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Analysis of Periodic Orbits with Smaller Primary As Oblate Spheroid

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Abstract— **We have studied closed periodic orbits with loops for two systems – Sun – Mars and Sun – Earth systems – using Poincare surface section (PSS) technique. Perturbation due to oblateness for the second primary (Mars or Earth) is taken in to consideration and obtained orbits with loops varying from one to five around both primaries. It is found that the oblateness coefficient A_2 and Jacobi constant C has non- negligible effect on the position of the orbits. The model may useful for designing space mission for low – energy trajectories.**

Keywords— restricted three body problem; low energy trajectory design; periodic orbits; oblateness; Poincare surface of section

I. Introduction

The 'low-energy' transfer trajectory is used to describe the space trajectories that consume less fuel compared to Hohman transfer (Hohmann, 1925) . A Hohmann transfer is an orbital maneuver that transfers a satellite or spacecraft between two coplanar circular orbits sharing a common focus. The transfer is through an elliptical orbit which is tangent to both circular orbits. The periapse and apoapse of the transfer ellipse are the radii of the initial and final circular orbits. The classical approach of Hohmann transfer needs considerable amount of fuel for space missions. So, the space missions become costly in terms of fuel. So, missions in which trajectory can be achieved using gravity only, is more desirable in terms of cost.

The motion of an infinitesimal particle in the plane of motion of two massive point masses , called the primaries, moving about their centre of mass in a circular orbit under their mutual gravitational attraction is referred to as planar restricted three body problem (PRTBP).The masses of primary bodies are taken as m_1 and m_2 with $m_1 > m_2$.

The coordinates of the bigger and smaller primaries are, respectively, $(1-\mu, 0)$and $(\mu, 0)$, where μ denotes the number $\frac{m_2}{m_1+m_2}$.
[1] is fundamental book on the RTBP. Various aspects of PRTBP using Poincare surface of section (PSS) method have been discussed by a number of researchers [2], [3], [4], [5] and [6]. Periodic orbits of spacecraft around two primary bodies are required to construct low energy trajectory. [7] established three classes with orbits around both primaries depending on motion of spacecraft is prograde or retrograde in the rotating system as well as fixed system. In this paper we analyzed periodic orbits around both primaries with retrograde motion in rotating system. This family of orbit is orbit with different number of loops. In this paper we have analyzed periodic orbits having number of loops from 1 to 5 for different pair of oblateness coefficient $A_2$and Jacobi constant C for Sun- Mars and Sun – Earth system. It has been found that A_2 and C has non-negligible effect on the position of the orbits. Irrespective of the number of loops, the second primary is always lies inside one of the loops and infinitesimal body orbits both the primaries. The closest approach of the secondary body to the second secondary body occurs for the orbit with a single loop. Therefore, such orbits can be used as low-energy orbits space mission design in solar system.

II.EQUATION OF MOTION

The perturbed mean motion n of second primary body is given by,

$$n^2 = 1 + \frac{3}{2}A_2 \tag{1}$$

where

$$A_2 = \frac{\rho_e^2 - \rho_p^2}{5R^2}. \tag{2}$$

Here A_2 is oblateness coefficient of second primary. ρ_e and ρ_p represent equatorial and polar radii of second primary and R is the distance between two primaries. The unit of mass is chosen equal to the sum of the primary masses and the unit of length is equal to their separation. The unit of time is such that the Gaussian constant of gravitation is unity in the unperturbed case. In the usual dimensionless synodic coordinate system, the equations of motion of the spacecraft are given by [8]

$$\ddot{x} - 2n\dot{y} = \frac{\partial\Omega}{\partial x}, \quad \ddot{y} + 2n\dot{x} = \frac{\partial\Omega}{\partial y}. \tag{3}$$

where

$$\Omega = \frac{n^2}{2}[(1-\mu)r_1^2 + \mu r_2^2] + \frac{(1-\mu)}{r_1} + \frac{\mu}{r_2} + \frac{\mu A_2}{2r_2^3}. \tag{4}$$

Here $$r_1^2 = (x-\mu)^2 + y^2, \tag{5}$$

And $$r_2^2 = (x+1-\mu)^2 + y^2. \tag{6}$$

Equations (3) and (4) lead to the first integral

$$\dot{x}^2 + \dot{y}^2 = 2\Omega - C \ , \tag{7}$$

where C is Jacobi constant of integration given by

$$C = n^2\big((1-\mu)r_1^2 + \mu r_2^2\big) + 2\frac{(1-\mu)}{r_1} + 2\frac{\mu}{r_2} + \frac{\mu A_2}{r_2^3} - \dot{y}^2. \tag{8}$$

These equations of motion are integrated in (x, y) variables using a Runge- Kutta Gill fourth order method.We have constructed Poincare surface section (PSS) on the x, $\dot{x}$plane. The initial values were selected along the Ox-axis by using intervals of length 0.001. By giving different value of C we can plot the trajectories, and then analysis of orbits can be done.

III. RESULTS AND DISCUSSON

We shall consider two systems, the Sun – Mars system and the Sun-Earth system. For the first system we take the mass of Sun and Mars as $m_1 = 1.9881 \times 10^{30}$ kg. and $m_2 = 6.4185 \times 10^{23}$ kg., respectively, whereas for the second system mass of Earth is taken as $m_2 = 5.972\times 10^{24}$ kg. [3]. Thus, for the Sun – Earth and Sun- Mars systems, mass factor are 0.000003002 and 0.0000003212 respectively. Equatorial and polar radii of Earth are 6378.1 kms. and 6356.8 kms. and that of Mars are 3396.2 kms. and 3376.2 kms., respectively. The distance between Sun and Earth is taken as 149600000 kms. and distance between Sun and Mars is 227940000 kms. So, oblateness coefficient calculated from equation (2) for Sun- Earth and Sun- Mars have values $A_2 = 2.42405\times 10^{-12}$ and $A_2 = 5.21389 \times 10^{-13}$ respectively. These values of oblateness are negligible. So, in order to study the non trivial effect of oblateness on periodic orbit around both primaries, we take different values of oblateness that can make observable changes in different parameters. We have studied the effect of oblateness on the location and period of Sun- Mars system for different values of Jacobi constant C using PSS. Fig. 1 is PSS constructed for Sun – Mars system when (A_2 , C) is (0.05, 2.94) by taking value of x from [0.7 , l] with interval of x differencing is 0.001. Also, time span t = 10000 and interval of time differencing is also 0.001. So, for each x equations of motions are integrated using Runge –Kutta – Gill method for 1000000 times. Each solution is plotted as a point in the Fig. 1. The arcs of PSS are known as islands whose center gives periodic orbirts.

In a similar way , we can obtain PSS for Sun – Earth which is shown in Fig. 2. This PSS is also constructed for (0.05, 2.94). So that we can compare both PSS. Mass factor of Sun – Earth is greater than Sun – Mars.

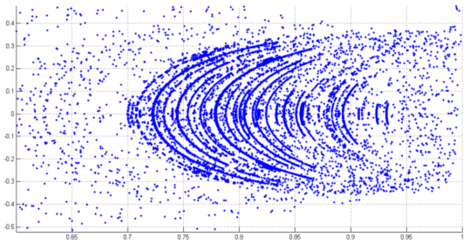


Fig. 1 PSS for $A_2 = 0.05$ and C = 2.94, for x = [0.7, 1], t = 10000 for Sun-Mars system.

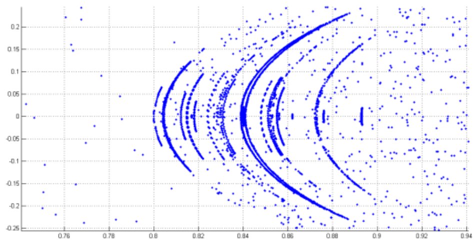


Fig. 2 PSS for A_2 = 0.05 and C= 2.94, for *x* = [0.8, 1], *t* = 10000 for Sun - Earth system.

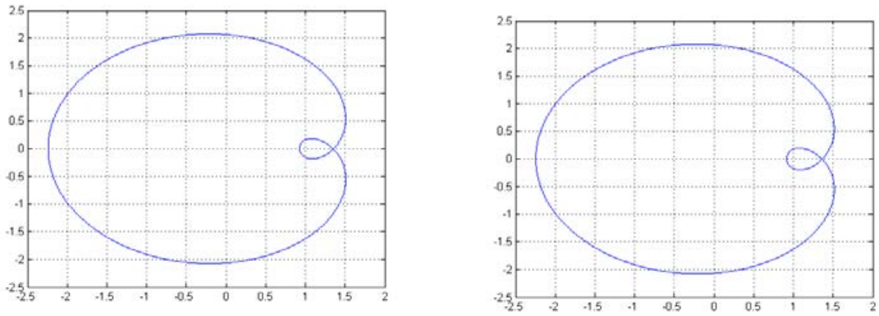
The numerical values of location and period of periodic orbit of spacecraft for C = 2.93, 2.94, 2.95, 2.96 and for oblateness A_2 = 0.001, 0.005, 0.01 and 0.05are displayed in Table 1. It is observed from the table that a change in C in the range (2.93, 2.96) affects the location but has no effect on the period and number of loops of the orbit. Oblateness also affects the location and period of the orbit. Similarly, the effects of C and A_2 in the location and period for the second system are studied and the numerical estimates of the changes are displayed in Table 2. It can be seen that, for both the systems, the period of orbit increases with increase in the number of loops.

A noticeable difference observed in both the systems is that for C = 2.96, A_2 = 0.001and 0.005, single- loop periodic orbit does not exist for Sun – Earth system.

The periodic orbits starting from single - loop to five loops around the Sun - Mars system for A_2 = 0.005 and 0.01 are shown in Figure 3(a) through Figure 3(j). It can be observed, from Figures 4 and 5, that the width of the orbit decreases continuously as the number of loops increases. Further in all cases the secondary body orbits around the second primary (Mars) in addition to orbiting both primaries. Further the secondary body is closest to Mars in the single loop closed orbit. Such orbits may be useful in the study of different aspects of both primaries. In many models available in literature not many closed orbits posses this kind of nature. We can observe similar nature in the orbits shown in Fig. 4(a) through Fig. 4(j) in the case of Sun - Earth system. Further the position of the orbit in the case of odd number of loop approaches the first primary, namely the Sun. This is true in the case of even number of loops also.

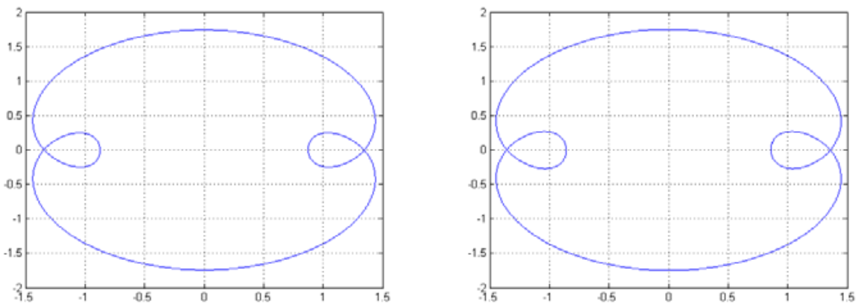
Figures 3 and 4 depict periodic orbits with number of loops referring from 1 to 5. These orbits are given for various A_2 for Sun - Mars and Sun - Earth systems. For both of these figures value of Jacobi constant C is 2.93. We have studied the variation of position of periodic orbits around Sun - Mars and Sun - Earth system due to the variation in oblateness and Jacobi constants C. In Fig. 5 we have shown the variation of position of closed periodic orbit with one loop for oblateness in the range (0, 0.05) for Sun – Mars system corresponding to Jacobi constants C = 2.93, 2.94, 2.95 and 2.96. From the Fig. 5 it is clear that the position of the orbits recedes away from Mars when the oblateness increases and C decreases.

Similar kind of conclusion can be drawn from Fig. 6 for the Sun – Earth system. From Figures 7 and 8, it can be observed that for given oblateness, location of periodic orbit moves away from second primary as number of loops in periodic orbit increase. Also, as oblateness increases, location of periodic orbit moves away from second primary. The variations of the eccentricity *e* with respect to oblateness A_2 and Jacobi constant C for single and two loops periodic orbits are shown in Fig. 9-12 for both Sun – Mars and Sun – Earth systems. The eccentricity decreases as A_2 increases and decreases as C increases for single loop periodic orbit for both systems Sun – Mars and Sun – Earth as shown in Fig. 9 and Fig. 10. Whereas, the eccentricity increase as

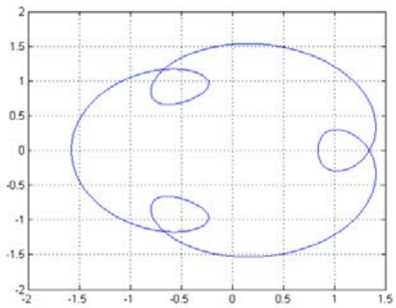


(a) A_2 = 0.005, single loop periodic orbit. (b) A_2 = 0.01, single loop periodic orbit.

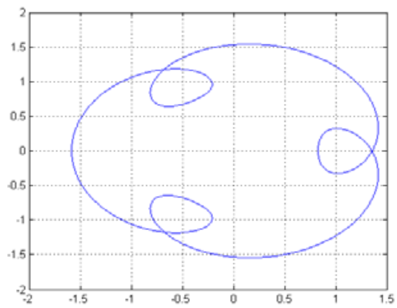
increases and decreases as C increase for two loops periodic orbit for both systems Sun – Mars and Sun – Earth as shown in Fig. 11 and 12. The variation in eccentricity due to oblateness is exactly opposite in nature for two loops orbit in comparison to single loop orbit for Sun – Mars and Sun – Earth system.



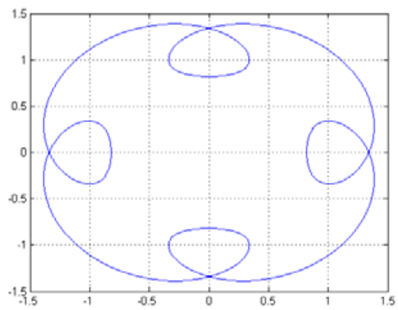
(c) A_2 = 0.005, two loops periodic orbit. (d) A_2 = 0.01, two loops periodic orbit.



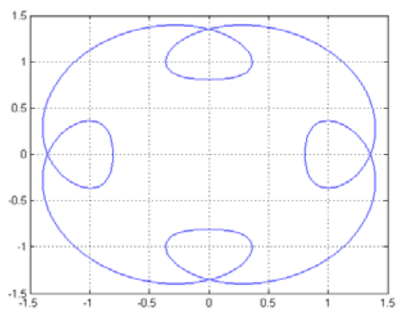
(e) $A_2 = 0.005$, three loops periodic orbit.



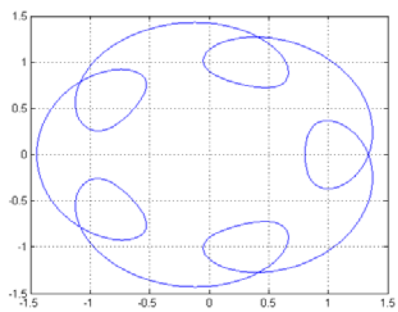
(f) $A_2 = 0.01$, three loops periodic orbit.



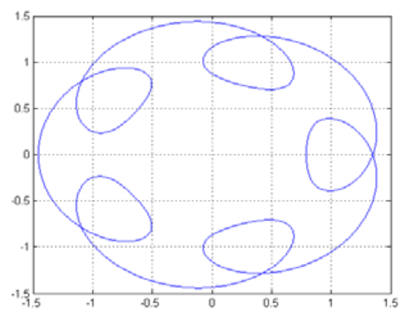
(g) $A_2 = 0.005$, four loops periodic orbit.



(h) $A_2 = 0.01$, four loops periodic orbit.

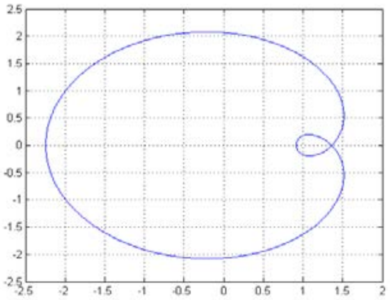


(i) $A_2 = 0.005$, five loops periodic orbit.

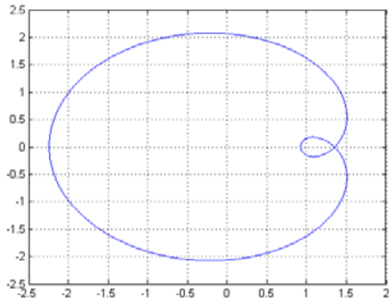


(j) $A_2 = 0.01$, five loops periodic orbit.

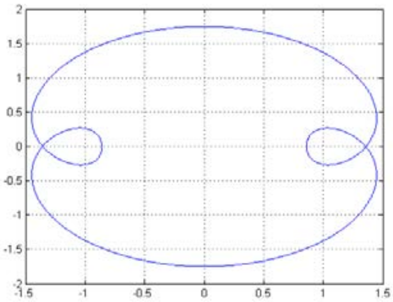
Fig. 3 Periodic orbits around both primaries for Sun - Mars system for C = 2.93.



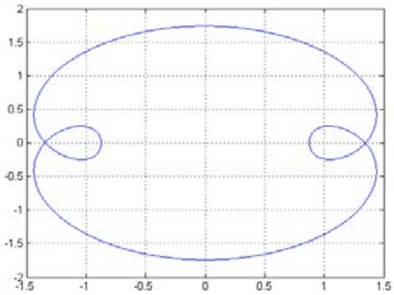
(a) A_2 = 0.005, single loop periodic orbit.



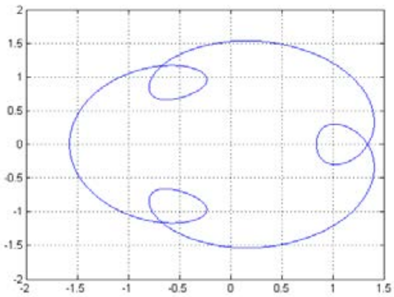
(b) A_2 = 0.01, single loop periodic orbit.



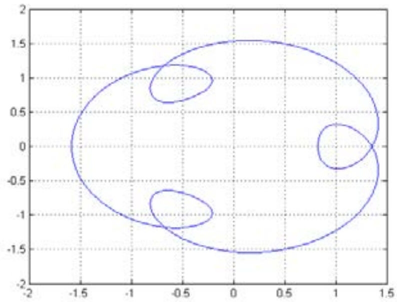
(c) A_2 = 0.005, two loops periodic orbit.



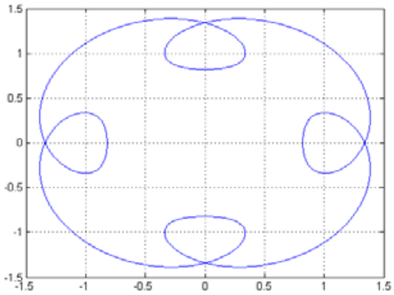
(d) A_2 = 0.01, two loops periodic orbit.



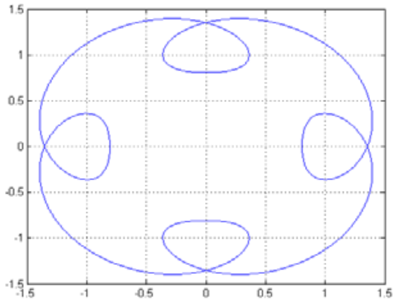
(e) A_2 = 0.005, three loops periodic orbit.



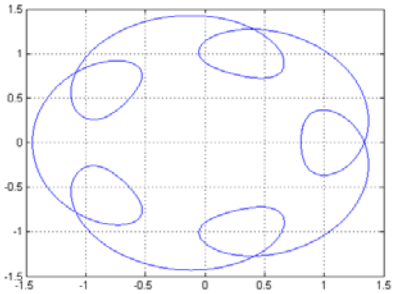
(h) A_2 = 0.01, three loops periodic orbit.



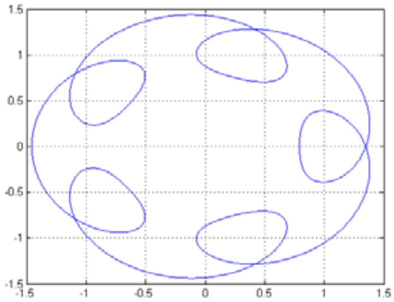
(g) $A_2 = 0.005$, four loops periodic orbit.



(h) $A_2 = 0.01$, four loops periodic orbit.



(i) $A_2 = 0.005$, five loops periodic orbit.



(j) $A_2 = 0.01$, five loops periodic orbit.

Fig. 4 Periodic orbits around both primaries for Sun - Earth system for C = 2.93.

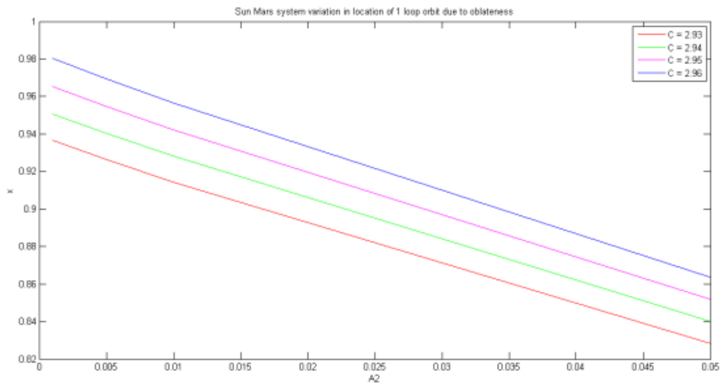


Fig. 5 Variation in location of single loop periodic orbit around Sun - Mars system due to oblateness.

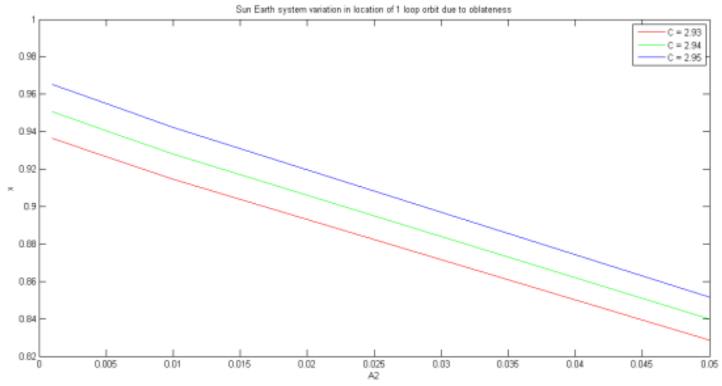


Fig. 6 Variation in location of one loop periodic orbit around Sun- Earth due to oblateness.

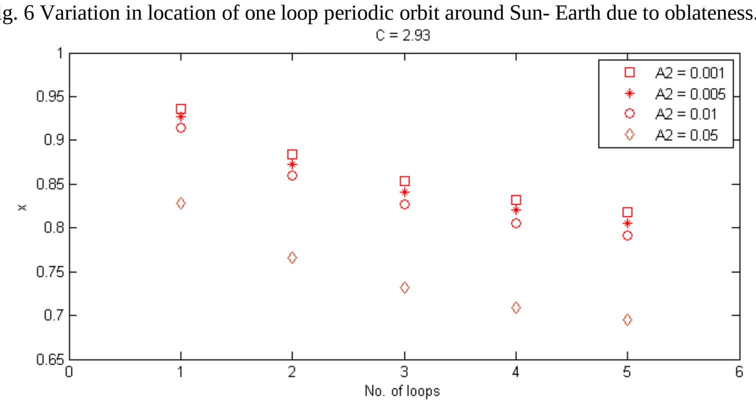


Fig. 7 Variation in location of periodic orbit of second primary around Sun and Mars for C = 2.93 due to number of loops for different oblateness A_2.

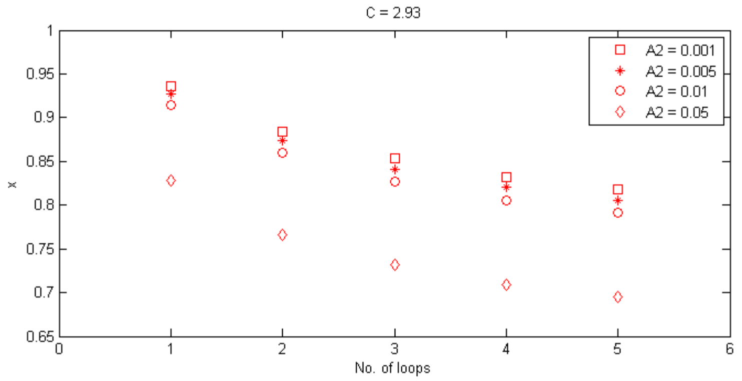


Fig. 8 Variation in location of periodic orbit of second primary around Sun – Earth system for C = 2.93 due to number of loops for different oblateness A_2.

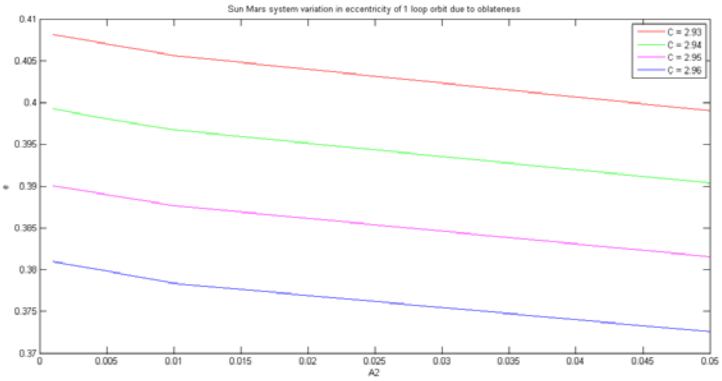


Fig. 9 Variation in eccentricity of single loop periodic orbit around Sun – Mars system due to oblateness.

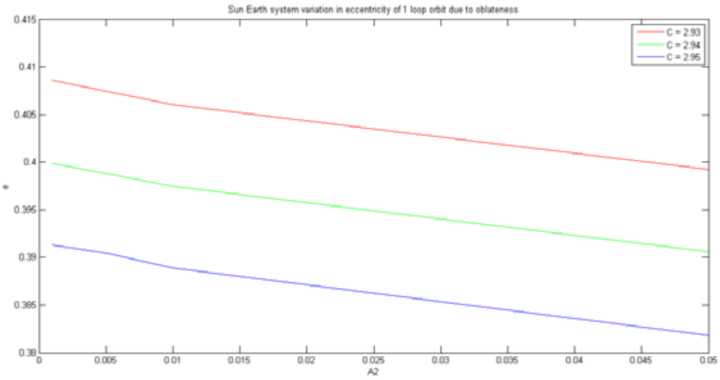


Fig. 10 Variation in eccentricity of single loop periodic orbit around Sun and Earth due to oblateness.

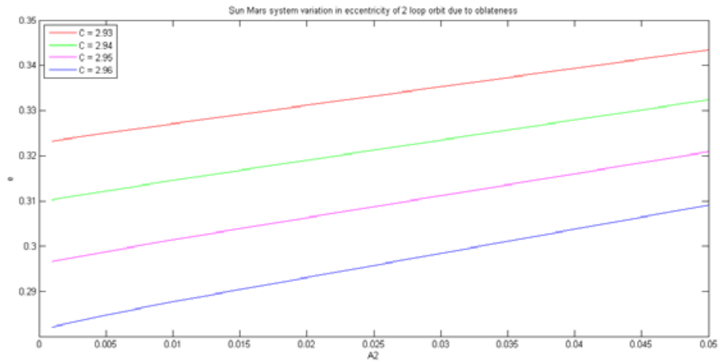


Fig. 11 Variation in eccentricity of two loops periodic orbit around Sun – Mars system due to oblateness.

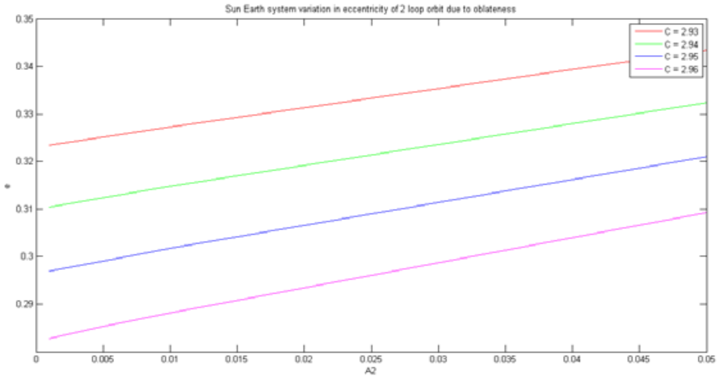


Fig. 12 Variation in eccentricity of two loops periodic orbit around Sun – Earth system due to oblateness.

Table 1: Analysis of periodic orbit for different pairs of A_2 and C for Sun - Mars system.

Number of loops	C	$A_2 = 0.001$		$A_2 = 0.005$		$A_2 = 0.01$		$A_2 = 0.05$	
		Location (x)	Period (T)	Location (x)	Period (T)	Location (x)	Period (T)	Location (x)	Period (T)
1	2.96	0.98028	13	0.96945	13	0.9563	13	0.86369	13
	2.95	0.96525	13	0.95474	13	0.94198	13	0.85165	13
	2.94	0.95068	13	0.94045	13	0.928	13	0.83988	13
	2.93	0.93647	13	0.92653	13	0.9144	13	0.82835	13
2	2.96	0.93819	19	0.92515	19	0.90953	19	0.80504	19
	2.95	0.91938	19	0.90699	19	0.8921	19	0.79159	19
	2.94	0.90151	19	0.88969	19	0.87545	19	0.77857	19
	2.93	0.88448	19	0.87315	19	0.85949	19	0.76593	19
3	2.96	0.91166	26	0.89746	26	0.88061	25	0.7715	25
	2.95	0.89087	26	0.87757	26	0.86174	25	0.75762	25
	2.94	0.87144	26	0.8589	26	0.84392	25	0.74425	25
	2.93	0.85315	26	0.84127	26	0.82703	25	0.73133	25
4	2.96	0.89323	32	0.87841	32	0.86095	32	0.750015	31
	2.95	0.87139	32	0.85766	32	0.84138	32	0.73601	31
	2.94	0.8512	32	0.83835	32	0.82305	32	0.72254	31
	2.93	0.83233	32	0.82022	32	0.80575	32	0.70957	31
5	2.96	0.87973	38	0.86457	38	0.8468	38	0.735165	37
	2.95	0.85732	38	0.84337	38	0.82688	38	0.72113	37
	2.94	0.83673	38	0.82373	38	0.8083	38	0.70767	37
	2.93	0.81758	38	0.80538	38	0.79083	38	0.69471	37

Table 2: Analysis of periodic orbit for different pairs of A_2 and C for Sun - Earth system.

Number of loops	C	$A_2 = 0.001$		$A_2 = 0.005$		$A_2 = 0.01$		$A_2 = 0.05$	
		Location (x)	Period (T)	Location (x)	Period (T)	Location (x)	Period (T)	Location (x)	Period (T)
1	2.96	-	-	-	-	0.9565	13	0.86375	13
	2.95	0.9653	13	0.95485	13	0.9421	13	0.8517	13
	2.94	0.9507	13	0.9405	13	0.9281	13	0.83994	13
	2.93	0.9365	13	0.92657	13	0.91446	13	0.8284	13
2	2.96	0.93825	19	0.92523	19	0.90961	19	0.805085	19
	2.95	0.91942	19	0.90704	19	0.89215	19	0.791635	19
	2.94	0.90154	19	0.88973	19	0.8755	19	0.7786	19
	2.93	0.8845	19	0.87319	19	0.85953	19	0.76596	19
3	2.96	0.91172	26	0.89753	26	0.88069	25	0.77155	25
	2.95	0.89091	26	0.87763	26	0.86179	25	0.75766	25
	2.94	0.87149	26	0.85895	26	0.84397	25	0.74428	25
	2.93	0.853183	26	0.84131	26	0.82706	25	0.73136	25
4	2.96	0.89329	32	0.87847	32	0.86103	32	0.750055	31
	2.95	0.87145	32	0.85771	32	0.84143	32	0.73605	31
	2.94	0.85124	32	0.83839	32	0.82309	32	0.72258	31
	2.93	0.83237	32	0.82026	32	0.80579	32	0.7096	31
5	2.96	0.8798	38	0.864634	38	0.84687	38	0.7352	37
	2.95	0.857375	38	0.84342	38	0.82693	38	0.72117	37
	2.94	0.83677	38	0.82378	38	0.80834	38	0.7077	37
	2.93	0.81762	38	0.8054	38	0.79087	38	0.69474	37

IV. CONCLUSION

In this paper we have studied the effect of oblateness on the position closed periodic orbit with loops varying from 1 to 5 for Sun - Mars and Sun - Earth systems, respectively. A noticeable change observed between two systems is that for C = 2.96, A_2 = 0.001 and 0.005, single loop periodic orbit does not exist for Sun – Earth system. All closed orbits with loops varying from 1 to 5, the second primary (here Mars or Earth) is always inside one of the loops and the trajectory orbits both primaries. For both the systems, the period of orbit increases with increase in the number of loops. Also, for both the systems, for the given C, as oblateness increases, location of periodic orbit moves towards Sun. Whereas, for the given oblateness as C increases, location of the periodic orbit moves away from the Sun. Further the eccentricity for both Sun – Mars and Sun – Earth systems decrease with increase in oblateness for single loop orbits, whereas, for orbits with loops two, the eccentricity increases with increase in oblateness. The variation in eccentricity due to oblateness is exactly opposite in nature for two loops orbit in comparison to single loop orbit for Sun – Mars and Sun – Earth system.

Thus, the present analysis of the two systems – Sun – Mars and Sun – Earth systems – using PSS technique reveals that A_2 and C has non negligible effect on the position of the obit. Since the second primary body is always inside one of the loops and the secondary body orbits both the primaries, these orbits can be used for designing low – energy space mission in solar system.

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