub-space of the motion associated with the *interface* of the subsystem. It is always possible to decouple the interface subspace and its orthogonal complement space and transform the dynamics equations (1) into [1]:

$$\left(\mathbf{A}_\mathrm{i}\mathbf{M}^{-1}\mathbf{A}_\mathrm{i}^\mathrm{T}\right)^{-1}\dot{\mathbf{w}}_\mathrm{i} + \left[\left(\mathbf{A}_\mathrm{i}\mathbf{M}^{-1}\mathbf{A}_\mathrm{i}^\mathrm{T}\right)^{-1}\left(\mathbf{A}_\mathrm{i}\mathbf{M}^{-1}\mathbf{c} - \dot{\mathbf{A}}_\mathrm{i}\mathbf{v}\right)\right] = \left[\left(\mathbf{A}_\mathrm{i}\mathbf{M}^{-1}\mathbf{A}_\mathrm{i}^\mathrm{T}\right)^{-1}\mathbf{A}_\mathrm{i}\mathbf{M}^{-1}(\mathbf{f}_\mathrm{o} + \mathbf{A}_\alpha^\mathrm{T}\boldsymbol{\lambda}_\alpha + \mathbf{A}_\mathrm{t}^\mathrm{T}\boldsymbol{\lambda}_\mathrm{t} + \mathbf{A}_\mathrm{p}^\mathrm{T}\boldsymbol{\lambda}_\mathrm{p})\right] + \boldsymbol{\lambda}_\mathrm{i} \tag{2}$$

Now, if we calculate $\boldsymbol{\lambda}_\alpha$ using the dynamic equations associated with the active constraint motion and substitute it in Eq. (2), it will give the reduced dynamics equations:

$$\underbrace{\left[\mathbf{A}_\mathrm{i}(\mathbf{I} - \mathbf{P}_\alpha)\mathbf{M}^{-1}\mathbf{A}_\mathrm{i}^\mathrm{T}\right]^{-1}}_{\mathbf{M}_\mathrm{eff}}\dot{\mathbf{w}}_\mathrm{i} = \boldsymbol{\lambda}_\mathrm{i} + \underbrace{\mathbf{M}_\mathrm{eff}\left[\mathbf{A}_\mathrm{i}(\mathbf{I} - \mathbf{P}_\alpha)\mathbf{M}^{-1}\left(\mathbf{f}_\mathrm{o} - \mathbf{c} + \mathbf{A}_\mathrm{t}^\mathrm{T}\boldsymbol{\lambda}_\mathrm{t}\right) + \left(\dot{\mathbf{A}}_\mathrm{i} - \mathbf{A}_\mathrm{i}\mathbf{M}^{-1}\mathbf{A}_\alpha^\mathrm{T}(\mathbf{A}_\alpha\mathbf{M}^{-1}\mathbf{A}_\alpha^\mathrm{T})^{-1}\dot{\mathbf{A}}_\alpha\right)\mathbf{v}\right]}_{\mathbf{f}_\mathrm{eff}} + \mathbf{M}_\mathrm{eff}\mathbf{H}_\mathrm{ip}\boldsymbol{\lambda}_\mathrm{p} \tag{3}$$

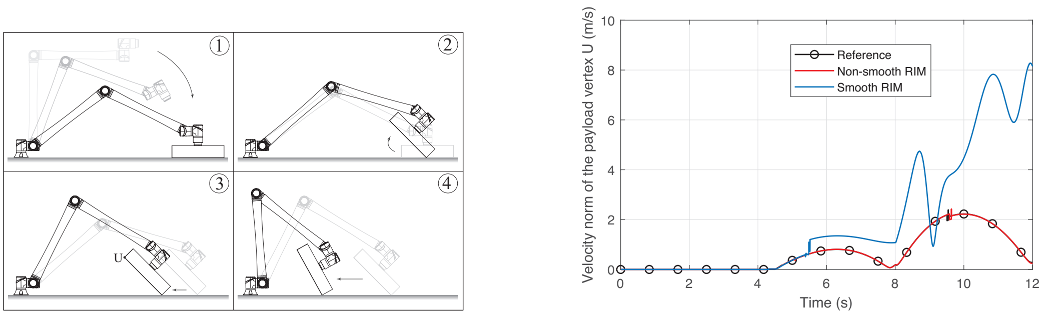
where $\mathbf{M}_{\text{eff}}$ and $\mathbf{f}_{\text{eff}}$ are the effective mass and force terms. Moreover, $\mathbf{H}_{\text{ip}} = \mathbf{A}_{\text{i}}(\mathbf{I}-\mathbf{P}_{\alpha})\mathbf{M}^{-1}\mathbf{A}_{\text{p}}^{\text{T}}$ is a generalized inverse mass matrix and, $\mathbf{P}_{\alpha} = \mathbf{M}^{-1}\mathbf{A}_{\alpha}^{\text{T}}(\mathbf{A}_{\alpha}\mathbf{M}^{-1}\mathbf{A}_{\alpha}^{\text{T}})^{-1}\mathbf{A}_{\alpha}$ is projector matrix. If no potential contacts are taken into consideration, i.e. $\boldsymbol{\lambda}_{\text{p}} = \mathbf{0}$, then Eq. (3) describes the dynamics of *smooth-RIM* that shows a reduced order model subjected only to bilateral constraints. However, we can include the complementarity of potential contacts in the formulation, resulting in

$$\begin{cases} \begin{bmatrix} \mathbf{M}_{\text{eff}} & -\mathbf{1} & -\mathbf{M}_{\text{eff}}\mathbf{H}_{\text{ip}} \\ \mathbf{0} & \mathbf{H}_{\text{pi}} & \mathbf{H}_{\text{pp}} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{w}}_{\text{i}} \\ \boldsymbol{\lambda}_{\text{i}} \\ \boldsymbol{\lambda}_{\text{p}} \end{bmatrix} + \begin{bmatrix} -\mathbf{f}_{\text{eff}} \\ \mathbf{b}_{\text{p}} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \dot{\mathbf{w}}_{\text{p}} \end{bmatrix} \\ \mathbf{0} \leq \dot{\mathbf{w}}_{\text{p}} \perp \boldsymbol{\lambda}_{\text{p}} \geq \mathbf{0} \end{cases} \tag{4}$$

where $\mathbf{H}_{\text{pi}}$ and $\mathbf{H}_{\text{pp}}$ are the generalized inverse mass matrices and $\mathbf{b}_{\text{p}}$ is the generalized array of remaining terms. Eq. (4) describes the non-smooth-RIM formulation, which allows for the transmission of contact states within the macro time step. The non-smooth-RIM formulation will be transferred to other subsystems and solved along with their dynamic equations as a coupled system within the macro time step.

## 2 Example

A 7-jointed robotic arm was used to compare the performance of the two approaches in a co-simulation setup. Here, the multibody system is divided into two subsystems. The end-effector (EE) and the environment are considered as the first subsystem while the remaining parts of the robotic arm form the second subsystem. The maneuver is shown in Fig. 1a where first, the robotic arm starts from a resting pose and moves toward a payload resting on the ground (part 1 in the figure). After reaching the boxed-shaped payload, the robot EE will grasp the payload and try to lift and rotate it so that one edge of the box remains in contact with the ground (part 2 of the figure). As the next step, the robotic arm will slide the payload on the ground horizontally while two vertices of the box remains in contact with the ground (parts 3 and 4 in the figure). The contact between the box and the ground is considered by two contact pairs on the vertices of the box. The velocity of one of these vertices named U is captured and shown in Fig. 1b using the reference solution, smooth RIM, and non-smooth RIM co-simulation. The simulation is integrated with the macro time step of $\frac{1}{60}$s while each subsystem will be integrated with the reduced model of the other subsystem with the micro time step of $\frac{1}{600}$s. According to Fig. 1b, the co-simulation using non-smooth RIM will give results in reasonable agreement with the reference results while using the smooth RIM will produce inaccurate outputs. To explain this, we have to look at the maneuver where the robot tries to lift and rotate the payload and two vertices of the box are constantly touching the ground. Using non-smooth RIM, these two pairs are considered potential contact pairs and their effect on the robot motion is taken into account by solving the complementarity problem. However, using the smooth RIM approach, the two contact pairs are considered like bilateral constraints in the macro time step, and are grouped as active constraints. This will give 2 active redundant constraints leading to ill-conditioned effective mass for the reduced model which results in inaccuracy.




(a) (b)

Figure 1: (a) Manoeuvre of the case study (b) Velocity norm of the point U on the payload.

## References


[1] J. Kövecses. Dynamics of Mechanical Systems and the Generalized Free-Body Diagram—Part I: General Formulation. ASME. J. Appl. Mech., 75(6): 061012, 2008.