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Hybrid Algorithm for Analyzing Velocity Fields from Sequence Images

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Hybrid algorithm for analyzing velocity fields from sequence images

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Abstract: *In this study, a hybrid algorithm was developed for analyzing velocity fields from sequence images taken during particle image velocimetry (PIV) process. The algorithm was used cross-correlation method to obtain initial results for optical-flow processing. Optical-flow was applied to obtain refined results and the resolution of the image reached to unit of pixel. The hybrid algorithm was then applied for systematic images generated by in-house code. The results showed that the algorithm could overcome disadvantage of cross-correlation and optical-flow methods, which indicated by high resoultion and high accurate velocity fields.*

Keywords: *optical-flow, PIV, cross-correlation algorithm, Lamb-Oseen vortex*

1. Introduction

Quantitative measurement of velocity fields from sequence image is an important task in aerodynamics. The method allows to obtain global velocity fields. Algorithms for analyzing velocity could be optical-flow [1]–[5] or cross-correlation [6], [7]. For example, the optical-flow algorithm is applied in visualizing cloud motions from satellite images, surface motions of the water in river or blood flow inside human body. Meanwhile, cross-correlation algorithms are widely applied for analyzing unsteady behavior of velocity fields from particle image velocimetry (PIV) images.

Although the optical-flow and cross-correlation algorithms have many advantages in analyzing images, each of them has some limitation. In more details, cross-correlation algorithm applies for interrogation window, which reduces spatical resolution of velocity fields [8]. Oppositely, optical-flow algorithm processes each pixel in the image frame and the resolution is the same to the size of the images [9]. However, the algorithm is very sensitive to brightness of image, which is not suitable for particle image from PIV measurement. Additionally, the boundary condition is a big problem for optical-flow algorithm.

In this study, we propose to combine both algorithms to overcome their disadvantages. The cross-correlation algorithm is applied firstly to obtain initial results for optical-flow method. Then optical-flow algorithm is utilized to increase resolution of the results. The results of this study can provide good reference for extended previous study by numerical methods, which currently focuses by many other researchers [10], [11]. A detailed explanation of the technique is presented in section 2, while section 3 presents examples of the algorithm for systematic data set. Finally, we conclude our paper in section 4.

2. Methodology description

2.1. PIV measurement process

The cross-correlation method is applied to obtain initial results for optical-flow process. The working principle of PIV is to calculate the correlation coefficient for interrogation windows. In the details, the first image is divided into small interrogation areas (interrogation windows). After that, the cross-correlations of those windows with the second image are calculated. The position of maximum cross-correlation shows the displacement of the interrogation windows in the second images. Since the time interval between the first and second images was known and displacement of interrogation windows was calculated, the velocity fields can be obtained. The formula for cross-correlation is shown as:

$$R(s) = \int_W I_1(X) I_2(X+s) dX \tag{1}$$

In this study, we use open source code PIVLab by for PIV processing. The coded presented by Thielicke and Stamhuis [12]. Since the size of interrogation often ranges from 8 × 8 pixels to 64 × 64 pixels, the resolution of the velocity fields reduces by around 64 times of the image resolution. To increase resolution of the velocity fields, an interpolation algorithm is applied. Consequently, the cross-correlation algorithm provides initial results with some level of uncertainty.

2.2. Scalar transport equation for optical-flow process

The velocity fields of a scalar concentration C of the fluids could be expressed by the scalar transport equation [13].

$$\frac{\partial C}{\partial t} + \nabla.(C\boldsymbol{u}) - \frac{1}{ReSc}\Delta C = 0 \tag{2}$$

Where u is velocity of the concentration fields, Re is Reynolds number and Sc is Schmidt numbers.

Here, we assume that the fluid is incompressible and the concentration C is proposal to intensity of the image (C = εI), then Eq. (2) becomes:

$$\frac{\partial I}{\partial t} + \boldsymbol{u}\nabla I - \frac{1}{ReSc}\Delta I = 0 \tag{3}$$

Equation (3) shows the relation between velocity fields and spatical-temporal change of image intensity. However, it contains two unknown components of $\boldsymbol{u} = (u, v)$. Consequently, an additional condition is required. Here, we used regulation term as it was proposed by Horn and Schunck [1]:

$$J_R = \int_{\boldsymbol{\Omega}} \left\| \nabla \boldsymbol{u}(x,t) \right\|^2 dx \tag{4}$$

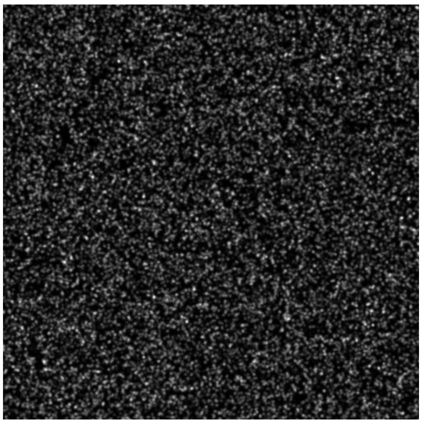
The velocity could be find by minimized the following function:

$$J(u) = \int_\Omega \left[\frac{\partial I}{\partial t} + \boldsymbol{u} \bullet \nabla I + \frac{1}{ReSc}\Delta I \right]^2 dx_1 dx_2 + \alpha \int_{\boldsymbol{\Omega}} \left\| \nabla \bullet \boldsymbol{u}(x,t) \right\|^2 dx_1 dx_2 \tag{5}$$

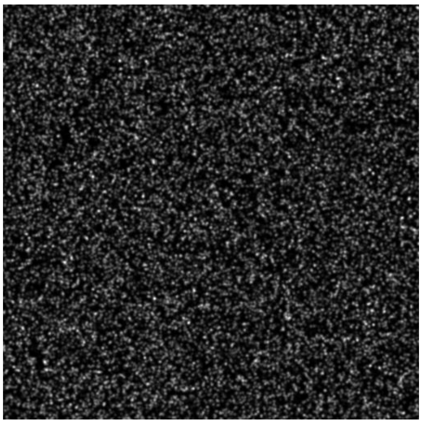
Where α is Lagrange multiplier. A discrete scheme was used to solve Equation (5). For more details of the method, reader can refer to [9]. For faster convergence, results of cross-correlation method were used as initial approximation. The method allows velocity fields fom a pair image, which is called a snapshot solution. A program written in Matlab environment was built to obtain the solution.

3. An example of velocity field extraction

To examize the algorithm, we applied it to recover Lamb-Oseen vortexes from systematic image. Since the solution of the velocity fields is known, the uncertainty of the method could be calculated. In this study, the particle images dataset with Lamb-Oseen vortex were created using method of Brady et al. [14]. The images have size of 512 × 512 pixels.The velocity fields are predetermined in the particle images with two vortexes. The core radius of core vortexes is located at (x, y) = (0, ±85). The diameter of the image is 5 pixel with Gaussian distribution. Concentration of particles on the image is 20% and the maximum displacement of the particles is around 5 pixels. A uniform flow fields is also added to the images. The two systematic images for data processing is shown in Fig. 1.



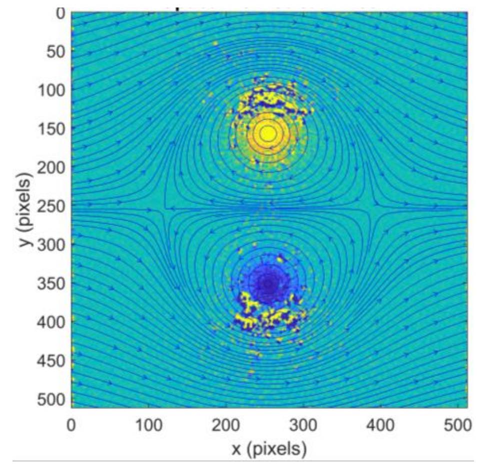
First image



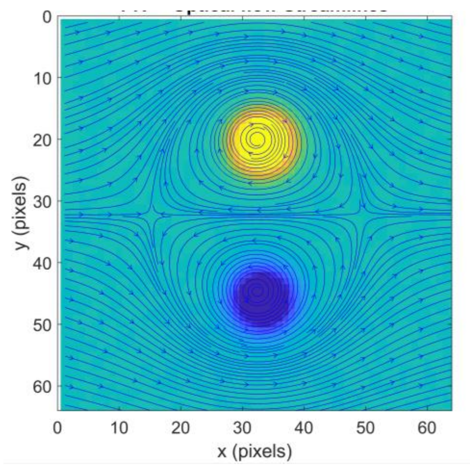
Second image

Fig. 1 Two systematic images for data processing

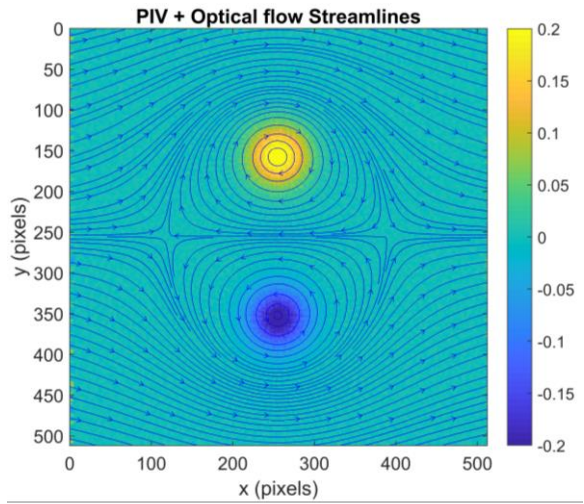
Figure 2 shows velocity fields and streamline by three methods. Clearly, the optical-flow algorithm provides unclear results near the center of vortex. One reason is that optical-flow algorithm can not deal with high movement of particles. Cross-correlation algorithm could overcome well this problem. However, the resolution of the method is reduced. The results from combination of cross-correlation and optical-flow algorithms show in Fig. 2c. Clearly, the method overcomes well problem of both two methods. Here, we obtain clear velocity fields. Additionally, resolution is much more improved.



a) Optical flow algorithm



b) Cross-correlation algorithm



c) Combined cross-correlation and optical-flow algorithms

Figure 2. Velocity fields by three algorithms

Figure 3 shows velocity at the centerline by optical-flow, combination of two methods and exacted solution of velocity profile. The hybrid method provides close results to the exacted velocity field and it could overcome the disadvantage of the optical-flow algorithm.

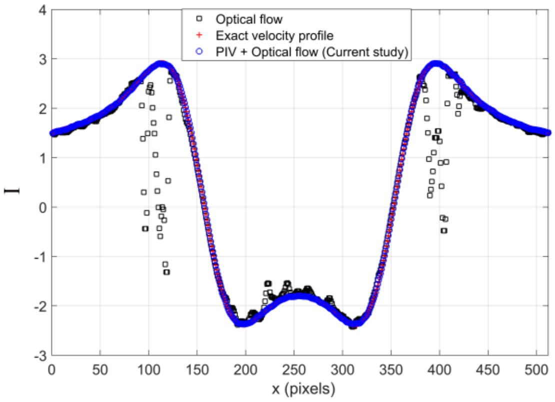
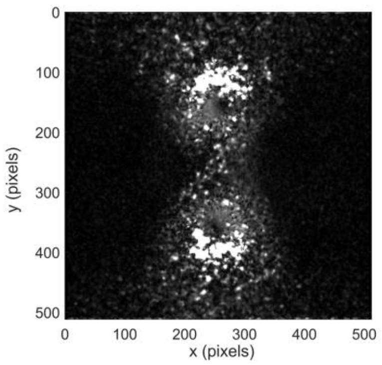


Figure 3. Velocity at centerline

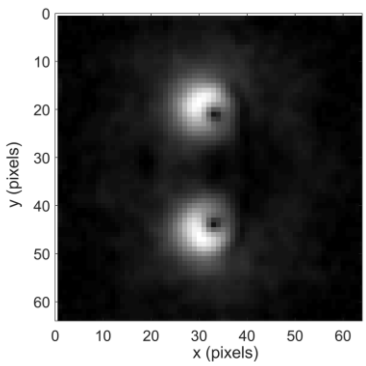
To evaluate the measurement uncertainty, root mean square error (RMSE) are calculated as the equation below:

$$RMSE = \frac{1}{N}\sqrt{\sum_{n=1}^{N}\left\| \boldsymbol{u}_c(x,t) - \boldsymbol{u}_e(x,t) \right\|^2} \tag{6}$$

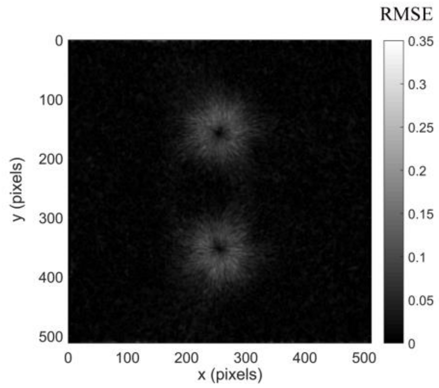
The results of root mean square error for diffirent methods are presented in Fig. 4. Interestingly, the RMSE in optical-flow algorithm is the highest, which occurs around the center of vortex rings. In the proposed method, RMSE is much smaller than those by the optical-flow and cross-correlation algorithm.



Optical flow algorithm



Cross-correlation algorithm



Combined cross-correlation and optical-flow algorithms

4. Conclusion

A hybrid algorithm was developed for analyzing velocity fields from sequence images taken during PIV measurement process. In the proposed algorithm, cross-correlation algorithm was applied firstly to obtain initial results. An interpolation method was applied to increases resolution of the initial results to image size. The optical-flow algorithm was applied with initial results by cross-correlation algorithm to obtain high resolution of velocity fields. Application of systematic images, the proposed algorithm overcomes disadvantage of both methods in solving global velocity fields. For further study, we will examine the algorithm for systematic data with different conditions of smoke particles such as diameter, concentration and intensity. Additionally, application to the real image will be conducted to verify the proposed algorithm.

References

[1] B. K. Horn and B. G. Schunck, "Determining Optical Flow Berthold," *Tech. Appl. Image Underst.*, vol. 0281, pp. 319–331, 1981.

[2] T. H. Tran, T. Ambo, T. Lee, L. Chen, T. Nonomura, and K. Asai, "Effect of boattail angles on the flow pattern on an axisymmetric afterbody surface at low speed," *Exp. Therm. Fluid Sci.*, vol. 99, no. May, pp. 324–335, 2018, doi: 10.1016/j.expthermflusci.2018.07.034.

[3] T. H. Tran, T. Ambo, L. Chen, T. Nonomura, and K. Asai, "Effect of boattail angle on pressure distribution and drag of axisymmetric afterbodies under low-speed conditions," *Trans. Jpn. Soc. Aeronaut. Space Sci.*, vol. 62, no. 4, pp. 219–226, 2019, doi: 10.2322/tjsass.62.219.

[4] T. H. Tran and L. Chen, "Optical-Flow Algorithm for Near-Wake Analysis of Axisymmetric Blunt-Based Body at Low-Speed Conditions," *J. Fluids Eng.*, vol. 142, no. 11, pp. 1–10, 2020, doi: 10.1115/1.4048145.

[5] T. H. Tran, M. Anyoji, T. Nakashima, K. Shimizu, and A. D. Le, "Experimental Study of the Skin-Friction Topology Around the Ahmed Body in Cross-Wind Conditions," *J. Fluids Eng.*, vol. 144, no. 3, 2022, doi: 10.1115/1.4052418.

[6] T. Kawata and S. Obi, "Velocity-pressure correlation measurement based on planar PIV and miniature static pressure probes," *Exp. Fluids*, vol. 55, no. 7, 2014, doi: 10.1007/s00348-014-1776-7.

[7] T. H. Tran, "The Effect of Boattail Angles on the Near-Wake Structure of Axisymmetric Afterbody Models at Low-Speed Condition," *Int. J. Aerosp. Eng.*, vol. 2020, 2020, doi:

10.1155/2020/7580174.

[8] T. T. Hung, "Single-pixel ensemble correlation algorithm for boundary measurement on axisymmetric boattail surface," *J. Sci. Tech. VOL - 208*, 2020.

[9] T. H. Tran and L. Chen, "Wall shear-stress extraction by an optical flow algorithm with a sub-grid formulation," *Acta Mech. Sin. Xuebao*, vol. 37, no. 1, pp. 65–79, 2021, doi: 10.1007/s10409-020-00994-9.

[10] A. D. Le, H. Phan Thanh, and H. Tran The, "Assessment of a Homogeneous Model for Simulating a Cavitating Flow in Water Under a Wide Range of Temperatures," *J. Fluids Eng.*, vol. 143, no. 10, p. 101204, 2021, doi: 10.1115/1.4051078.

[11] T. H. Tran, H. Q. Dinh, H. Q. Chu, V. Q. Duong, C. Pham, and V. M. Do, "Effect of boattail angle on near-wake flow and drag of axisymmetric models: a numerical approach," *J. Mech. Sci. Technol.*, vol. 35, no. 2, pp. 563–573, Feb. 2021, doi: 10.1007/s12206-021-0115-1.

[12] W. Thielicke and E. Stamhuis, "PIVlab–towards user-friendly, affordable and accurate digital particle image velocimetry in MATLAB," *J. open Res. Softw.*, vol. 2, no. 1, 2014.

[13] C. Cassisa, S. Simoens, V. Prinet, and L. Shao, "Subgrid scale formulation of optical flow for the study of turbulent flow," *Exp. Fluids*, vol. 51, no. 6, pp. 1739–1754, 2011, doi: 10.1007/s00348-011-1180-5.

[14] M. R. Brady, S. G. Raben, and P. P. Vlachos, "Methods for digital particle image sizing (DPIS): comparisons and improvements," *Flow Meas. Instrum.*, vol. 20, no. 6, pp. 207–219, 2009.