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Abstract

This paper presents an optimization method to solve a multi-objective model of Bi-level linear programming problem with intuitionistic fuzzy coefficients. The idea is based on TOPSIS (technique for order preference by similarity to ideal solution) method. TOPSIS method is a multiple criteria method that identifies a satisfactory solution from a given set of alternatives on the basis of minimization of distance from an ideal point and maximization of distance from nadir point simultaneously. A new model of multi-objective bi-level programming problem in intuitionistic fuzzy environment has been considered. The problem is first reduced to a conventional multi-objective bi-level linear programming problem by using accuracy function and then modified TOPSIS method is proposed to solve the problem at both the leader and the follower level where various linear/non-linear membership functions are used to represent the flexibility in the approach of decision makers. The problem is solved in a hierarchical manner, i.e. first the problem at leader level is solved and then the feasible region is extended by relaxing the decision variables controlled by the leader. The feasible region is extended so as to obtain a satisfactory solution for the decision makers at both the levels. Finally, the application of the proposed approach in the production planning of a company has been presented. An illustrative numerical example is also given to illustrate the methodology of the approach defined in this paper.

Keywords:
Multi-objective optimization problem, Bi-level programming problem, TOPSIS, Intuitionistic fuzzy number, Fuzzy optimization.

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1. Introduction

A decentralized programming problem where a hierarchical administrative structure is used to arrange multiple decision makers can be easily modelled by using multi-level programming problem. Bi-level programming problem is one of the special cases of multi-level programming problem where only two decision makers, both controlling one subset of decision variables independently, with different and maybe conflicting objectives are located at two different hierarchical levels. In bi-level programming problem, the upper level decision maker is termed as leader and the lower level decision maker is termed as follower.

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Following the hierarchy, the leader makes a decision first and depending upon the decision of the leader; the follower chooses his/her strategy which may affect the objective of the leader. The behaviour of leader influences the strategy of follower; which in turn may affect the leader. As a result, decision deadlocks are natural to arise. In most of the practical decision making situations, the problem of proper distribution of decision powers to decision makers is also encountered.

The very first works regarding solution procedure of bi-level optimization problem was presented by Candler and Townsley in 1982 [9]. Various applications of bi-level optimization method in governmental planning such as changing the levels of punishment (taxation) for illegal drug imports, decisions on investment plan to improvise transportation and communication infrastructure and processing capacity, together with price level etc. have been presented in Candler and Townsley in [9]. In a typical set up of bi-level decision making problem, the central government or a central authority usually acts as the upper level decision maker which constructs policies and the state government or an organization working under the central authority acts as the lower level decision maker. This class of optimization problem has very wide range of applications and thus has attracted various researchers in the last few decades. Some surveys which presents both new solution approaches and theoretical results are [24], [11]. Bi-level linear programming problem has also been dealt in [8], [14], [22] and [25] and non linear bi-level programming problem has been discussed by Abo Sinna in [2] and [6]. Moitra and Pal in 2002 [16] solved the Bi-level programming problem by using fuzzy goal programming method. Degree of satisfaction for the optimality of objectives of decision makers at both the levels is to be maximized while keeping in mind the optimality of decision variables controlled by upper level decision maker. Arora and Gupta [4] in 2009 used the dynamic fuzzy goal programming approach and divide the problem into two phases where the feasible region is to be determined in the first phase and second phase deals with the attainment of objective function of leader and follower and optimization of decision variables controlled by the upper level decision maker. Wan et. al [23] in 2008 proposed an interactive fuzzy decision making method where a class of bi-level programming problem is solved where upper level decision maker and lower level decision maker have a common decision variable. An interactive fuzzy decision making method for solving bi-level programming problem was also presented by Zheng et. al [27] in 2014 and they introduced a balance function which tackles the problem of improper distribution of decision powers.

In most of the real world situations, decision maker at each level aims to optimize more than one objective and such real world situations give rise multi-objective bi-level programming problem when there are two levels in the administrative structure. In the presence of multiple conflicting objectives, usually one solution can not optimize all the objectives; so, an efficient solution is to be searched. TOPSIS method [13] is one such method that can be used for solving multi-objective decision making problems. The method is based on the concept that any solution which is closest to positive ideal solution and farthest from the negative ideal solution is a good choice for the efficient solution. Hwang and Yoon [21] were first to develop the method for solving multiple attribute decision making problem and further the method was extended by many researchers in [3], [1], [10] and [6] so as to solve multi-objective decision making problems.

Modelling real-life optimization problems present difficulties because of lack of knowledge of precise technological coefficients and the introduction of fuzzy set theory simplifies the task of representing imprecise coefficients with the help of fuzzy numbers. Decision making in an imprecise environment was introduced by Bellman and Zadeh [7] and researchers like Pramanik Roy [18], Sinha et. al [20] developed approaches based on fuzzy programming to solve multi-level linear programming problem. Zimmerman [28] proposed an approach for solving multi-objective optimization problem by using fuzzy programming. Mohamed in 1997 [15] explained the relationship between fuzzy programming and goal programming. In some situations, imprecise coefficients may be provided with some hesitation and such situations can be modelled effectively by using Intuitionistic fuzzy numbers. Various works related to LPP in intuitionistic fuzzy environment has been done in last decades and some of them are [17] where a crisp decision is used for defuzzification. Singh and Yadav [19] used accuracy function to defuzzify [26] the intuitionistic fuzzy number and then used fuzzy programming technique to solve multi-objective optimization problem. An application of multi-objective decision making in intuitionistic fuzzy environment in transportation model has been presented by Jana and Roy in [12].

In an organization, planning usually takes place at two levels. The first level or the upper level consists of decision makers who aim to minimize investment and control decision like capital investment in raw material, power, machinery, infrastructure and many others. The second level or lower level consists of executors who aim to minimize the labour hours and maximize the production and control decisions for the same. The availability of raw materials, machines, power and infrastructure influence the labour hours and hence the production. The executors can make the decision only when the capital investment has been decided by the upper level decision makers. Then the executors decide the labour hours and the production which in turn affect the revenue earned and the investment. In real life situations, coefficients like investment required, labour hours required for one units of any product and revenue earned by that unit can never be defined precisely and thus always have a scope of hesitation because of some uncontrollable factors. In order to model the production planning problem presented, objectives at two levels are to be optimized. The objectives at first level corresponds to maximization of the profit earned and minimization of investment. Notice that both the objectives are conflicting in nature. The objectives at executor level corresponds to minimization of labour hours and maximizing the production. An efficient solution of first level would be the maximum profit earned with minimum investment while at second level the efficient solution would correspond to the maximum production achieved with minimum labour hours. The problem presented can be easily modelled in the form of a multi-objective bi-level optimization problem with intuitionistic fuzzy numbers. In this paper, we have proposed a method for solving such a problem with the help of TOPSIS method. To the best of author's knowledge in the problem domain, the optimization of multiple objectives in two levels has never been formulated till date.

In this paper, multi-objective bi-level optimization problem under intuitionsitic fuzzy environment has been modelled and a solution procedure based on modified TOPSIS is proposed for finding the efficient solution. Various membership functions are used during the solution process so as to provide flexibility for making decisions and defining aspiration levels of the decision makers. This paper is organized as follows: Preliminaries and concepts regarding intuitionistic fuzzy set theory and distance measures have been defined in Section 2. In Section 3, a multi-objective bi-level linear programming model in an intuitionistic fuzzy environment has been formulated and its solution methodology has been discussed. In section 4, an algorithm for the same has been presented. Section 5 presents a numerical example which is used to explain the adopted methodology and the last section comprises of concluding remarks.

2. Preliminaries and Concepts

Definition 1. *Let X be a universe of discourse. Then an intuitionistic fuzzy set $\tilde{A}^I$ in X [5]is defined by*

$$\tilde{A}^I = \{< x, \mu_{\tilde{A}^I}(x), \vartheta_{\tilde{A}^I}(x) >: x \in X\} \tag{1}$$

where $\mu_{\tilde{A}^I}(x)$ and $\vartheta_{\tilde{A}^I}(x)$ represents the degree of membership [29] and degree of non-membership of element x in $\tilde{A}^I$, respectively. $h(x) = 1 - \mu_{\tilde{A}^I}(x) - \vartheta_{\tilde{A}^I}(x)$ represents the degree of hesitation for element x.

Definition 2. *A Triangular Intuitionistic fuzzy number (TIFN) [5] $\tilde{A}^I$ is an IFN with the membership function and non-membership function given by eq. 2 and eq. 3 respectively.*

$$\mu_{\tilde{A}^I}(x) = \begin{cases} \dfrac{x-a}{b-a} & if\, a \leq x \leq b \\ 1 & if\, x = b \\ \dfrac{c-x}{c-b} & if\, b \leq x \leq c \\ 0 & otherwise \end{cases} \tag{2}$$

and

$$\vartheta_{\tilde{A}^I}(x) = \begin{cases} \dfrac{b-x}{b-a'} & if\, a' \leq x \leq b \\ 1 & if\, x = b \\ \dfrac{x-c'}{c'-b} & if\, b \leq x \leq c' \\ 0 & otherwise \end{cases} \tag{3}$$

where $a' \leq a \leq b \leq c \leq c'$.

This TIFN is denoted by $(a, b, c; a', b, c')$. *Figure 1 represents the membership and non-membership functions of TIFN defined above. The set of all TIFNs is denoted by* $IF(R)$.

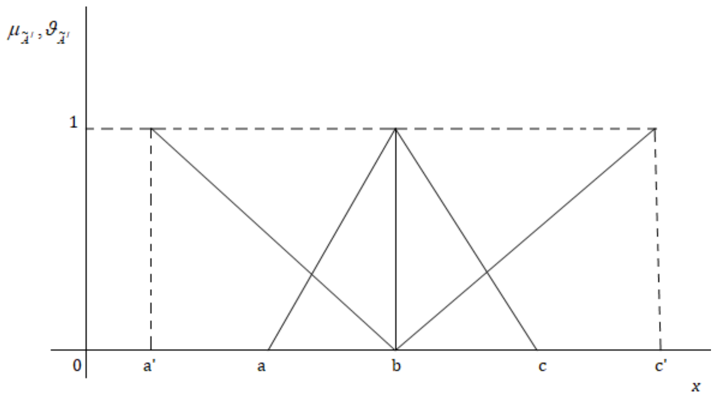


Figure 1: A Triangular Intuitionistic Fuzzy Number

Definition 3. *Accuracy Function: Let* $\tilde{A}^I = (a, b, c; a', b, c')$ *be a triangular intuitionistic fuzzy number. The score function for membership function and non-membership function is denoted by* $S(\mu_{\tilde{A}^I})$ *and* $S(\vartheta_{\tilde{A}^I})$ *respectively where*

$$S(\mu_{\tilde{A}^I}) = \frac{a + 2b + c}{4} \quad \text{and} \quad S(\vartheta_{\tilde{A}^I}) = \frac{a' + 2b + c'}{4}.$$

The accuracy function of $\tilde{A}^I$ *is denoted by* $f(\tilde{A}^I)$ *and is defined by eq.* (4)

$$f(\tilde{A}^I) = \frac{(a + 2b + c) + (a' + 2b + c')}{8} \tag{4}$$

Definition 4. *Distance measure: If* $F(x) = (f_1(x), f_2(x), \ldots, f_m(x))$ *is a vector of objective functions which is to be maximized. Then the* L_p *metric defines the distance between two points* $F(x)$ *and* F^* *as*

$$d_p = \left\{ \sum_{j=1}^{m} \lambda_j^p [f_j^* - f_j(x)]^p \right\}^{1/p} \tag{5}$$

where λ_j; $j = 1, 2, \ldots, m$ *is relative importance of objective* f_j *and* $F^* = ({f_1}^*, {f_2}^*, \ldots, {f_m}^*)$ *where* ${f_i}^* = \max_{x \in S} f_i(x)$. *Since the objectives are not commensurable, so a scaling function in the interval* $[0, 1]$ *should be used for every objective function. So, the following metric could be used.*

$$d_p = \left\{ \sum_{j=1}^{m} \lambda_j^p \left(\frac{f_j^* - f_j(x)}{f_j^* - f_j^-} \right)^p \right\}^{1/p} \qquad p = 1, 2, \ldots, m. \tag{6}$$

where ${f_i}^- = \min_{x \in S} f_i(x)$, *where* S *is the constraint space.*

3. Problem Formulation and Model Development

A standard form of multi-objective bi-level linear optimization problem with intuitionistic fuzzy coefficients is given by:

$$\begin{array}{lrcl} \underset{x}{\text{Maximize}} & \tilde{Z_1^I}(x,y) & = & [\tilde{Z_{11}^I}(x,y), \tilde{Z_{12}^I}(x,y), \ldots, \tilde{Z_{1k_1}^I}(x,y)] \\ \text{where y solves} & & & \\ \text{Maximize} & \tilde{Z_2^I}(x,y) & = & [\tilde{Z_{21}^I}(x,y), \tilde{Z_{22}^I}(x,y), \ldots, \tilde{Z_{2k_2}^I}(x,y)] \\ \text{subject to} & \tilde{A^I}x + \tilde{B^I}y & \leq & \tilde{r}^I \\ & x, y & \geq & 0 \end{array} \tag{7}$$

where $\tilde{Z_{ij}^I} = \tilde{c_{ij}^I}x + \tilde{d_{ij}^I}y$ for $i = 1, 2$ and $j = 1, 2, \ldots, k_i$ and $x = (x_1, x_2, \ldots, x_{n_1}) \in \mathbb{R}^{n_1}$ and $y = (y_1, y_2, \ldots, y_{n_2}) \in \mathbb{R}^{n_2}$ and $n_1 + n_2 = n$. In (7), $\tilde{A^I} \in IF(\mathbb{R}^{m \times n_1})$ and $\tilde{B^I} \in IF(\mathbb{R}^{m \times n_2})$ and $\tilde{r^I} \in IF(\mathbb{R}^m)$. Also, $\tilde{c_{ij}^I} \in IF(\mathbb{R}^{1 \times n_1})$ and $\tilde{d_{ij}^I} \in IF(\mathbb{R}^{1 \times n_2})$. Therefore, the problem in eq. (7) can be written as:

$$\begin{array}{lrcl} \underset{x}{\text{Maximize}} & \tilde{Z_1^I}(x,y) & = & [\tilde{Z_{11}^I}(x,y), \tilde{Z_{12}^I}(x,y), \ldots, \tilde{Z_{1k_1}^I}(x,y)] \\ \text{where y solves} & & & \\ \text{Maximize} & \tilde{Z_2^I}(x,y) & = & [\tilde{Z_{21}^I}(x,y), \tilde{Z_{22}^I}(x,y), \ldots, \tilde{Z_{2k_2}^I}(x,y)] \\ \text{subject to} & & & \\ & \tilde{a_{i1}^I}x_1 + \tilde{a_{i2}^I}x_2 + \ldots + \tilde{a_{in_1}^I}x_{n_1} + \tilde{b_{i1}^I}y_1 + \tilde{b_{i2}^I}y_2 + \ldots + \tilde{b_{in_2}^I}y_{n_2} & \leq & \tilde{r_i^I} \\ & x_1, x_2, \ldots, x_{n_1}, y_1, y_2, \ldots y_{n_2} & \geq & 0 \end{array} \tag{8}$$

where $\tilde{Z_{ij}^I}(x,y) = \tilde{c_{ij1}^I}x_1 + \tilde{c_{ij2}^I}x_2 + \ldots + \tilde{c_{ijn_1}^I}x_{n_1} + \tilde{d_{ij1}^I}y_1 + \tilde{d_{ij2}^I}y_2 + \ldots + \tilde{d_{ijn_2}^I}y_{n_2}$ for $i = 1, 2; j = 1, 2, \ldots n_i$.

First we use the concept of defuzzification and reduce every intuitionistic fuzzy coefficient to a crisp coefficient using accuracy function as defined in eq. (4). That is, a multi-objective bi-level optimization problem under intuitionistic fuzzy environment is reduced to its corresponding crisp form.

$$\begin{array}{lrcl} \underset{x}{\text{Maximize}} & Z_1'(x,y) & = & [Z_{11}'(x,y), Z_{12}'(x,y), \ldots, Z_{1k_1}'(x,y)] \\ \text{where y solves} & & & \\ \text{Maximize} & Z_2'(x,y) & = & [Z_{21}'(x,y), Z_{22}'(x,y), \ldots, Z_{2k_2}'(x,y)] \\ \text{subject to} & & & \\ & a_{i1}'x_1 + a_{i2}'x_2 + \ldots + a_{in_1}'x_{n_1} + b_{i1}'y_1 + b_{i2}'y_2 + \ldots + b_{in_2}'y_{n_2} & \leq & r_i' \\ & x_1, x_2, \ldots, x_{n_1}, y_1, y_2, \ldots y_{n_2} & \geq & 0 \end{array} \tag{9}$$

where

$Z'_{ij}(x,y) = c'_{ij1}x_1 + c'_{ij2}x_2 + \ldots + c'_{ijn_1}x_{n_1} + d'_{ij1}y_1 + d'_{ij2}y_2 + \ldots + d'_{ijn_2}y_{n_2}$ for $i = 1,2; j = 1,2,\ldots n_i$ where $a' = f(\tilde{a}^I)$. Following the hierarchy, first an efficient solution for upper level decision maker is to be obtained. The TOPSIS model used to solve upper level multi-objective optimization problem of eq. (7) is given by (10)

$$
\begin{aligned}
&\text{Minimize} && d_p^{PIS^u}(x,y) \\
&\text{Maximize} && d_p^{NIS^u}(x,y) \\
&\text{subject to} && \\
& && \begin{array}{rcll} a'_{i1}x_1 + a'_{i2}x_2 + \ldots + a'_{in_1}x_{n_1} + & & & \\ & \leq & r'_i & i = 1,2,\ldots,m \\ b'_{i1}y_1 + b'_{i2}y_2 + \ldots + b'_{in_2}y_{n_2} & & & \\ x_1, x_2, \ldots, x_{n_1}, y_1, y_2, \ldots y_{n_2} & \geq & 0 & \end{array}
\end{aligned}
\tag{10}
$$

where

$$d_p^{PIS^u} = \left\{ \sum_{i=1}^{k_1} w_i^p \left(\frac{Z_{1i}^* - Z_{1i}}{Z_{1i}^* - Z_{1i}^-} \right)^p \right\}^{1/p}, \tag{11}$$

$$d_p^{NIS^u} = \left\{ \sum_{i=1}^{k_1} w_i^p \left(\frac{Z_{1i} - Z_{1i}^-}{Z_{1i}^* - Z_{1i}^-} \right)^p \right\}^{1/p} \tag{12}$$

and $Z_{1i}^* = \max_{(x,y)\in S} Z_{1i}$, $Z_{1i}^- = \min_{(x,y)\in S} Z_{1i}$, $S = \{(x,y) = (x_1, x_2, \ldots x_{n_1}, y_1, y_2, \ldots y_{n_2}) \in \mathbb{R}^n : a'_{i1}x_1 + a'_{i2}x_2 + \ldots + a'_{in_1}x_{n_1} + b'_{i1}y_1 + b'_{i2}y_2 + \ldots + b'_{in_2}y_{n_2} \leq r'_i; i = 1,2,\ldots,m\}$ and $w_i, i = 1,2,\ldots k_1$ are the relative weights of the objective functions such that $w_i \geq 0 \quad \forall i = 1,2,\ldots,k_1$ and $\sum_{i=1}^{k_1} w_i = 1$.

The fuzzy goal programming model is used to solve the bi-objective optimization problem modelled in eq. (10). The corresponding fuzzy goal programming model is given below:

$$
\begin{aligned}
&\text{Find} && \{(x_i, y_j); i = 1,2,\ldots,n_1, j = 1,2,\ldots,n_2\} \\
&\text{subject to} && \\
& && \begin{array}{rcll} d_p^{PIS^u} & \approx & {d_p}^{PIS^{u*}} & \\ d_p^{NIS^u} & \approx & {d_p}^{NIS^{u*}} & \\ a'_{i1}x_1 + a'_{i2}x_2 + \ldots + a'_{in_1}x_{n_1} + & & & \\ & \leq & r'_i & i = 1,2,\ldots,m \\ b'_{i1}y_1 + b'_{i2}y_2 + \ldots + b'_{in_2}y_{n_2} & & & \\ x_1, x_2, \ldots, x_{n_1}, y_1, y_2, \ldots y_{n_2} & \geq & 0 & \end{array}
\end{aligned}
\tag{13}
$$

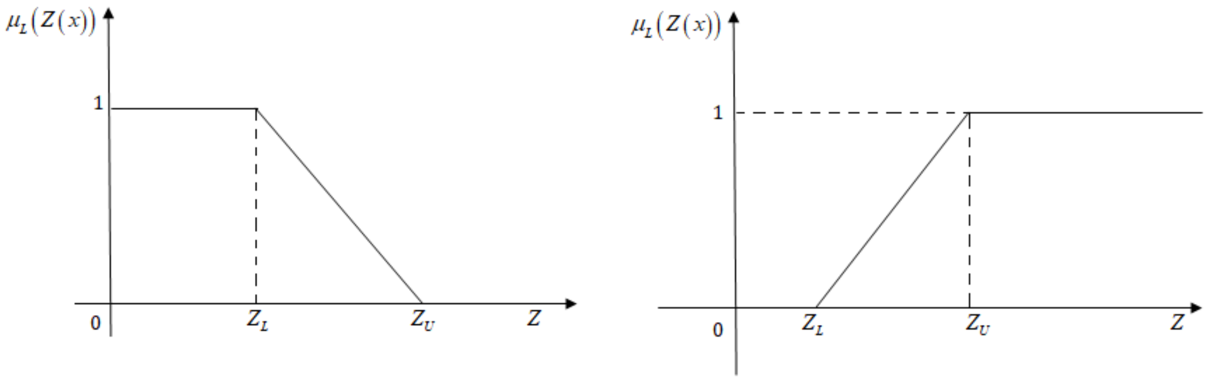
where $\approx$ is a fuzzy goal, which means some deviations are allowed in strict goal. Here, $d_p^{PIS^{u*}} = \min_{xinS} d_p^{PIS^u}$ and $d_p^{NIS^{u*}} = \max_{xinS} d_p^{NIS^u}$ To change the fuzzy goal programming model into a crisp LPP, different types of linear/non-linear membership functions can be used. A linear membership function is the most utilized function in decision making process while solving mathematical programming problems. A linear approximation is defined by fixing two points, the least and most desirable levels of acceptability of an objective function. In general fuzzy set theory, such an assumption is not always justified. Thus a justification should be made considering the fuzziness of goal in mind. From this point of view, several linear/non-linear shapes of membership functions are considered.

3.1. Linear Membership Function

Linear membership function for maximization and minimization of an objective are given by

$$\mu_L(Z(x)) = \begin{cases} 0 & if\, Z \leq Z_L \\ \dfrac{Z - Z_L}{Z_U - Z_L} & if\, Z_L \leq Z < Z_U \\ 1 & if\, Z \geq Z_U \end{cases} \quad \text{and} \quad \mu_L(Z(x)) = \begin{cases} 1 & if\, Z \leq Z_L \\ \dfrac{Z_U - Z}{Z_U - Z_L} & if\, Z_L \leq Z < Z_U \\ 0 & if\, Z \geq Z_U \end{cases} \tag{14}$$

respectively where Z_L and Z_U are the minimum and maximum values of the objective function Z respectively. Figure 2a and Figure 2b represent the linear membership function when the objective is to minimize and maximize an objective function Z respectively.



(a) Minimization objective

(b) Maximization of objective

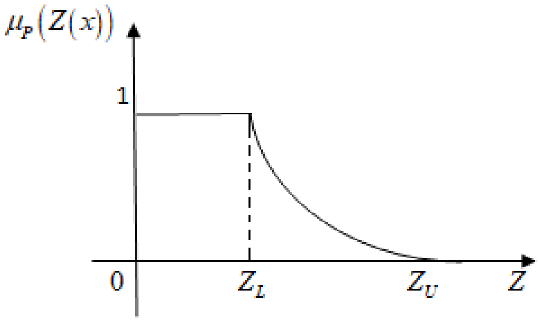
Figure 2: Linear Membership Functions

3.2. Parabolic Membership Function

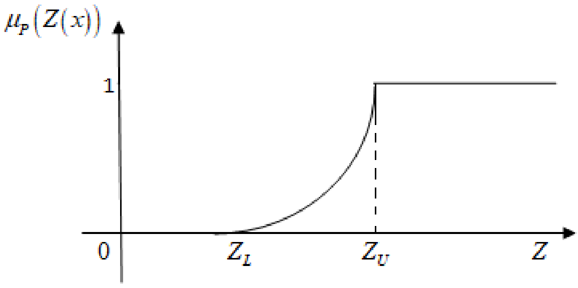
The parabolic membership function for maximization and minimization of an objective can be defined as

$$\mu_P(Z(x)) = \begin{cases} 0 & if\, Z \leq Z_L \\ \left(\dfrac{Z - Z_L}{Z_U - Z_L}\right)^2 & if\, Z_L \leq Z < Z_U \\ 1 & if\, Z \geq Z_U \end{cases} \quad \text{and} \quad \mu_P(Z(x)) = \begin{cases} 0 & if\, Z \leq Z_L \\ 1 - \left(\dfrac{Z_U - Z}{Z_U - Z_L}\right)^2 & if\, Z_L \leq Z < Z_U \\ 1 & if\, Z \geq Z_U \end{cases} \tag{15}$$

respectively. Figure 3a and Figure 3b represent the parabolic membership function when the objective is to minimize and maximize an objective function Z respectively.



(a) Minimization of objective



(b) Maximization of objective

Figure 3: Parabolic Membership Functions

3.3. Hyperbolic Membership Function

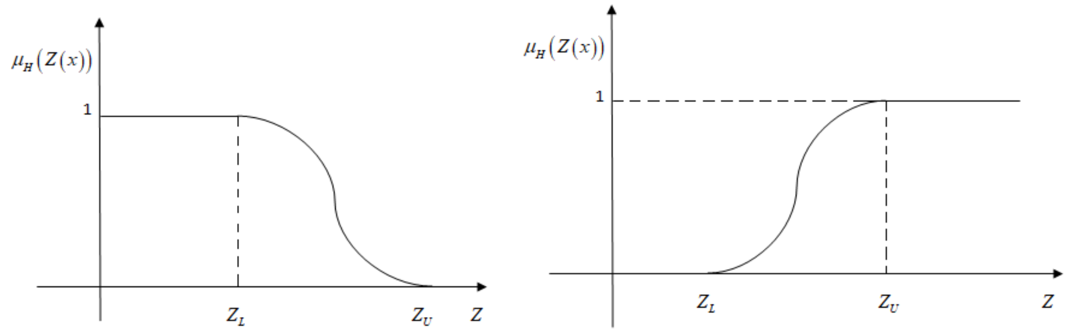
The hyperbolic membership function is concave over the part where the decision maker is performing better than the goal and he tends to have a small marginal rate of satisfaction and the membership function is convex over the part when the decision maker is worse off the goal and he tends to have a higher marginal rate of satisfaction. The complete functions in case of maximization and minimization of objectives are given by eq. 16 and eq. 17 respectively.:

$$\mu_H(Z_p(x)) = \begin{cases} 0 & if\, Z_p \leq L_p \\ \frac{1}{2} + \frac{1}{2}tanh(Z_p(x) - \frac{U_p + L_p}{2})\alpha_p & if\, 0 < L_p \leq Z_p < U_p \\ 1 & if\, Z_p \geq U_p \end{cases} \tag{16}$$

$$\mu_H(Z_p(x)) = \begin{cases} 1 & if\, Z_p \leq L_p \\ \frac{1}{2} + \frac{1}{2}tanh(Z_p(x) - \frac{U_p + L_p}{2})\alpha_p & if\, 0 < L_p \leq Z_p < U_p \\ 0 & if\, Z_p \geq U_p \end{cases} \tag{17}$$

Figure 4a and Figure 4b represent the hyperbolic membership function when the objective is to minimize and maximize an objective function Z respectively.

Various membership functions defined in eq. (14) - eq (17) can be used according to the satisfaction level of the decision maker and a crisp programming model of the fuzzy goal programming model is



(a) Minimization of objective (b) Maximization of objective

Figure 4: Hyperbolic Membership Functions

constructed. A corresponding crisp programming model is given by eq. (18):

$$
\begin{array}{lrcl}
Maximize & & \lambda & \\
\text{subject to} & & & \\
& \mu(d_p^{PIS^u}) & \geq & \lambda \\
& \mu(d_p^{NIS^u}) & \geq & \lambda \\
& \mu(d_p^{PIS^u}) & \leq & 1 \\
& \mu(d_p^{NIS^u}) & \leq & 1 \\
& \begin{array}{r} a'_{i1}x_1 + a'_{i2}x_2 + \ldots + a'_{in_1}x_{n_1} + \\ b'_{i1}y_1 + b'_{i2}y_2 + \ldots + b'_{in_2}y_{n_2} \end{array} & \leq & r'_i \quad i = 1, 2, \ldots, m \\
& x_1, x_2, \ldots, x_{n_1}, y_1, y_2, \ldots y_{n_2} & \geq & 0
\end{array}
\tag{18}
$$

An optimal solution of eq. (18) is a vector of the form $(\lambda, x_1^*, x_2^*, \ldots, x_{n_1}^*, y_1^*, x_1^*, y_2^*, \ldots, y_{n_2}^*)$ which implies that $(x_1^*, x_2^*, \ldots, x_{n_1}^*, y_1^*, x_1^*, y_2^*, \ldots, y_{n_2}^*)$ is an efficient solution of the upper level optimization problem. According to the concept of bi-level programming technique, which states that the leader sets the value of the decision variables controlled by him, assume t_k^L and t_k^R, $k = 1, 2, \ldots n_1$ be the maximum acceptable negative and positive relaxation values for decision variable x_k, respectively, controlled by the leader. The tolerances t_k^L and t_k^R are not necessarily the same. The tolerances are provided so as to extend the feasible region for the search of the satisfactory solution. The tolerance value of the decision variables controlled by the leader are modified in a way such that the feasible region for follower is non-empty.

3.4. Proposed TOPSIS method for Multi-objective Bi-level optimization problem

A satisfactory solution for the multi-objective bi-level optimization problem can be obtained by using the TOPSIS approach, where the objective is to minimize the distance from the positive ideal solution $(d_p^{PIS^B})$ and maximize the distance from the negative ideal solution $(d_p^{NIS^B})$, where$(d_p^{PIS^B})$ and $(d_p^{NIS^B})$

are given by eq. (19) and eq. (20) respectively.

$$d_p^{PIS^B} = \left\{ \sum_{i=1}^{k_1} w_i^p \left(\frac{Z_{1i}^* - Z_{1i}}{Z_{1i}^* - Z_{1i}^-} \right)^p + \sum_{j=1}^{k_2} w_{j+k_1}^p \left(\frac{Z_{2i}^* - Z_{2i}}{Z_{2i}^* - Z_{2i}^-} \right)^p \right\}^{1/p} \tag{19}$$

and

$$d_p^{NIS^B} = \left\{ \sum_{i=1}^{k_1} w_i^p \left(\frac{Z_{1i} - Z_{1i}^-}{Z_{1i}^* - Z_{1i}^-} \right)^p + \sum_{j=1}^{k_2} w_{j+k_1}^p \left(\frac{Z_{2i} - Z_{2i}^-}{Z_{2i}^* - Z_{2i}^-} \right)^p \right\}^{1/p} \tag{20}$$

where $w_j's$ represents the weights and $Z_{ij}^* = \max_{(x,y)\in S} Z_{ij}$, $Z_{ij}^- = \min_{(x,y)\in S} Z_{ij}$ for $i = 1, 2; j = 1, 2, \ldots k_i$.

In order to obtain an efficient solution, the problem defined in eq. (7) is reduced to the following bi-objective optimization problem with conflicting objectives

$$\begin{aligned}
&\text{Minimize} && d_p^{PIS^B}(x,y) \\
&\text{Maximize} && d_p^{NIS^B}(x,y) \\
&\text{subject to} && \\
&&& \begin{matrix} a'_{i1}x_1 + a'_{i2}x_2 + \ldots + a'_{in_1}x_{n_1} + \\ b'_{i1}y_1 + b'_{i2}y_2 + \ldots + b'_{in_2}y_{n_2} \end{matrix} \leq r'_i \qquad i = 1, 2, \ldots, m \\
&&& x_1, x_2, \ldots, x_{n_1}, y_1, y_2, \ldots y_{n_2} \geq 0
\end{aligned} \tag{21}$$

Since both the objectives are usually conflicting to each other, so we solve them separately and let $d_p^{PIS^{B*}} = \min_{x\in S} d_p^{PIS^B}$, $d_p^{NIS^{B*}} = \max_{x\in S} d_p^{NIS^B}$ and $d_p^{PIS^{B-}} = \max_{x\in S} d_p^{PIS^B}$, $d_p^{NIS^{B-}} = \min_{x\in S} d_p^{NIS^B}$. Then based on the preference concept, the solution with the shorter distance from PIS and with larger distance from NIS is assigned a larger degree of acceptance. The membership degree of $d_p^{PIS^B}$ and $d_p^{PIS^B}$ is defined by using various linear/non-linear membership functions as defined in eq. (14), eq. (15), eq. (16) and (17).

In order to generate a satisfactory solution of multi-objective bi-level optimization problem defined in eq. (7), a crisp model is to be solved which minimizes the distance from positive ideal solution and maximizes the distance from negative ideal solution while giving some relaxation in the decision variables controlled by the leader. A single objective optimization model corresponding to this situation is given by eq. (22).

$$\begin{array}{lrcll}
Maximize & & \delta & & \\
\text{subject to} & & & & \\
& \mu(d_p^{PIS^B}) & \geq & \delta & \\
& \mu(d_p^{NIS^B}) & \geq & \delta & \\
& \mu(d_p^{PIS^B}) & \leq & 1 & \\
& \mu(d_p^{NIS^B}) & \leq & 1 & \\
& \dfrac{x_k - (x_k^* - t_k^L)}{t_k^L} & \geq & \delta & \\
& \dfrac{(x_k^* + t_k^R) - x_k}{t_k^R} & \geq & \delta & \\
& a'_{i1}x_1 + a'_{i2}x_2 + \ldots + a'_{in_1}x_{n_1} + & & & \\
& b'_{i1}y_1 + b'_{i2}y_2 + \ldots + b'_{in_2}y_{n_2} & \leq & r'_i & i = 1, 2, \ldots, m \\
& x_1, x_2, \ldots, x_{n_1}, y_1, y_2, \ldots y_{n_2} & \geq & 0 & \\
& \delta & \in & [0, 1] &
\end{array} \tag{22}$$

A single objective optimization problem is then modelled by using various membership functions explained in eq. (14) - eq. (17) and an optimal solution to the problem is obtained with the help of various software packages. An optimal solution of eq. (22) of the form $(\delta, x_1^*, x_2^*, \ldots, x_{n_1}^*, y_1^*, y_2^*, \ldots, y_{n_2}^*)$ implies that an efficient solution of eq. (7) is given by $(x_1^*, x_2^*, \ldots, x_{n_1}^*, y_1^*, y_2^*, \ldots, y_{n_2}^*)$.

4. Proposed TOPSIS Algorithm

To solve the multi-objective bi-level programming problem under intuitionistic fuzzy environment, the proposed TOPSIS algorithm is given as follows:

Step 1: Use accuracy function as defined in eq. (4) and reduce every intuitionistic fuzzy coefficient to a crisp coefficient.

Step 2: Find the maximum and minimum values of all the objective functions at both leader and follower level under the given constraints.

Step 3: Construct a pay off table of positive ideal solution (PIS) of leader and obtain $Z_{1i}^* = (Z_{11}^*, Z_{12}^*, \cdots, Z_{1k_1}^*)$, the individual optimal solutions, where $Z_{1i}^* = \max\limits_{(x,y)\in S} Z_{1i}(x, y)$.

Step 4: Construct a pay off table of negative ideal solution (NIS) of leader and obtain $Z_{1i}^- = (Z_{11}^-, Z_{12}^-, \cdots, Z_{1k_1}^-)$, the individual negative ideal solutions, where $Z_{1i}^- = \min\limits_{(x,y)\in S} Z_{1i}(x, y)$.

Step 5: Construct equations for $d_p^{PIS^u}$ and $d_p^{NIS^u}$ as defined in eq. (11) and eq. (12) respectively.

Step 6: Ask the decision maker to select p and w_i, $i = 1, 2, \ldots k_1$.

Step 7: Reduce the leader problem to a bi-objective optimization problem as defined in eq. (10).

Step 8: Optimize both the objectives of model (10) separately and denote them by $(d_p^{PIS^u})^*$ and $(d_p^{NIS^u})^*$.

Step 9: Convert the bi-objective optimization model into a fuzzy goal programming model, where the goal is to find the value of decision variables such that $(d_p^{PIS^u})$ is approximately equal to $(d_p^{PIS^u})^*$ and $(d_p^{NIS^u})$ is approximately equal to$(d_p^{NIS^u})^*$.

Step 10: Use various linear/non-linear membership function to convert the fuzzy goal programming model into a crisp LPP as defined in eq. (18).

Step 12: The optimal solution of eq. (18); $(\lambda, x_1^{u*}, x_2^{u*}, \ldots, x_{n_1}^{u}{}^*, y_1^{u*}, y_2^{u*}, \ldots, y_{n_2}^{u}{}^*)$ implies that the efficient solution of the upper level optimization problem is given by $(x_1^{u*}, x_2^{u*}, \ldots, x_{n_1}^{u}{}^*, y_1^{u*}, y_2^{u*}, \ldots, y_{n_2}^{u}{}^*)$ based on which the maximum left and right tolerance values; t_k^L and t_k^R on the decision variables controlled by the leader are to be defined.

Step 13: Construct a pay off table of positive ideal solution (PIS) of multi-objective bi-level linear programming problem and obtain $Z_{ij}^*, i = 1,2; j = 1,2,\ldots n_i$, the individual optimal solutions, where $Z_{ij}^* = \max\limits_{(x,y)\in S} Z_{ij}(x,y)$.

Step 14: Construct a pay off table of negative ideal solution (NIS) of multi-objective bi-level optimization problem and obtain $Z_{ij}^-; i = 1,2; j = 1,2,\ldots,n_i$, the individual negative ideal solutions, where $Z_{ij}^- = \min\limits_{(x,y)\in S} Z_{ij}(x,y)$.

Step 15: Construct equations for $d_p^{PIS^B}$ and $d_p^{NIS^B}$ as defined in eq. (19) and eq. (20) respectively.

Step 16: Reduce the multi-objective bi-level optimization problem to a bi-objective optimization problem, as defined in eq. (21).

Step 17: Optimize both the objectives of (21) separately and denote them by $d_p^{PIS^B{}^*}$ and $d_p^{NIS^B{}^*}$ respectively.

Step 18: Formulate the model (22) for multi-objective bi-level optimization problem by using various membership functions.

Step 19: Solve model (22) to get $(x^*, y^*) = (x_1^*, x_2^*, \ldots, x_{n_1}^*, y_1^*, y_2^*, \ldots, y_{n_2}^*)$

Step 20: If leader and follower are satisfied with solution in **Step 19**, go to **Step 21**, else go to **Step 22**.

Step 21: Stop with the satisfactory solution, $(x^*, y^*) = (x_1^*, x_2^*, \ldots, x_{n_1}^*, y_1^*, y_2^*, \ldots, y_{n_2}^*)$.

Step 22: Modify the maximum negative and positive tolerance values on the decision variables $(x_1^{u*}, x_2^{u*}, \ldots, x_{n_1}^{u}{}^*)$, t_k^L and t_k^R, $k = 1,2,\ldots n_1$ and go to **Step 18**.

5. An Illustrative Example

The following numerical example is considered to illustrate the production planning problem of a manufacturing company. Assume that the company produces three different commodities namely, x_1, x_2 and x_3. The objective is to determine the number of units of each commodity to be produced so that the decisions like profit maximization and power consumption minimization can be achieved by the central authority and the objectives of executors is to minimize the labour hours and maximize the returns earned on manufacturing commodities while keeping in mind the constraints related to raw material availability, space availability and machine hours availability.

For the ease of solving, every objective has been modified into a maximization type and a model for the explained production planning problem in an intuitionistic fuzzy environment is given by eq. (23) which is then solved by using Proposed TOPSIS algorithm.

$$\begin{array}{lrcl}
\underset{x_1}{\text{Maximize}} & \tilde{Z}_1^I(x_1,x_2,x_3) & = & [\tilde{Z}_{11}^I(x_1,x_2,x_3), \tilde{Z}_{12}^I(x_1,x_2,x_3)] \\
\text{where } x_2, x_3 \text{solves} & & & \\
\text{Maximize} & \tilde{Z}_2^I(x_1,x_2,x_3) & = & [\tilde{Z}_{21}^I(x_1,x_2,x_3), \tilde{Z}_{22}^I(x_1,x_2,x_3)] \\
\text{subject to} & & & \\
 & (1,2,3;0,2,4)x_1 - (2,4,6;0,4,8)x_2 + (2,3,4;1,3,5)x_3 & \leq & (80,84,88;76,84,92) \\
 & (1,2,3;0,2,4)x_1 + (2,3,4;1,3,5)x_2 + (1,2,3;0,2,4)x_3 & \leq & (100,105,110;95,105,115) \\
 & (4,6,8;2,6,10)x_1 - (1,2,3;0,2,4)x_2 + (2,3,4;1,3,5)x_3 & \leq & (19,21,23;15,21,27) \\
 & x_1,x_2,x_3 & \geq & 0
\end{array} \tag{23}$$

where

$$\begin{array}{rcl}
\tilde{Z}_{11}^I(x_1,x_2,x_3) & = & (3,5,7;1,5,9)x_1 + (1,2,3;0,2,4)x_2 + (6,8,10;4,8,12)x_3 \\
\tilde{Z}_{12}^I(x_1,x_2,x_3) & = & (1,2,3;0,2,4)x_1 - (2,3,4;1,3,5)x_3 \\
\tilde{Z}_{21}^I(x_1,x_2,x_3) & = & (1,3,5;0,3,6)x_1 - (1,2,3;0,2,4)x_2 + (1,2,3;0,2,4)x_3 \\
\tilde{Z}_{22}^I(x_1,x_2,x_3) & = & (3,5,7;1,5,9)x_2 + (2,4,6;0,4,8)x_3
\end{array} \tag{24}$$

Using the accuracy function, every intuitionistic fuzzy coefficient can be reduced to a crisp coefficient and then (23) becomes:

$$\begin{array}{lrcl}
\underset{x_1}{\text{Maximize}} & \tilde{Z}_1^I(x_1,x_2,x_3) & = & [5x_1+2x_2+8x_3, 2x_1-3x_3] \\
\text{where } x_2, x_3 \text{solves} & & & \\
\text{Maximize} & \tilde{Z}_2^I(x_1,x_2,x_3) & = & [3x_1-2x_2+2x_3, 5x_2+4x_3] \\
\text{subject to} & & & \\
 & 2x_1-4x_2+3x_3 & \leq & 84 \\
 & 2x_1+3x_2+2x_3 & \leq & 105 \\
 & 6x_1-2x_2+3x_3 & \leq & 21 \\
 & x_1,x_2,x_3 & \geq & 0
\end{array} \tag{25}$$

The individual maximum and minimum value of all the objectives in two levels are given in Table (1).

Table 1: Individual maximum and minimum values

	Z_{11}	Z_{12}	Z_{21}	Z_{22}
Maximum	210	24.81818	14	189
Minimum	0	-63	-70	0

The positive ideal solution pay off table for the leader is shown by Table (2) and thus $(Z_{11}^*, Z_{12}^*) = (210, 24.81818)$.

Table 2: PIS pay off for Leader

Z_{11}	Z_{12}	x_1	x_2	x_3
210*	-63	0	21	21
115.500	24.81818*	12.40909	26.72727	0

The negative ideal solution pay off table for the leader is shown by Table (3) and thus $(Z_{11}^-, Z_{12}^-) = (0, -63)$.

Table 3: NIS pay off for Leader

Z_{11}	Z_{12}	x_1	x_2	x_3
0^-	0	0	0	0
210	-63^-	0	21	21

Formulating the equation for $d_p^{PIS^u}$ and $d_p^{NIS^u}$ gives

$$d_p^{PIS^u} = \left\{ w_1^p \left(\frac{210 - (5x_1 + 2x_2 + 8x_3)}{210 - 0} \right)^p + w_2^p \left(\frac{24.81818 - (2x_1 - 3x_3)}{24.81818 + 63} \right)^p \right\}^{1/p} \tag{26}$$

and

$$d_p^{NIS^u} = \left\{ w_1^p \left(\frac{(5x_1 + 2x_2 + 8x_3) - 0}{210 - 0} \right)^p + w_2^p \left(\frac{(2x_1 - 3x_3) + 63}{24.81818 + 63} \right)^p \right\}^{1/p} \tag{27}$$

Taking $w_1 = w_2 = \frac{1}{2}$ and $p = 2$, we get

$$d_p^{PIS^u} = \left\{ \frac{1}{4} \left(\frac{210 - (5x_1 + 2x_2 + 8x_3)}{210 - 0} \right)^2 + \frac{1}{4} \left(\frac{24.81818 - (2x_1 - 3x_3)}{24.81818 + 63} \right)^2 \right\}^{1/2} \tag{28}$$

and

$$d_p^{NIS^u} = \left\{ \frac{1}{4} \left(\frac{(5x_1 + 2x_2 + 8x_3) - 0}{210 - 0} \right)^2 + \frac{1}{4} \left(\frac{(2x_1 - 3x_3) + 63}{24.81818 + 63} \right)^2 \right\}^{1/2} \tag{29}$$

Then the TOPSIS model formulation for the upper level decision maker is given by eq. (30):

$$\begin{aligned}
&\text{Minimize} && d_p^{PIS^u}(x,y) \\
&\text{Maximize} && d_p^{NIS^u}(x,y) \\
&\text{subject to} && \\
& && 2x_1 - 4x_2 + 3x_3 \leq 84 \\
& && 2x_1 + 3x_2 + 2x_3 \leq 105 \\
& && 6x_1 - 2x_2 + 3x_3 \leq 21 \\
& && x_1, x_2, x_3 \geq 0
\end{aligned} \tag{30}$$

The maximum and minimum values of $d_p^{PIS^u}$ are 0.5166396 and 0.2041313 at $(0,0,0)$ and $(10.33969, 25.77216, 3.502068)$ respectively. The maximum and minimum values of $d_p^{NIS^u}$ are 0.5697151 and 0.2725840 at $(12.40909, 26.72727, 0)$ and $(0, 0.9793644, 7.652910)$ respectively. Thus, $(d_p^{PIS^u*}, d_p^{NIS^u*}) = (0.2041313, 0.5697151)$.

Converting the bi objective optimization problem to a goal programming model gives:

$$\begin{aligned}
&\text{Find} && (x_1, x_2, x_3) \\
&\text{subject to} && \\
& && d_p^{PIS^U} \approx 0.2041313 \\
& && d_p^{NIS^U} \approx 0.5697151 \\
& && 2x_1 - 4x_2 + 3x_3 \leq 84 \\
& && 2x_1 + 3x_2 + 2x_3 \leq 105 \\
& && 6x_1 - 2x_2 + 3x_3 \leq 21 \\
& && x_1, x_2, x_3 \geq 0
\end{aligned} \tag{31}$$

The equations for membership functions of $d_p^{PIS^u}$ and $d_p^{NIS^u}$ by using linear membership functions are given by:

$$\mu_{d_p^{PIS^u}}(x) = \begin{cases} 1 & if\, d_p^{PIS^u} \leq 0.2041313 \\ \dfrac{0.5166396 - d_p^{PIS^u}}{0.5166396 - 0.2041313} & if\, 0.2041313 \leq d_p^{PIS^u} < 0.5166396 \\ 0 & if\, d_p^{PIS^u} \geq 0.5166396 \end{cases} \tag{32}$$

and

$$\mu_{d_p^{NIS^u}}(x) = \begin{cases} 0 & if\, d_p^{NIS^u} \leq 0.2725840 \\ \dfrac{d_p^{NIS^u} - 0.2725840}{0.5697151 - 0.2725840} & if\, 0.2725840 \leq d_p^{NIS^u} < 0.5697151 \\ 1 & if\, d_p^{NIS^u} \geq 0.5697151 \end{cases} \tag{33}$$

The corresponding crisp LPP model is given by:

$$\begin{array}{lrcl} Maximize & & & \lambda \\ \text{subject to} & & & \\ & 1.653202 - 3.1999 d_p^{PIS^u} & \geq & \lambda \\ & -0.917386299 + 3.36551 d_p^{NIS^u} & \geq & \lambda \\ & 1.653202 - 3.1999 d_p^{PIS^u} & \leq & 1 \\ & -0.917386299 + 3.36551 d_p^{NIS^u} & \leq & 1 \\ & 2x_1 - 4x_2 + 3x_3 & \leq & 84 \\ & 2x_1 + 3x_2 + 2x_3 & \leq & 105 \\ & 6x_1 - 2x_2 + 3x_3 & \leq & 21 \\ & x_1, x_2, x_3 & \geq & 0 \\ & \lambda & \in & [0,1] \end{array} \tag{34}$$

Solving (34), we get $(\lambda, x_1^*, x_2^*, x_3^*) = (0.9608319, 11.96506, 26.52233, 0.751441)$. Let the upper level decision maker decides $x_1 = 11.96506$ with 0.5 as positive and negative tolerance limits.

The positive ideal solution pay off table and the negative ideal solution pay off table for the follower is shown by Table (4) and Table (5) and thus $(Z_{21}^*, Z_{22}^*) = (14, 189)$ and $(Z_{21}^-, Z_{22}^-) = (-70, 0)$.

Table 4: PIS pay off for follower

Z_{21}	Z_{22}	x_1	x_2	x_3
14^*	28	0	0	7
0	189^*	0	21	21

Table 5: NIS pay off for follower

Z_{21}	Z_{22}	x_1	x_2	x_3
-70^-	175	0	35	0
0	0^-	0	0	0

Formulating the equation for $d_p^{PIS^B}$ and $d_p^{NIS^B}$ gives

$$d_P^{PIS^B} = \left\{ w_1^p \left(\frac{210 - (5x_1 + 2x_2 + 8x_3)}{210 - 0} \right)^p + w_2^p \left(\frac{24.81818 - (2x_1 - 3x_3)}{24.81818 + 63} \right)^p + w_3^p \left(\frac{14 - (3x_1 - 2x_2 + 2x_3)}{14 + 70} \right)^p + w_4^p \left(\frac{189 - (5x_2 + 4x_3)}{189 - 0} \right)^p \right\}^{1/p} \tag{35}$$

and

$$d_P^{NIS^B} = \left\{ w_1^p \left(\frac{(5x_1 + 2x_2 + 8x_3)}{210 - 0} \right)^p + w_2^p \left(\frac{(2x_1 - 3x_3) + 63}{24.81818 + 63} \right)^p + w_3^p \left(\frac{(3x_1 - 2x_2 + 2x_3) + 70}{14 + 70} \right)^p + w_4^p \left(\frac{(5x_2 + 4x_3)}{189 - 0} \right)^p \right\}^{1/p} \tag{36}$$

Taking $w_1 = w_2 = w_3 = w_4 = \frac{1}{4}$ and $p = 2$, we get

$$d_p^{PIS^B} = \left\{ \frac{1}{16} \left(\frac{210 - (5x_1 + 2x_2 + 8x_3)}{210 - 0} \right)^2 + \frac{1}{16} \left(\frac{24.81818 - (2x_1 - 3x_3)}{24.81818 + 63} \right)^2 + \frac{1}{16} \left(\frac{14 - (3x_1 - 2x_2 + 2x_3)}{14 + 70} \right)^2 + \frac{1}{16} \left(\frac{189 - (5x_2 + 4x_3)}{189 - 0} \right)^p \right\}^{1/p} \quad (37)$$

and

$$d_p^{NIS^B} = \left\{ \frac{1}{16} \left(\frac{(5x_1 + 2x_2 + 8x_3)}{210 - 0} \right)^2 + \frac{1}{16} \left(\frac{(2x_1 - 3x_3) + 63}{24.81818 + 63} \right)^2 + \frac{1}{16} \left(\frac{(3x_1 - 2x_2 + 2x_3) + 70}{14 + 70} \right)^2 + \frac{1}{16} \left(\frac{(5x_2 + 4x_3)}{189 - 0} \right)^2 \right\}^{1/2} \quad (38)$$

TOPSIS method is now used and the multi-objective bi-level optimization problem is reduced to a bi-objective optimization problem:

$$\begin{array}{lrcl} \text{Minimize} & d_p^{PIS^B}(x, y) & & \\ \text{Maximize} & d_p^{NIS^B}(x, y) & & \\ \text{subject to} & & & \\ & 2x_1 - 4x_2 + 3x_3 & \leq & 84 \\ & 2x_1 + 3x_2 + 2x_3 & \leq & 105 \\ & 6x_1 - 2x_2 + 3x_3 & \leq & 21 \\ & x_1, x_2, x_3 & \geq & 0 \end{array} \quad (39)$$

The maximum and minimum values of $d_p^{PIS^B}$ are 0.3624506 and 0.141611 at $(0, 0, 0)$ and $(9.063417, 25.18312, 5.661909)$ respectively. The maximum and minimum values of $d_p^{NIS^B}$ are 0.3576674 and 0.2395355 at $(11.10674, 25.45848, 0.2687355)$ and $(0, 14.65093, 0)$ respectively. Thus, $(d_p^{PIS^{B*}}, d_p^{NIS^{B*}}) = (0.141611, 0.3576674)$.

The bi-objective programming problem can be converted into a crisp programming model as follows:

$$\begin{array}{lrcl} Maximize & \delta & & \\ \text{subject to} & & & \\ & \mu(d_p^{PIS^B}) & \geq & \delta \\ & \mu(d_p^{NIS^B}) & \geq & \delta \\ & 2x_1 - 4x_2 + 3x_3 & \leq & 84 \\ & 2x_1 + 3x_2 + 2x_3 & \leq & 105 \\ & 6x_1 - 2x_2 + 3x_3 & \leq & 21 \\ & \mu(x_1) & \geq & \delta \\ & x_1, x_2, x_3 & \geq & 0 \\ & \delta & \in & [0, 1] \end{array} \quad (40)$$

Applying various models from eq (14) - eq (17) and solving by LINGO, the optimal solution to the problem (40) by using various membership functions is given by Table (6).

Table 6: Solutions

	Linear Function	**Parabolic Function**	**Hyperbolic Function**
(x_1, x_2, x_3)	(11.93146, 26.50683,0.8083040)	(11.80166, 25.7691, 0.5752819)	(11.789773, 22.44511, 1.034612)
δ	0.9327935	0.4532053	0.9948509
Z_{11}	119.1374	115.1464	112.1558
Z_{12}	21.4380	21.8775	20.4916
Z_{21}	-15.6027	-14.9803	-7.4278
Z_{22}	135.7674	131.1407	116.3640
μ_{11}	0.5673	0.32182929	0.5143
μ_{12}	0.9615	0.92448225	0.5342
μ_{21}	0.6548	0.42876304	0.5357
μ_{22}	0.7183	0.51595489	0.5159

6. Conclusion

In this paper, a concept for solving multi-objective bi-level linear optimization problem in intuitionistic fuzzy environment is introduced. The concept of intuitionistic fuzzy environment not only allows one to define a degree of membership but also a degree of non-membership which is not simply the complement of membership degree. The defuzzification of intuitionistic fuzzy coefficients is done by using accuracy function and a conventional multi-objective bi-level optimization problem is obtained. A satisfactory solution for the multi-objective problems at both the levels is acquired by using TOPSIS method. After obtaining a satisfactory solution for the leader level optimization problem, the decision variables controlled by leader are relaxed with the help of several linear and non-linear membership functions so as to extend the feasible region for the follower. These membership functions provide flexibility to the decision maker so as to choose the function which better fits the problem and provides greater satisfaction. Here, only linear membership function is used by the upper level decision maker whereas various linear/non-linear membership functions are used while solving multi-objective bi-level optimization problem. It is discovered that the satisfaction of the decision maker follows the order, Hyperbolic > Linear > Parabolic, in the case of given numerical example. Moreover, a real world application of the proposed problem has been explained in the production planning process of a company. The proposed method has been used to model multi-objective bi-level optimization problem in an uncertain environment where a component of non-membership is also involved in deciding impreciseness. In future, the proposed method can be extended for solving multi-objective multi-level optimization problem under intuitionistic fuzzy environment.

7. Conflict of Interest

On behalf of all authors, the corresponding author states that there is no conflict of interest.

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