Descending auction model using mari considering the consumer market

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Abstract

Japanese flower markets employ a descending auction with special rule called "mari". In the descending auction, seller offers an initial price which is enough high. The auction decrease the price when a buyer stops. Then the auction decides the buyer as a winner and allocates the item to the buyer. The mari is a transaction spped up rule, which the buyers without the winner choose to join a coalition for buying the items as the winner's price. Our study extend this model to widely market including consumer transaction. We evaluate our proposed model by simulations. The result shows that our model reduce the number of auctions compered with the previous model.

1 Introduction

In recent years, with the spread of the Internet, it has become possible to conduct auctions easily, and economic transactions by auctions have become familiar. There are many types of auctions depending on the number of goods sold and purchased, and the method of price determination. For example, in the case of one kind of single goods, there are auctions where the buyer bids many times and raises the price gradually, and conversely, the auction which the price is lowered gradually and the buyer who made the first bid buys. The Japanese flower market uses a special auction method. It uses a slightly modified version of the standard descending auction. The auction is currently conducted using a computer, and a large display board in front of the auction hall displays information such as the number and price of goods exhibited at the auction. When the auction starts, the offer price is gradually lowered, and the buyer presses a button when the price he wants to buy is reached. The first buyer to press a button can buy goods at the stopped price [1]. In the Japanese flower market, there is a rule "mari" which allows other buyers to buy goods at the same price as the first winner if the buyer who won the auction purchases the goods and the goods still remain[2].

When buying goods by mari, the payment is the same price as the first winner of the down auction. Since the buyers participating in mari are the buyers who can not win in the auction, the payment amount is larger than their own bids. When using mari, the buyer's gain may be negative, such as if the bid is higher than her valuation or if the initial winner pays too much. On the other hand, such a market is a part of the economic transaction including procurement

process and customer transaction process, and the profit of the buyer is represented by the sum of the profit from the auction and the profit from consumer market. Therefore, we extend the standard descending auction model with mari to a model that includes procurement process and customer transaction process, and examines the efficiency of trading speed and the allocation efficiency of goods by mari participation.

We describe the composition of this paper. In the second chapter, we explain desending auction and mari. In the third chapter, we extend the model of the existing research and propose the model including the consumer market. In chapter 4, to analyze the model, we show some numerical simulation result.

2   Descending auction and "mari"

An auction is one of the economical environments which decides a price of good and an allocation to buyers. We consider a single item multi unit auction, which a seller wish to sell multi unit of the single object to buyers. An auction mechanism decides both an allocation of items to buyers and a payment of each buyer. Our study focuses on a descending auction mechanism. The procedure of a standard descending auction is as follows.

step1 The seller provides the multi unit single item to the auction with initial price. The item price keeps falling down from the initial price during the auction.

step2 If a first buyer bids an offer to stop the price, the seller fixes the item price and allocates the item to the first buyer. In this time, the first buyer is called the first winner.

In this auction mechanism, only one winner can be decided at each auction, and it is necessary to repeat the auction until there are no items or no buyers. To solve this issue, Japanese flower markets employ a special rule called "mari" where buyers without the winner in the previous auction can apply for purchasing the remain items at the same price as in the previous auction. When there are a lot of number of buyers participating in mari, i.e. the number of remain items is less than the number of mari participators, the mari rule decides the winners at uniform random.   The procedure of a standard descending auction with mari [3] is:

step1 The seller provides the multi unit single item to the auction with initial price. The item price keeps falling down from the initial price during the auction.

step2 If a first buyer bids an offer to stop the price, the seller fixes the item price and allocates the item to the first buyer. In this time, the first buyer is called the first winner.

step3 After the decision of first winner, the seller asks to other buyers whether you purchase the item at the same price of first winner or not.

step4 If the number of items grater than or equal to the number of mari participators, all participators get the item with the same price of the first winner. Otherwise the seller select the winner at random.

step5 If goods remain, a standard descending auction is continued.

If the number of mari participators is few, the seller needs to repeat the descending auction until the goods remain. To repeat the descending auction is not efficient in context of time. On the other hand, it is known that mari rule decreasing the allocation efficiency [3]. These two efficiency relationship is a tradeoff. In this study, we focus on an efficiency of trading time.

When the buyer participates to mari, the buyer consider an own utility. If the buyer purchase the item with the first winner's price, she may get a negative utility in this auction, i.e. the price will exceed her valuation. The rational buyers will participate to mari when her utility will nonnegative. However, in the flower market, the procurement process is only part of economy.

  We should consider the transaction between the buyer and customers. The buyer's utility is a sum of a utility from the auction and a utility of transaction to customers. Considering the total utility in the economy increases the incentive to participate to mari.

3   Descending auction model using mari considering the consumer market

We introduce a procurement economy including the auction and transaction to customers. The auction consists of a seller who wishes to sell k homogeneous goods and n single minded buyers. Let $N = \{1, 2, \ldots, n\}$ be a set of buyers. A buyer $i \in N$ has a valuation $v_i \in \mathbb{R}_+$ for a good. Let the buyer i's bidding strategy be $\beta_i \in \mathbb{R}_+$ which represents a distance between a bid value and the valuation v_i. For example, if β_i is greater than 1, the buyer tells an overbid, and if β_i less than 1, the buyer tells an underbid. Thus buyer i's bid value is $\beta_i v_i$. Let $r \in \mathbb{R}_+$ be the reserved price, which is the price that the seller will not sell if it is less than this price. Let $\boldsymbol{x} \in \{0, 1\}^n$ be an allocation vector, if $x_i = 1$, the buyer i got a good. Let $p_i \in \mathbb{R}_+$ be a price where the buyer i pays at the auction.Particularly, let $p \in \mathbb{R}_+$ be a price where the first descending auction winner pays.

We consider a following transaction model between the buyer and customers. The buyer i has a sales price $s_i \in \mathbb{R}_+$ to the customers. Let the buyer i's sales strategy be $\gamma_i(\beta_i, v_i, p_i) \in \mathbb{R}_+$. The sales strategy represents how far from the sales price in which the buyer sells. The final sales price is $\gamma_i(\beta_i, v_i, p_i)s_i$. The probability of a sale a good of the buyer i is represented by $q_i(\beta_i, v_i, p_i, \gamma_i, s_i) \in [0, 1]$, and $\lambda_i \in [0, 1]$.By the definition, buyer i's sales strategy γ_i is influenced by own bid and payment at the auction side. Since we are widely considering the sourcing economy, we can discuss strategies to bid higher than the valuation to buy reliable good. It shows that even if a negative utility occurs in an auction, it can be transferred to the sale price of a good. From the definition, q_i depends on the selling price and the market value. Since we know the allocation and prices of all goods from the auction results, we can predict the number and price of goods in the consumer market. However, when a buyer think q_i, the auction is not finished and she does not know the bid of other buyers, so she predicts the market price only by the buyer's own bid and payment.

Descending auction using mari is defined as follows:

Input : For each buyer $i \in N$, number of goods k, bidding strategy β_i, valuation v_i, reserved price r, sales price s_i, and acceptable sales probability λ_i for the buyer i.

Output : A payment vector $\boldsymbol{p}$ and an allocation vector $\boldsymbol{x}$.

step1 In the descending auction, suppose the buyer c with the highest bid $\beta_c v_c$ is winner. $p \leftarrow \beta_c v_c$.

step2   (i) If $r > p$,
            End of auction.
       (ii) Else ,
            Take $p_c \leftarrow p, x_c \leftarrow 1, k \leftarrow k - 1$ and remove the buyer c from N, i.e. $N \leftarrow N \setminus \{c\}$.

step3 Select a set M of mari paticipants from N.

    (i) If $|M| > k$,
        Choose a mari winner subset $M' \subseteq M$ with $|M'| = k$.
        Take $p_i = p, x_i = 1 \, \forall i \in M'$.
        End of auction.

    (ii) Else,
        Take $p_i = 1, x_i = 1 \, \forall i \in M$, $k \leftarrow k - |M|$, and $N \leftarrow N \setminus (M \cup \{c\})$.

step4 Repeat the following while $k > 0$.

    (a) Each bidder $i \in N$ redetermines own bidding strategy β_i.

    (b) Buyer c' with the highest bid $\beta_{c'} v_{c'}$ won auction. $p \leftarrow \beta_{c'} v_{c'}$

      (i) If $r > p$,
        End of auction.

      (ii) Else,
        Take $p_{c'} \leftarrow p, x_{c'} \leftarrow 1, k \leftarrow k - 1$ and remove the buyer c' from N.

At the auction, the buyer determines the following four strategies:

1. bidding strategy β in step1,

2. mari participation strategy,

3. bidding strategy β in step4,

4. sales strategy γ for the consumer market.

An expected payoff U_i of the buyer i at the end of the auction can be expressed as follows.

$$U_i = (v_i - p_i) + q_i(\beta_i, v_i, p_i, \gamma_i, s_i)(\gamma_i(\beta_i, v_i, p_i)s_i - v_i).$$

The buyer's expected payoff is the sum of gains from the auction and the consumer market. In this model, it is considered that the good's valuation remains even though it can not be sold.

We describe how to decide to participate in mari. If the buyer i participates to mari, her payment p_i is fixed to p. Then the buyer i decides on a sales strategy that maximizes gains as $\mathrm{argmax}\{U_i\}$.
$$\gamma_i \in \mathbb{R}_+$$

The conditions of mari participation are the follows: (1) $U_i \geq 0$ and (2) $q_i(\beta_i, v_i, p, \gamma_i, s_i) \geq \lambda_i$. The buyer joins mari when the expected payoff becomes nonnegative and the probability of selling the goods at that time is higher than the acceptable sales probability.

4 Analysis of the model

4.1 Definitions for analyzing the model

Definition 1. *A total surplus of the auction is the sum of the buyers' and seller's payoffs, which is*

$$\sum_{i \in N}(v_i - p_i)x_i + \sum_{i \in N} p_i x_i.$$

The following two criteria measure a trade speed and allocation efficiency of goods.Kitahara and Ogawa [3] used this to analyze the descending auction model with mari.

Definition 2. *Efficiency loss rate (LR) is the ratio of the allocation efficiency of the goods lost by mari and the maximum surplus.*

$$LR = \frac{maximum\ surplus\ \text{-}\ surplus\ when\ allocated\ using\ mari}{maximum\ surplus}$$

If $LR = 0$, it indicates the most efficient allocation situation when allocated using mari and if $LR = 1$, it indicates a situation where all participants did not get anything when allocated using mari.

LR is used to confirm the allocation efficiency of goods. If it is small, it indicates that the auction achieves efficiently allocation. The most efficient allocation is that k goods are allocated to the buyer by descending order of valuation. If thee auction employs mari, the auction may not be able to allocate goods in order from high valued buyers.

Definition 3. *Round reduction rate (RR) indicates the ratio between the number of goods allocated by mari and the number of goods remaining at start step3.*

$$RR = \frac{Number\ of\ goods\ sold\ by\ mari}{Number\ of\ goods\ remaining\ at\ start\ step3}$$

If $RR = 0$, then there is no mari participant and all goods are sold in standard descending auctions, if $RR = 1$, it indicatethe situation where all goods are sold in mari.

RR is used to confirm a trade speed.If it is large,it indicates that trade speed is fast.If there is no mari participant, seller repeats descending auction k times. The number of descending auctions can be reduced by selling goods in mari.

4.2   Numerical simulation

We create a simulator for the proposed model and Kitahata and Ogawa model [3]. The difference between the two models is the buyer's strategy. The proposed model calculates $\max_\gamma U_i$ using successive least squares method when deciding whether to join mari. Kitahara and Ogawa model calculates expected payoff and use them to determine mari participation strategies.

First, v_i is uniformly selected from a closed interval $[0,1]$, s_i is uniformly selected from a closed interval $[v_i,1]$, and λ_i is uniformly selected from a closed interval $[0,1]$. When considering whether to participate in mari, the values on which q_i depends are fixed except for γ. Then we assume the probability of selling goods as follows.

$$q_i = \begin{cases} -a\gamma + b \\ 0(\text{if} -a\gamma + b < 0), \end{cases}$$

$$a > 0, b \in [0,1], \gamma > 0.$$

The parameter a indicates how much the probability of selling a good changes when the buyer i changes the price of the good, and the parameter b indicates the probability of selling when the good's price is 0. This is an assumption that simply raising the price reduces the probability of selling a good. We find γ analytically. The mari participation condition holds:

$$\max_\gamma \{U_i(\gamma) = (v_i - p_i) + (-a\gamma + b)(\gamma s_i - v_i)\}$$
$$\Leftrightarrow \min_\gamma \{-U_i(\gamma) = -(v_i - p_i) - (-a\gamma + b)(\gamma s_i - v_i)\}.$$

Transform $-U_i(\gamma) = as_i\gamma^2 - (av_i + bs_i)\gamma + bv_i - (v_i - p_i)$. Differentiate the objective function and consider the minimum solution.

$$(-U_i(\gamma))' = 2as_i\gamma - (av_i + bs_i) = 0.$$

So the minimum γ is

$$\gamma = \frac{av_i + bs_i}{2as_i} = \frac{b}{2a} + \frac{v_i}{2s_i}.$$

Because of $0 \le v_i \le 1, 0 \le s_i \le 1$

$$\frac{b}{2a} \le \gamma \le \frac{b}{2a} + \frac{1}{2}.$$

Assign γ to the objective function.

$$as_i \left(\frac{av_i + bs_i}{2as_i}\right)^2 - (av_i + bs_i)\left(\frac{av_i + bs_i}{2as_i}\right) + bv_i - (v_i - p_i)$$

$$= \frac{(av_i + bs_i)^2}{4as_i} - \frac{(av_i + bs_i)^2}{2as_i} + bv_i - (v_i - p_i)$$

$$= -\frac{(av_i - bs_i)^2}{4as_i} - (v_i - p_i).$$

Therefore, the maximum U_i under this assumption is

$$U_i = (v_i - p_i) + \frac{(av_i - bs_i)^2}{4as_i}.$$

This indicates that the buyer can lose up to $\frac{(av_i - bs_i)^2}{4as_i}$.

For example, if $a = \frac{1}{2}, b = 1$, then $q_i = -\frac{1}{2}\gamma + 1 (\gamma \in [0, 2])$. This is the case where the buyer i thinks that if $\gamma = 0$ the goods will surely sell and $\gamma = 2$ they will never sell. The sales strategy of the buyer i is $1 \le \gamma \le 1.5$. At this time, $0.25 \le q_i \le 0.5$, so it is known that buyers who participated in mari under this assumption have a low probability of selling goods.

We assume four other probabilities of selling goods. First,

$$q_i = \begin{cases} -\gamma^2 + 2\gamma \\ 0 (\text{if} -\gamma^2 + 2\gamma < 0). \end{cases}$$

This is the case where the buyer i thinks that if $\gamma = 1$, the goods will surely sell, and if $\gamma = 0, 2$, it will never sell. If goods are significantly cheap, consumers will not buy with anxiety. This is an assumption that considers not only the price but also the buyer's anxiety. Second,

$$q_i = \begin{cases} -0.8\gamma^2 + 1.6\gamma \\ 0 (\text{if} -0.8\gamma^2 + 1.6\gamma < 0). \end{cases}$$

The assumption is that the probability of selling goods is reduced in the same situation as above. Third,

$$q_i = \begin{cases} (\frac{1}{2})^\gamma \\ 0 (\text{if} (\frac{1}{2})^\gamma < 0). \end{cases}$$

This is the assumption that if you raise the price, the probability of selling the goods falls gently. Last,

$$q_i = \begin{cases} \frac{0.8}{1+\mathrm{e}^{50(\gamma-1)}} \\ 0 (\text{if} \frac{0.8}{1+\mathrm{e}^{50(\gamma-1)}} < 0). \end{cases}$$

This is known as a sigmoid function. This is the assumption that the probability of selling goods suddenly drops below a certain price. It is assumed that similar goods are displayed in order of price on the net.

Using these five sales probabilities, we simulate a descending auction with mari. The experimental environment is as follows. Table 1 shows an environment of our simulator coded by Python 3.6.3.

Table 1: The experimental environment

CPU	Memory	OS
2.6 HHz Intel Core i5	8 GB 1600 MHz DDDR3	macOS Sierra 10.12.6

First, we prepare 10 instances for the number of goods k and the number of buyers n. After that, set the other inputs as shown in Table 2 below. The bidding strategy is the same as the existing research.

Table 2: Input to the experiment

	Existing model[3]	Proposed model 1	Proposed model2
v_i	$U[0,1]$	$U[0,1]$	$U[0,1]$
β_i	$\frac{n-k}{n}$	$\frac{n-k}{n}$	$\frac{n-k}{n}$
r	0	0	0
q_i		$q = -\frac{\gamma}{2} + 1$	$q = -\frac{\gamma}{2} + 1$
s_i		$U[v_i,1]$	$U[v_i,1]$
λ_i		$U[0,1]$	$\lambda = 0$

	Proposed model 3	Proposed model4	Proposed model 5
v_i	$U[0,1]$	$U[0,1]$	$U[0,1]$
β_i	$\frac{n-k}{n}$	$\frac{n-k}{n}$	$\frac{n-k}{n}$
r	0	0	0
q_i	$q = -\gamma^2 + 2\gamma$	$q = -\gamma^2 + 2\gamma$	$q = -0.8\gamma^2 + 1.6\gamma$
s_i	$U[v_i,1]$	$U[v_i,1]$	$U[v_i,1]$
λ_i	$U[0,1]$	$\lambda = 0$	$U[0,1]$

I input the values in Table 2 into the existing research model and the proposed model and compare them. We also compare cases where we do not consider acceptable sales probability among the proposed models. The number of LR, RR, and mari participants (MP for short) are calculated for each model for the number of goods of 10 instances and the number of buyers. That is Tables 3 and 4. The values in Tables 3 and 4 are each 100 instances created and the average value written with three significant figures. In order to eliminate the bias, valuation v_i, sales price s_i, and acceptable sales probability λ_i are made the same value in the pattern of the number of same goods and the number of buyers.

	Proposed model 6	Proposed model 7	Proposed model 8
v_i	$U[0,1]$	$U[0,1]$	$U[0,1]$
β_i	$\frac{n-k}{n}$	$\frac{n-k}{n}$	$\frac{n-k}{n}$
r	0	0	0
q_i	$q = -0.8\gamma^2 + 1.6\gamma$	$q = (\frac{1}{2})^x$	$q = (\frac{1}{2})^x$
s_i	$U[v_i,1]$	$U[v_i,1]$	$U[v_i,1]$
λ_i	$\lambda = 0$	$U[0,1]$	$\lambda = 0$

	Proposed model 9	Proposed model 10
v_i	$U[0,1]$	$U[0,1]$
β_i	$\frac{n-k}{n}$	$\frac{n-k}{n}$
r	0	0
q_i	$q = \frac{0.8}{1+e^{50(\gamma-1)}}$	$q = \frac{0.8}{1+e^{50(\gamma-1)}}$
s_i	$U[v_i,1]$	$U[v_i,1]$
λ_i	$U[0,1]$	$\lambda = 0$

4.3   Discussion

From the experiments, our model has faster transaction speed and less allocation efficiency compared with previous model. The reason for the increase in mari participants is considered to be that the conditions for mari participation were relaxed by the incentive to purchase goods at a price that exceeds the valuation by considering the consumer market. It is clearly understand that the decreasing allocation efficiency is caused by the increasing the number of mari participants. If the number of mari participants is greater than the number of goods, the auction decides the winner at random. Thus the probability that a buyer with a high valuation will win is reduced. This result is as same as previous research.

   The number of mari participants in proposed model 1 is smaller than in previous research model, and the number of mari participants in proposed model 2 is larger than previous model. Since the difference between the proposed model 1 and 2 is whether or not to consider the sales acceptance probability, the proposed model 2 participates in mari if the expected gain is not negative. Since the data of the instance uses the same value, in the proposed model 1, it can be seen that there are many buyers whose probability of selling a good has not reached the sales acceptance probability even if the expected gain is not negative. It is as same as proposed model 7. The proposed models 3, 4, 5, 6, 9, and 10 have more mari participants than existing research models, even considering the sales tolerance. It can be seen that the assumption of q_i, the probability of selling goods, is greatly influenced.

5   Conclusion

In this research, we proposed descending auction model using mari considering the consumer market, and confirm the difference between the existing research model and the proposed model by simulation. As a result, under the assumption, it was found how much the buyer could pay at the auction site than the valuation of the goods. The simulation also showed that almost all goods are sold in mari in the proposed model. we were able to confirm the properties of mari in a model closer to reality than existing research.

   As a future works, it is possible to make q different value depending on the buyer and extend it to a model to consider how the buyer participates in the auction. Also, it is considered to

Table 3: Simulation result

		k=2,n=3	k=2,n=4	k=3,n=4	k=2,n=5	k=3,n=5
	LR	0.0186	0.0101	0.0139	0.00994	0.00956
Existing model [3]	RR	0.700	0.670	0.765	0.760	0.730
	MP	0.98	1.03	1.74	1.29	1.76
	LR	0.499	0.0373	0.0534	0.0540	0.0830
Prop. model 1	RR	0.580	0.570	0.495	0.530	0.555
	MP	0.710	0.780	1.01	0.880	1.19
	LR	0.0665	0.0794	0.0905	0.0925	0.112
Prop. model 2	RR	0.99	0.99	0.99	0.98	0.99
	MP	1.79	2.26	2.82	2.70	3.41
	LR	0.103	0.124	0.105	0.141	0.152
Prop. model 3	RR	0.960	0.980	0.930	1.00	0.955
	MP	1.56	2.27	2.25	2.85	2.92
	LR	0.0832	0.124	0.105	0.137	0.137
Prop. model 4	RR	1.00	1.00	1.00	1.00	1.00
	MP	1.91	2.82	2.94	3.56	3.77
	LR	0.0487	0.107	0.0904	0.134	0.140
Prop. model 5	RR	0.87	0.90	0.82	0.93	0.87
	MP	1.21	1.69	1.85	2.16	2.33
	LR	0.0980	0.0994	0.0864	0.144	0.140
Prop. model 6	RR	1.00	1.00	1.00	1.00	1.00
	MP	1.88	2.70	2.93	3.36	3.77
	LR	0.0452	0.0400	0.0422	0.0553	0.0613
Prop. model 7	RR	0.500	0.460	0.380	0.480	0.395
	MP	0.53	0.56	0.77	0.64	0.85
	LR	0.0749	0.0898	0.107	0.0967	0.115
Prop. model 8	RR	1.00	0.990	0.995	0.990	0.995
	MP	1.86	2.40	2.87	2.91	3.54
	LR	0.0682	0.0970	0.103	0.130	0.135
Prop. model 9	RR	0.920	0.920	0.865	0.890	0.870
	MP	1.39	1.72	2.01	2.08	2.39
	LR	0.0813	0.111	0.105	0.0993	0.108
Prop. model 10	RR	1.00	1.00	0.995	1.00	0.990
	MP	1.86	2.45	2.89	2.97	3.54

change the bidding strategy β not to depend only on the number of goods k and the number of buyers n, but to change the bid considering the consumer market.

Acknowledgement

Table 4: Simulation result

		k=4,n=5	k=4,n=20	k=10,n=20	k=18,n=20	k=20,n=30
Existing model[3]	LR	0.0142	0.00660	0.00873	0	0.00508
	RR	0.787	0.786	0.820	0.546	0.862
	MP	2.58	3.05	7.78	9.28	16.71
Prop. model 1	LR	0.0547	0.0426	0.0695	0.0111	0.0528
	RR	0.480	0.623	0.486	0.380	0.468
	MP	1.45	2.26	4.37	6.46	8.89
Prop. model 2	LR	0.0935	0.0720	0.144	0.0730	0.160
	RR	1.00	1.00	0.997	1.00	1.00
	MP	3.9	6.99	13.9	18.8	25.9
Prop. model 3	LR	0.104	0.275	0.272	0.0607	0.235
	RR	0.930	1.00	1.00	0.851	0.994
	MP	3.16	11.4	13.3	14.5	21.9
Prop. model 4	LR	0.0942	0.254	0.251	0.0819	0.210
	RR	1.00	1.00	1.00	1.00	1.00
	MP	3.97	14.6	17.3	19.0	28.0
Prop. model 5	LR	0.0824	0.241	0.242	0.0307	0.179
	RR	0.790	1.00	0.960	0.674	0.886
	MP	2.49	8.24	10.2	11.5	17.2
Prop. model 6	LR	0.101	0.229	0.233	0.0848	0.208
	RR	1.00	1.00	1.00	1.00	1.00
	MP	3.96	13.1	16.8	19.0	27.6
Prop. model 7	LR	0.0371	0.0655	0.0559	0.00697	0.0381
	RR	0.327	0.383	0.590	0.274	0.333
	MP	0.98	2.07	3.45	4.66	6.33
Prop. model 8	LR	0.0806	0.112	0.182	0.0771	0.177
	RR	1.00	1.00	0.999	1.00	1.00
	MP	3.92	9.12	15.0	18.9	26.5
Prop. model 9	LR	0.0851	0.106	0.221	0.0397	0.196
	RR	0.830	0.927	0.944	0.729	0.904
	MP	2.65	4.20	9.69	12.4	17.6
Prop. model 10	LR	0.0843	0.0867	0.192	0.0800	0.186
	RR	1.00	0.997	1.00	1.00	1.00
	MP	3.91	6.65	14.9	18.9	26.6

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